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MACHINEBUILDING

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# THE INVESTIGATION OF PROCESS CONTACT DEFORMATION OF SINTERED PM CYLINDERS

The work concerns to the investigation of the elastic problem of surface contact deformation of two cylinders, from which one is made from Sintered Powder Materials (PM). It is considered that the substance of PM is homogeneous, isotropic and the pores are distributed uniformly. The solution of the problem is realized on the foundation of real mechanical properties of PM and the surface contact theory of H. Hertz. For strength problem solving of PM the conception of equivalent stress for porous material is used. The calculated data show that in case of taking into account a very important factor for PM hydrostatic pressure the place of dangerous point is changed: from inside of usual (pore-free) material to surface for PM.

Keywords: contact deformation, sintered material, strength, design stress, hydrostatic pressure, real mechanical properties.

It is well known that through the methods of Powder Metallurgy sintered materials and machine parts with high physico-mechanical and exploitational properties are obtained. For strength problem solving of PM first of all we must know the complete information about following mechanical properties [1]: Young's, shear and bulk moduli, as well as Poisson's ratio, which depend on relative density of material  $\gamma = \rho/\rho_0$ , where  $\rho, \rho_0$  are densities of porous and pore-free materials. Then we get the components of stress-strain state of part of machine and use the corresponding strength conditions. The elastic contact deformation process of cylinders from usual material is quite well known: in plane strain case the H. Hertz formulae [2] are used. It is shown in [3] that one can use for fatique problem solving of PM Hertzian pressure and components of stress state. Accordingly the dangerous point is inside the material and high hydrostatic pressure in case of contact deformation does not play any role. It seems to us that strength problem of contact deformation process of PM is different from the usual material. Therefore, the present work was initiated to solve the problem of contact deformation process of PM cylinders.

The elastic deformation process of two cylinder contacts is examined. One of cylinders was manufactured from usual material, the other - PM with initial porosity  $v_0$ . To define PM cylinder mechanical properties, we use the following formulae [1]:

$$E_2 = E_0 (1 - v_0)^{3.4}, \qquad \mu_2 = (1 - \mu_0)(1 - v_0)^{0.16} - 1,$$
 (1)

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where  $E_2$ ,  $E_0$  and  $\mu_2$ ,  $\mu_0$  are Young's modulus and Poisson's ratio of porous material and substance of PM cylinder.

If  $E_0 = 211.6 kn / mm^2$ ,  $\mu_0 = 0.288$  [1] and initial porosity of material  $v_0$  are known from formulae (1), we may define values  $E_2$  and  $\mu_2$ . For example, if  $v_0 = 0.1$  we get  $E_2 = 147.9 kn / mm^2$ ,  $\mu_2 = 0.266$  and if  $v_0 = 0.2$  we get  $E_2 = 99.1 kn / mm^2$ ,  $\mu_2 = 0.243$ .

For definition distribution of stresses at axis oz in case of plane strain condition ( $\varepsilon_{11} = 0$ ) the following formulae are used [2]:

$$\sigma_{11} = -2\mu_2 p_0 \left( \left( 1 + \left(\frac{z}{b}\right)^2 \right)^{0.5} - \frac{z}{b} \right), \tag{2}$$

$$\sigma_{22} = -p_0 \left( \left(1 + 2\left(\frac{z}{b}\right)^2\right) / \left(1 + \left(\frac{z}{b}\right)^2\right)^{0.5} - 2\frac{z}{b} \right),\tag{3}$$

$$\sigma_{33} = -p_0 / (1 + (\frac{z}{b})^2)^{0.5}, \tag{4}$$

where  $\sigma_{11}, \sigma_{22}, \sigma_{33}$  are stresses at direction x, y, z (Fig. 1),  $p_0$  is maximum pressure on the area of contact (if y = 0), b is half of contact area width:

$$p_0 = 0.418(2qE_1E_2(R_1 + R_2)/(R_1R_2(R_1 + R_2)))^{0.5},$$
(5)

$$b = 1.075(qR_1R_2(E_1 + E_2)/(E_1E_2(R_1 + R_2)))^{0.5},$$
(6)

q is load at unity length of contact area.

