Armenian Journal of Physics, 2022, vol. 15, issue 1, pp. 1-6

Investigation of the Inflationary Process of the Theory of JBD

H.K. Teryan, A.S. Kotanjyan, G.H. Harutunyan

Yerevan State University, 1 Alex Manoogian, 0025, Yerevan, Armenia

E-mail: hripsime.teryan95@gmail.com

(Received: February 16, 2022; Revised: March 1, 2022; Accepted: March 15, 2022)

Abstract. The cosmological model is considered in the presence of a conformally coupled field in the evolution of the early Universe. Such a field, satisfying the condition of general covariance, leads to the appearance of an additional term in the functional of the action, which connects the scalar field with scalar curvature, which can apparently serve as an analogue of the potential energy of a scalar field. Such a variant of action arises in the Jordan-Bruns-Dicke theory as a result of the well-known conformal transformations of Bekenstein. The problem can be represented as an autonomous dynamical system, and qualitative methods can be used to analyze such a system.

Keywords: Inflation, scalar-tensor theory of gravity, Λ constant, conformal coupled field

https://doi.org/10.54503/18291171-2022.15.1-1

1. Introduction

Historically, the first successful model of inflation by Starobinsky [1, 2] was based on the theory of gravity with a Lagrangian containing both the first and the highest degrees of scalar curvature. In such a theory, the Friedmann equation is modified for large values of the Hubble parameter, which leads to a cosmological solution with an exponentially growing scale factor. The Starobinsky model also falls into the category of cosmological solutions, when the inflationary regime arises due to the presence of specific matter since, as it is known, the theory of gravity with a Lagrangian nonlinear in the scalar of curvature is equivalent to general relativity with a scalar field. Subsequently, Starobinsky developed an inflationary model entirely based on the effects of vacuum polarization, since it was shown that the exponential expansion can be caused by quantum corrections to the pulse energy tensor [Gurevich, Zel'dovich, Starobinsky [3, 4]]. In a cosmological situation, it becomes necessary to take into account the interaction of fields of scalar matter with gravity, i.e. describe them against the background of a non-stationary metric corresponding to curved space-time. The problem of the joint evolution of gravitational and material fields is being solved [5-7].

In this paper, a cosmological model is investigated in which gravity is described on the basis of the Jordan-Brans-Dicke (JBD) theory, and the inflationary regime arises due to the presence of a specific scalar in the Universe. One of the conformal transformations of the modified theory (Bekenstein [8]) is considered, in which a scalar field, satisfying the general covariance condition, leads to the appearance of an interference term $\psi^2 R/12$ in the action functional (ψ - scalar field, R - scalar curvature of space). As for the cosmological constant Λ, it is known that it was introduced for the first time by Einstein and qualified as a global-scale phenomenon that manifests itself in the dynamics of cosmological expansion. It is associated with antigravity, which manifests itself only within the framework of the scale, where a regular cosmological expansion according to the Hubble law is actually observed. In Friedmann's model of the expanding Universe, the numerical value Λ does not follow from the theory, it is subject to measurement in special cosmological observations.

2. Conform-related scalar field in the presence of a cosmological constant

Equations of the theory of JBD transformations

$$\tilde{g}_{\mu\nu}=\frac{1}{4}z^{(n+1)/n}\left(1+z^{-n}\right)^2 g_{\mu\nu}$$

$$\psi=\frac{6(z^n-1)}{k(z^n+1)},\quad n=\sqrt{\frac{3+2\varsigma}{3}},\quad z=\left(\frac{y}{y_0}\right)^n \tag{1}$$

used by Bekenstein, are reduced to equations of general relativity with a source of non-gravitational fields and a conform coupled massless scalar field satisfying the Penrose-Chernikov equation [9]

$$g^{\alpha\beta}\nabla_\alpha\psi_\beta-\frac{1}{6}R\psi=0 \tag{2}$$

In the presence of a cosmological constant, the JBD action functional takes the form

$$W=\int\left[-\frac{1}{2k}\left(R+2\Lambda\right)+\frac{\psi^2 R}{12}+\frac{1}{2}\left(\bar{\nabla}\psi\right)^2+L_m\right]\sqrt{-g}\,d^4x\,, \tag{3}$$

