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Interferometric Method for Determination of X–ray Train Length

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Abstract. It is elaborated, made and tested a special interferometer–train metering device. A method for interferometric determination of X–Ray train length is offered. It is proved that the interference pattern disappears, when the path difference between imposing waves is more than the monochromatic X–Ray train length. The limit of disappearance of interference pattern depending on the value of path difference is determined. The X–Ray train length is determined which is close to the meaning determined theoretically.

Keywords: interference, intensity, wave train, train length, duration of coherency, dynamic scattering

1. Introduction

It is known [1–4] that for calculation of scattered wave intensity and diffraction widths of spectral lines it is necessary first to find out the relation of X–ray train length to dimensions of perfection areas of single crystals. The wave train is defined as the vibration of the form described at some length by a simple sinusoidal curve (with constant or slightly varying amplitude), the amplitude of which is zero outside this length. For this reason, the determination of X–ray train length is important not only from the view point of interference theory, but also of X–ray structural analysis in general. It is especially important for X–ray interferometry and holography, as well as for verification of theoretical ideas about train [5].

In our opinion the interferometric method is fairly appropriate for measuring the train length of X–ray coherent radiation. This aim in view the radiation from X–ray source is split into two beams and these are made to intersect after passage of different paths and produce an interference pattern. Then the path difference of these two beams is increased till the disappearance of obtained interference pattern. It is evident that the interference pattern disappears from the moment when the path difference between these two beams exceeds the train length of monochromatic X–rays. So, by measuring the path difference at the moment of pattern disappearance one measures the train length, i.e., the duration of coherent radiation producing this interference pattern. This experiment is rather complicated and requires the employment of methods of X–ray interferometry.

It is known [6] that in optics the interference pattern disappears at sufficiently large path differences between the waves. This effect, however, was not observed at the interference of X–rays until X–ray interferometers appeared. X–ray interferometers appeared as a result of the discovery of anomalous passage (the Borman effect) and of following rapid development of the dynamical theory of X–ray interference [7–13]. Based on the experience of making interferometers of different design and applications [14–19], we have designed and made special interferometer and suggested new interferometric method for measurement of X–ray train length (duration of coherent radiation), which was the aim of this work.

When carrying out these experimental works, we kept in our mind that the interference pattern disappears as the path difference between the interfering waves is gradually increased and when the amplitudes of these waves vary considerably, i.e., the sum and difference of amplitudes of these waves insignificantly differ from one another (the visibility diminishes).

2. Problem statement and methodology justification

At first sight it may seem that the phase (path) difference between interfering waves may be easily increased by placing in the path of one of the waves a medium with refractive index different from unity. However, that may entail new difficulties. As the refractive index of X–rays for media (other than vacuum or air) are slightly different from unity, to obtain differences in optical path lengths in excess of train length it is necessary to introduce a medium of about 10 cm extent, owing to which first, the dimensions of interferometer would extremely increase and, second, due to the absorption in medium the wave would so attenuate that the interference pattern would disappear on account of the above reasoning. Detailed discussions of this problem have shown that for measuring X–ray train length it is necessary to produce a phase difference between interfering waves by providing a difference in their geometrical paths.