If the values of q, characteristics of materials  $v_0, E_1, E_2, \mu_2$  and radii of  $R_1, R_2$  are known, from formulae (1) – (6), we may define values of  $p_{0,b}, \sigma_{11}, \sigma_{22}$  and  $\sigma_{33}$ , which allow us to determine the effective  $\sigma_i$  and average or hydrostatic  $\sigma_0$  stresses

$$\sigma_i = (0.5((\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2))^{0.5}, \tag{7}$$

$$\sigma_0 = (\sigma_{11} + \sigma_{22} + \sigma_{33})/3. \tag{8}$$

They play a very important role for the process of plastic deformation and strength analysis of porous material.

It is convenient to introduce the problem solving by dimensionless values. So, the following denotations  $\overline{\sigma}_{11} = \sigma_{11}/p_0$ ,  $\overline{\sigma}_{22} = \sigma_{22}/p_0$ ,  $\overline{\sigma}_{33} = \sigma_{33}/p_0$ ,  $\overline{\sigma}_i = \sigma_i/p_0 \ \overline{\sigma}_0 = \sigma_0/p_0$  are given. For strength problem solving of sintered PM the conception of equivalent stress for porous material  $\sigma_{eq}$  is used [4], taking into account initial porosity of material  $v_0$ , effective  $\sigma_i$  and average  $\sigma_0$  stresses. The

formulae for the definition of dimensionless design stress value  $\overline{\sigma}_d = \overline{\sigma}_{eq}$  and limit conditions are introduced as follows:

$$\overline{\sigma}_d = \sigma_d / p_0 = (\overline{\sigma}_i^2 + 9\alpha_0^m \overline{\sigma}_0^2)^{0.5} / \beta_0^{n+0.5} = \sigma_y / p_0, \qquad (9)$$

where  $\sigma_y$  – yield stress, *m* and *n* - parameters of real porosity of sintered PM,  $\alpha_0$  and  $\beta_0$  are the values of  $\alpha$  and  $\beta$  porosity functions in case of initial porosity of material is  $v_0$  [4].

Different formulae for definition of values of  $\alpha_0$  and  $\beta_0$  are known. Very simple functions of porosity are given in [5]:

$$\alpha_0 = v_0, \ \beta_0 = 1 - v_0. \tag{10}$$

In [5] for different initial porosity sintered steel, which contains the components Ni(4%), Cu(1.5%) and Mo(0.5%) in case of using the formulae (10), the main values of *m* and *n* parameters are determined equal to m = 1.1 and n = 1.4.

For usual material  $v_0 = 0$ ;  $\mu = 0.3$ ;  $\alpha_0 = 0$ ;  $\beta_0 = 1$  and from formula (9) we obtained  $\overline{\sigma}_d = \overline{\sigma}_i$ .

The dimensionless calculated values of stress state components  $\overline{\sigma}_{ij}$ , effective  $\overline{\sigma}_i$ , average  $\overline{\sigma}_0$  and design  $\overline{\sigma}_d$  stresses for usual ( $v_0 = 0$ ;  $\mu = 0.3$ ) and porous ( $v_0 = 0.1$ ;  $\mu_2 = 0.266$ ;  $v_0 = 0.2$ ;  $\mu_2 = 0.243$  and m=1.1; n=1.4) material are given in Table.

The components of stress state  $\overline{\sigma}_{ij}$ , values of  $\overline{\sigma}_i$  and  $\overline{\sigma}_0$  alteration sheets at axis oz in case of  $v_0 = 0.1$  are shown in Fig. 1.