and the corresponding equations

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\Lambda g_{\mu\nu}=kT_{\mu\nu}\,, \tag{4}$$

$$T_{\mu\nu}=\nabla_\mu\psi\nabla_\nu\psi-\frac{1}{2}g_{\mu\nu}\left(\nabla\psi\right)^2-\frac{1}{6}\nabla_\mu\nabla_\nu\psi^2+\frac{1}{6}g_{\mu\nu}\nabla^2\psi^2+\frac{\psi^2}{6}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right), \tag{5}$$

where $\nabla^2\psi=g^{\mu\nu}\nabla_\mu\nabla_\nu\psi,\qquad \left(\nabla\psi\right)^2=g^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi\,.$

Meaning

$$\frac{1}{6}\nabla_\mu\nabla_\nu\psi^2=\frac{1}{3}\nabla_\mu\psi\nabla_\nu\psi+\frac{1}{3}\nabla_\mu\nabla_\nu\psi, \tag{6}$$

$$\frac{1}{6}g_{\mu\nu}\nabla^2\psi^2=\frac{1}{3}g_{\mu\nu}g^{\alpha\beta}\nabla_\beta\psi\nabla_\alpha\psi+\frac{1}{3}g_{\mu\nu}\nabla^2\psi\,, \tag{7}$$

finally get

$$\left(1-\frac{k\psi^2}{6}\right)\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)-\Lambda g_{\mu\nu}=k\tau_{\mu\nu} \tag{8}$$

$$\tau_{\mu\nu}=\frac{2}{3}\nabla_\mu\psi\nabla_\nu\psi-\frac{1}{6}g_{\mu\nu}\left(\nabla\psi\right)^2-\frac{\psi}{3}\nabla_\mu\nabla_\nu\psi+\frac{\psi}{3}g_{\mu\nu}\nabla^2\psi\,, \tag{9}$$

whose convolution gives $\psi\nabla^2\psi$, and, accordingly, convolution (8) leads to $R=-4\Lambda$. A conform coupled field ψ satisfies the equation

$$\nabla^2\psi+\frac{2}{3}\Lambda\psi=0 \tag{10}$$

and the analogue of the De-Sitter cosmological problem is determined by the basic relations

$$\ddot{\psi}+3H\dot{\psi}+\frac{2}{3}\Lambda\psi=0 \tag{11}$$

$$\dot{H}+2H^2=\frac{2}{3}\Lambda \tag{12}$$

$$3\left(1-\frac{k\psi^2}{6}\right)H^2-k\psi\dot{\psi}H-\left(\Lambda+\frac{k\dot{\psi}^2}{2}\right)=0 \tag{13}$$

3. Qualitative consideration of the cosmological problem

Insertion of dimensionless variables $\tau=t/t_0$, $\psi=x/\sqrt{k}$, $\dot{\psi}=y/(t_0\sqrt{k})$, $\ddot{\psi}=dy/(t_0^2\sqrt{k}d\tau)$, $L=\Lambda t_0^2$ [10, 11] allows (13) to be rewritten in the form

$$3\left(1-\frac{x^2}{6}\right)H^2t_0^2-xyHt_0-\left(L+\frac{y^2}{2}\right)=0, \tag{14}$$

as a result

$$Ht_0=\frac{\left(\frac{xy}{2}\pm\sqrt{3L\left(1-\frac{x^2}{6}\right)+\frac{3}{2}y^2}\right)}{3\left(1-\frac{x^2}{6}\right)} \tag{15}$$

In this case, the system of dynamic equations is represented as

$$\frac{dx}{d\tau}=y, \tag{16}$$

$$\frac{dy}{d\tau}=-\frac{\left(\frac{y^2x}{2}\pm y\sqrt{3L\left(1-\frac{x^2}{6}\right)+\frac{3y^2}{2}}\right)}{\left(1-\frac{x^2}{6}\right)}-\frac{2}{3}\Lambda x, \tag{17}$$

and the phase trajectory has the form

$$\frac{dy}{dx}=-\frac{\frac{yx}{2}\pm\sqrt{3L\left(1-\frac{x^2}{6}\right)+\frac{3}{2}y^2}}{\left(1-\frac{x^2}{6}\right)}-\frac{2}{3}L\frac{x}{y} \tag{18}$$