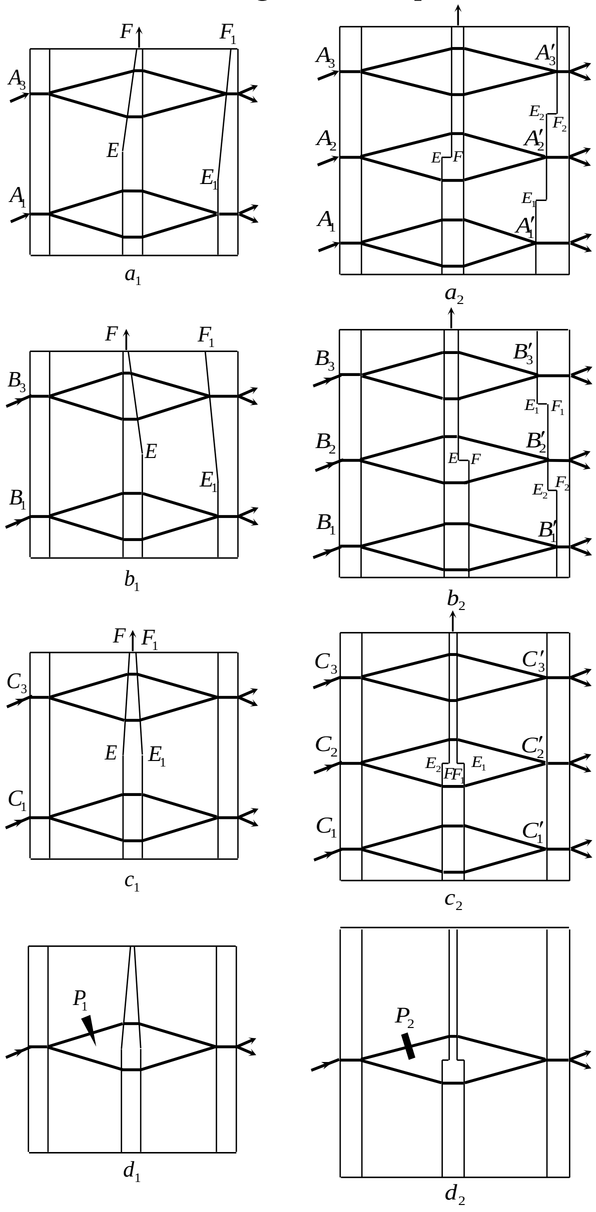

Fig. 1. Some variants of interferometers for obtaining phase changes of superimposed waves.

Thus, for solution of this task, we first consider some variants of three–block interferometers (Fig. 1), ensuring phase changes of superimposed waves and designed so, that the waves superimposed in these interferometers overlap exactly on the entrance surface of the third block, regardless of their exit points from the first block.

When the primary wave is incident on the first blocks of interferometers a$_2$ a_1, b_1 and c_1 (Fig. 1) in the points A_1 and A_3, B_1 and B_3, C_1 and C_3, the path differences in points A_1' and A_3', B_1' and B_3', C_1' and C_3' of the third blocks are zero. In the same interferometers when the primary wave is incident in the points A_2, B_2 and C_2, path differences arise in the A_2', B_2' and C_2' points of the third blocks. This is due to the fact that in case of falling on the points A_1 and A_3, B_1 and B_3, C_1 and C_3 the distances between blocks in this part of interferometer are equal, while at the incidence on the points A_2, B_2 and C_2, they are not equal.

In the interferometers a_1, b_1 and c_1 (Fig. 1) when the primary wave falls on the points A_1, B_1 and C_1 of the first blocks, the phase differences between the interfering waves at the entrances of the third blocks are equal to zero. In the same interferometers when the primary wave is incident in the points A_3, B_3 and C_3 of the first blocks, the phase differences between the interfering waves first preserve zero values, then gradually increase and finally acquire their maximum and constant value.

It should be noted that in the interferometer b$_1$ the surfaces EF and E_1F_1 slope symmetrically. In the interferometers a_2 and b_2 the steps satisfy the conditions $E_1F_1 = E_2F_2$ and $E_1F_1 + E_2F_2 = EF$, and in the interferometers c_2 the steps are equal $(EF = E_1F_1)$.

Obviously, that in the interferometers a_1, b_1 and c_1 (Fig.1) the paths passed in the second block by waves that interfere in the third block are different and, hence, these waves are differently attenuated and there arises a difference in their amplitudes. However, the contrast of interference patterns shall not be visibly affected as, first, the paths in the second block differ one from another by shares of millimeter, and, second, as the passage is anomalous (especially in thicker parts of the second block), the absorptions of interfering waves in the second block differ insignificantly. At last, if in separate cases the differences in amplitudes of differently absorbed waves that emerge from the second block and interfere in the third block are remarkable, one may reduce these differences to zero by placing additional absorbers in their paths (Fig. 1 d_1, d_2). In a_1, b_1 and c_1 interferometers the absorber is a wedge P_1 (Fig. 1 d_1), while in the a_2, b_2 and c_2 ones, the absorber is a plane–parallel plate P_2 (Fig. 1 d_2). These absorbers insignificantly change the phase of waves, but equalize the absorptions. If needed, the shifts of beams due to the presence of these absorbers may be also taken into account.

Thus, to measure the length of the X–ray train, it is advisable to use interferometers with wedge–shaped blocks, i.e., interferometers of the types a_1, b_1 и c_1 (Fig. 1). The advantage of the a_1, b_1 and c_1 interferometers over a_2, b_2 and c_2 ones is that in former interferometers the path difference between interfering waves is continuously changeable, owing to which it will be possible to detect the moment when the path difference exceeds the train length. At first sight the disadvantage of these interferometers consists in the fact that owing to the inclination of incidence surfaces the Bragg angles may change. But the inclination of surfaces is very small (1–2 angular degrees) and for this reason the values of reflection angles are preserved with high accuracy and the blocks retain the reflecting positions.