Table

	$-\overline{\sigma}_{22}$	- <del>-</del>	$v_0 = 0, \mu = 0.3$			$v_0 = 0.1, \mu_2 = 0.266$				$\nu_0 = 0.2, \mu_2 = 0.243$			
Z						$\alpha^m = 0.08, \beta^{n+0.5} = 0.8$				$\alpha^m = 0.17, \beta^{n+0.5} = 0.65$			
b			$-\overline{\sigma}_{11}$	$-\overline{\sigma}_0$	$\bar{\sigma}_i$ ,	$-\overline{\sigma}_{11}$	$-\overline{\sigma}_0$	$\overline{\sigma_i}$	$\bar{\sigma}_d$	$-\overline{\sigma}_{11}$	$-\overline{\sigma}_0$	$\overline{\sigma_i}$	$\sigma_d$
					$\sigma_d$								
0	1.00	1.00	0.60	0.87	0.40	0.53	0.84	0.46	1.04	0.49	0.83	0.51	1.8
0.2	0.66	0.98	0.49	0.71	0.42	0.44	0.69	0.47	0.92	0.40	0.68	0.50	1.5
0.4	0.43	0.93	0.41	0.58	0.51	0.36	0.57	0.54	0.88	0.33	0.56	0.56	1.4
0.6	0.27	0.86	0.34	0.49	0.55	0.30	0.48	0.57	0.86	0.27	0.47	0.59	1.3
0.8	0.19	0.79	0.29	0.42	0.56	0.26	0.41	0.56	0.81	0.24	0.41	0.58	1.2
1.0	0.13	0.71	0.25	0.36	0.53	0.22	0.35	0.54	0.75	0.20	0.35	0.55	1.1
1.2	0.09	0.64	0.22	0.32	0.50	0.19	0.31	0.51	0.70	0.18	0.30	0.51	1.0

Values of stress state components for different  $v_0$  of PM (m = 1.1, n = 1.4)



Fig. 1. The sheets of component alteration of elastic stress state  $\overline{\sigma}_{ij}, \overline{\sigma}_i$  and  $\overline{\sigma}_0$  at axis oz in case of  $v_0 = 0.1$ 

The calculated data of Table and Fig.1 show that the components of  $\overline{\sigma}_{ij}$  and  $\overline{\sigma}_0$  quickly decrease at axis oz. The effective stress  $\overline{\sigma}_i$  in the zone z/b = 0.8 is maximum, that is why dangerous point for usual material is present.

In case of PM the values of Poisson's ratio  $\mu$ , absolute values of the first principal  $|\overline{\sigma}_{11}|$  and average  $|\overline{\sigma}_{0}|$  stresses, as compared with the usual material decrease, while the values of effective stress  $\overline{\sigma}_{i}$  increase.

The design stress alteration sheets at axis oz for different initial porosity of material are shown in Fig. 2. Since the maximum design stress  $(\overline{\sigma}_d)^{\text{max}}$  acts in the dangerous points of material, we may define the places of dangerous points and their values  $(\overline{\sigma}_d)^{\text{max}}$  (Fig. 3). For usual material  $v_{02} = 0$  (curve 1) the internal point *a* is

dangerous, where  $(\overline{\sigma}_d)_1^{\max} = 0.56$ . For porous materials  $v_{02} = 0.1$  (2) and  $v_{03} = 0.2$  (3) the external points *b* and *c*, where  $(\overline{\sigma}_d)_2^{\max} = 1.036$  and  $(\overline{\sigma}_d)_3^{\max} = 1.76$  correspondingly are dangerous.

Thus, for PM the dangerous point from inside (if  $v_0 = 0$ ) moves on the surface of material, where the maximum contact pressure  $p_0$  acts. The comparison of curves in Fig.2 show that for high porosity material the maximum design stress becomes greater. It is the result of the high hydrostatic pressure and functions of porosity  $\alpha$  and  $\beta$ . Consequently, for high porosity materials the values  $p_0$  in accordance with (9) will be less.