3. Theoretical analysis

Let us assume that the second block of the interferometer has a wedge shape form, or more precisely, its lower part is a parallelepiped and the upper part is a wedge (Fig. 2), moreover, the reflecting planes are perpendicular both to the large surfaces of the blocks and to the surface of the base.

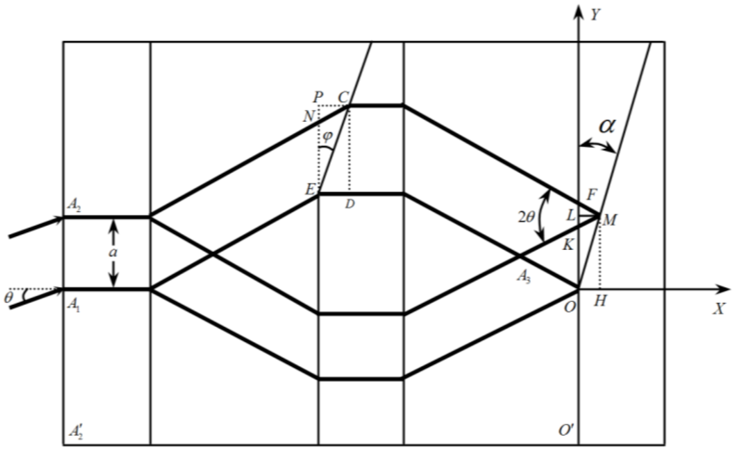

Fig. 2. Scheme of the X–ray interferometer with wedge–shaped blocks, and the path of rays in it.

Let the monochromatic X–ray beam falls at Bragg angle on the first block of interferometer. The beam incident on A_1 point of the first block passes two first blocks and is focused on O point, and the beam incident on A_2 point is focused on M point. When the incidence point on the entrance surface of the first block travels from A_1 to A_2, the focal points on the entrance surface of the third block travel from O to M, i.e., the OM straight line is the locus of foci, and, like the second block, the third block also has a parallelepiped (lower part) and wedge–shaped (upper part) shape.

Now let's find the coordinates of the point M. In Fig.2. we choose the point O as the origin of coordinates, the OY axis – parallel to the first block, and OX axis – perpendicular to that. Then the coordinates of the point M are defined by the following expressions:

$$X = \frac{a}{2} \cdot \frac{tg\varphi}{1 - tg\theta \cdot tg\varphi}, \tag{1}$$

$$Y = \frac{a}{2} \cdot \frac{2 - tg\theta \cdot tg\varphi}{1 - tg\theta \cdot tg\varphi}, \tag{2}$$

where a is the distance between A_1 and A_2 points on the entrance surface of the first block, θ is the Bragg angle, φ is the wedge angle of the second block.

Having in mind (1) and (2) from the OLM triangle for the wedge angle $\propto$ of the third block we obtain:

$$tg\alpha = \frac{X}{Y} = \frac{tg\varphi}{2 - tg\theta \cdot tg\varphi}. \tag{3}$$

It is seen from (3) that the angle α does not depend on the distance a between incidence points A_1 and A_2, i.e., OM line is the locus of focal points between O and M points. It is also easy to see that when the incidence point A_1 moves down to $A_2^{'}$, the focus O also moves down to O' point.

Thus, the shapes of the third and second blocks are the same except for the fact that, in general, the angles at their wedges are not equal and related as (3). As the ∝ and φ angles are small, formula (3) is transformed to the form:

$$\alpha = \frac{\varphi}{2-\varphi\cdot tg\theta} \tag{4}$$

whence it is seen that when $\theta < \frac{\pi}{2}$, angle φ exceeds ∝.

For calculation of the path difference that arises between the interfering waves in the interferometer, we use the scheme shown in Fig. 3. From the Fig. 2 seen, that when the incidence points are between A_1 and A_2', the path differences of interfering waves in points located between O and O' are zero.

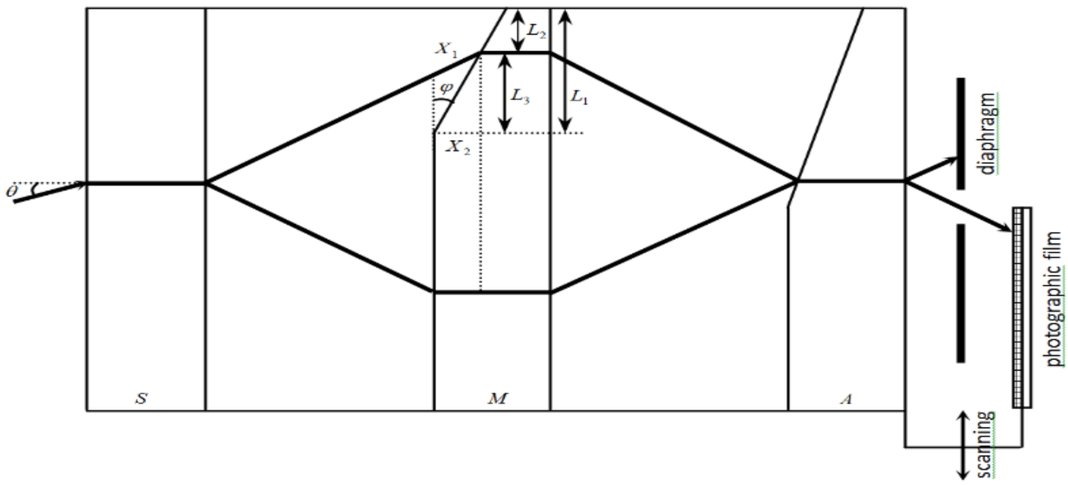

Fig. 3. Scheme for calculating the path difference between interfering waves.

When the incidence point is between A_1 and A_2, the path differences of interfering waves Δ in the points located between O and M differ from zero. These differences Δ arise only due to the wedge shape surface of the second block (Fig. 3). For Δ the following simple relation is obtained:

$$\Delta = L_3 tg\varphi \cdot (\sec\theta - 1), \tag{5}$$

where L_3 is the distance of incidence point on the surface of the second block from the base of the wedge.

As is seen from (5), in case of $\varphi=0$ the path differences are zero and increase with increasing of a distance L_3. When deriving the relation (5), the path differences are calculated as $\Delta = X_1 - X_2(1-\delta) = X_1 - X_2 + X_2\delta$, where δ is the unit decrement of refractive index (δ ~10^{-6}). Keeping in mind the smallness of the $X_2\delta$ this difference was replaced by expression $\Delta = X_1 - X_2$.

4. Experimental results and their discussion

For measurement of X–ray train length, a special interferometer was prepared from dislocation–free silicon single crystal (Fig. 3). In it $(1\bar{1}0)$ planes were perpendicular to the large surfaces and the base of interferometer. The interferometer geometry was as follows: the distance between the interferometer blocks was 12.5 mm, the blocks had 0.6 mm thickness, and the width and height were respectively 17.5 mm and 11.7 mm; L_1 = 4.84 mm, angles $\varphi = 2^{\circ}20'20''$, $\propto = 1^{\circ}10'50''$ in $(2\bar{2}0)$ reflection of CuK_{α_1} radiation.

In the experiment, an increase in the paths difference of interfering waves is achieved by a scanning. The diagram of interferometer scanning and the corresponding interference topogramm are presented in Figs. 3 and 4.

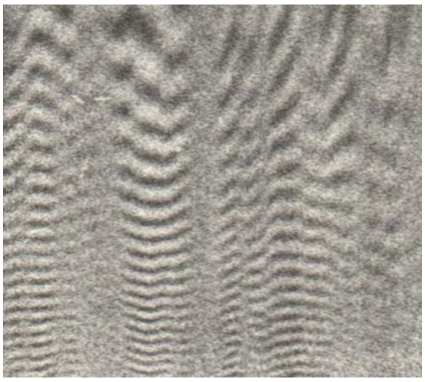

Fig. 4. Interference topogramm obtained by scanning.

The value of parameter of L_3 = 0.098 mm corresponding to a disappearance of the interference pattern was determined using the formula (5), and for the value of the train length during the CuK_{α_1} it is being obtained $l = 3.76 \cdot 10^{-5} cm$.

It is known [5] that $l = c\tau$ where c is the propagation speed of electromagnetic waves, τ is the duration of coherent radiation. As $\tau = 4.53\lambda^2 sec$ (see [5]), and $\lambda = 1.54 \cdot 10^{-8} cm,$ then theoretically for the train length we get $l = 3.23 \cdot 10^{-5} cm$.

5. Conclusion

Thus, as a result of the conducted experimental and theoretical studies, we can state the following:

1. The developed interferometer of a special type is indeed an interferometer – X–ray train length meter.
2. The proposed interferometric method can be used to measure the length of an X–ray train with high accuracy.
3. Using the scanning method, gradually increasing the path difference between the interfering waves, one can obtain an interference pattern, and, depending on the size of the path differences, determine the point of disappearance of this pattern, i.e., measure the X–ray train length.
4. Comparison of the experimental results obtained for the length of the X–ray train with the theoretical value allows us to confirm that the proposed method provides high accuracy, and reliable results.

Author contributions

Author H.R. Drmeyan raised the idea, carried out data processing and theoretical calculations; author S.A. Mkhitaryan developed the experimental activities and participated in the data processing.

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