Fig. 2. The sheets of alteration of dimensionless design stress  $\overline{\sigma}_d$  for different porosity of material. Curve  $1 - \nu_0 = 0;$  $2 - \nu_0 = 0.1; \quad 3 - \nu_0 = 0.3$ 

Summary. The elastic problem of surface contact deformation of two cylinders is investigated and one of them was manufactured from sintered PM. The initial porosity of material  $v_0$  is taken into account. The real mechanical properties of materials are used. The plane strain case of elastic deformation is examined. for which the H.Hertz formulae are used. For strength problem solving of sintered material the conception of equivalent stress for porous material  $\sigma_{cq}$  is used, taking into account initial porosity of

material  $v_0$ , effective  $\sigma_i$  and average  $\sigma_0$  stresses. It is shown that in case of PM the values of Poisson's ratio  $\mu$ , absolute values of the first principal  $|\overline{\sigma}_{11}|$  and average  $|\sigma_0|$  stresses decrease, while the value of effective stress  $\overline{\sigma_i}$  increases, and then decreases. Based on the conception of equivalent stress  $\overline{\sigma_{eq}}$  the place of dangerous point of PM is found, where  $\overline{\sigma_d} = \overline{\sigma_{eq}}$  is maximum. For pore-free cylinder the dangerous point is inside the material. For PM cylinder the place of the dangerous point moves on the surface, where maximum contact pressure  $p_0$  acts.

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### ՓՈՇԵՆՅՈՒԹԵՐԻՑ ԵՌԱԿԱԼՎԱԾ ԳԼԱՆՆԵՐԻ ԿՈՆՏԱԿՏԱՅԻՆ ԴԵՖՈՐՄԱՑՄԱՆ ԳՈՐԾԸՆԹԱՑՆԵՐԻ ՀԵՏԱԶՈՏՈՒՄԸ

Հետազոտված են երկու գլանների կոնտակտային դեֆորմացման խնդիրները, որոնցից մեկը պատրաստված է եռակալված փոշենյութից (ՓՆ)։ Ենթադրվում է, որ ՓՆ-ի հիմնանյութը համասեռ է, իզոտրոպ և ծակոտիները բաշխված են հավասարաչափ։ Խնդրի լուծումը իրականացվում է ՓՆ-ի իրական մեխանիկական հատկությունների և Հ. Հերցի գլանային մակերևույթների կոնտակտային տեսության հիման վրա։ ՓՆ-ի ամրության խնդիրներ լուծելու համար օգտագործվել է ծակոտկեն նյութերի համար համարժեքային լարման գաղափարը։ Հաշվարկային տվյալները ցույց են տվել, որ այն դեպքում, երբ հաշվի ենք առնում ՓՆ-ի համար շատ կարևոր գործոն՝ հիդրոստատիկ Ճնշումը, նյութի վտանգավոր կետի դիրքը փոխվում է՝ ներսից (սովորական նյութի համար) տեղափոխվում է դեպի մակերևույթը (ՓՆ-ի համար)։

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#### ИССЛЕДОВАНИЕ ПРОЦЕССОВ КОНТАКТНОГО ДЕФОРМИРОВАНИЯ СПЕЧЕННЫХ ПОРОШКОВЫХ ЦИЛИНДРОВ

Проведено исследование задач контактного деформирования двух цилиндров, один из которых изготовлен из спеченных порошковых материалов (ПМ). Принимается, что вещество ПМ однородное, изотропное и поры распределены равномерно. Решение задачи осуществляется на основании реальных механических характеристик ПМ и контактной теории цилиндрических поверхностей Герца. Вводится понятие эквивалентного напряжения для пористых материалов. Расчетные данные показывают, что в случае, когда учитывается гидростатическое давление, изменяется положение опасной точки материала: изнутри для обычных материалов перемещается на поверхность для ПМ.