Taking into account that equation (11) can be assigned a simple physical analogy-mechanical rolling in a potential well with a profile $\mathrm{v}(x)=2L\left(1-x^2/6\right)$, $\mathrm{v}_x^{/}=-2Lx/3$ and the time-dependent coefficient of friction $3H\dot{\psi}$ [10, 11], and the energy-momentum tensor of such a scalar field coincides with the energy-momentum tensor of an ideal fluid with the equation of state $P\approx-\varepsilon$, option is being considered

$$y=\sqrt{2L\left(1-\frac{x^2}{6}\right)}sh\chi(x), \tag{19}$$

The function $\chi(x)$ is determined from the system of dynamic relations

$$Ht_0 = \frac{xy/2 \pm \sqrt{\frac{3}{2}}\,\mathrm{v}(x)ch\chi(x)}{3\left(1 - x^2/6\right)}, \tag{20}$$

$$\frac{dy}{d\tau} = -\frac{\left(\frac{y^2 x}{2} \pm y\sqrt{\frac{3}{2}}\,\mathrm{v}(x)ch\chi(x)\right)}{\left(1 - \frac{x^2}{6}\right)} + \mathrm{v}'(x). \tag{21}$$

From (16), (19), (20) we obtain an equation that defines the function $\chi(x)$

$$\frac{d\chi}{dx} = -\frac{\left(\frac{x}{3}cth\chi(x) \mp \sqrt{\frac{3}{2}}\right)}{1 - \frac{x^2}{6}}, \tag{22}$$

as well as

$$Ht_0 = \sqrt{2L}\,\frac{\left(\frac{x}{2}sh\chi(x) \pm \frac{3}{2}ch\chi(x)\right)}{3\left(1 - \frac{x^2}{6}\right)}, \tag{23}$$

which, together with (16), (19), and (21), makes it possible to describe the inflationary stage of the Universe with the initial condition (19).

4. Option with the choice of a dimensionless parameter t_0

Integration of equation (12) can be represented as [12, 13]

$$H = H_0\sqrt{\overset{0}{\Omega}_\Lambda}\,th\left[2H_0\sqrt{\overset{0}{\Omega}_\Lambda}\,t + \delta\right], \tag{24}$$

where

$$e^{2\delta} = \left(1 + \sqrt{\overset{0}{\Omega}_\Lambda}\right)\Big/\left(1 - \sqrt{\overset{0}{\Omega}_\Lambda}\right), \quad \overset{0}{\Omega}_\Lambda = \Lambda / \left(3H_0^2\right)$$

($H_0 -$ the value of the Hubble parameter at the stage under consideration). The solution to the problem is greatly simplified if we choose as the dimensionless τ parameter $\tau = t/t_0$, $t_0 = 1\Big/\left(H_0\sqrt{\overset{0}{\Omega}_\Lambda}\right)$ which leads to using

$$Ht_0 = th\left(2\tau + \delta\right), \tag{25}$$

$$\frac{d^2\left(\psi a^3\right)}{dt^2} = 3Ht_0\frac{d\left(\psi a^3\right)}{d\tau} + 2\left(\psi a^3\right)\left(2 - 3H^2 t_0^2\right), \tag{26}$$

$$\frac{a^2}{a_0^2} = \frac{ch\left(2\tau + \delta\right)}{ch\delta}, \tag{27}$$

$$q = \frac{2}{\left[th\left(2\varepsilon + \delta\right)\right]^2} - 1. \quad (28)$$

The system of equations of item 3 takes the following form

$$\frac{dx}{d\tau} = y, \quad (29)$$

$$\frac{dy}{d\tau} = -3yth\left(2\tau + \delta\right) - 2x. \quad (30)$$

5. Results and Discussions

In Figures 1, 2, 3 time dependencies of H, a, x, q are presented.

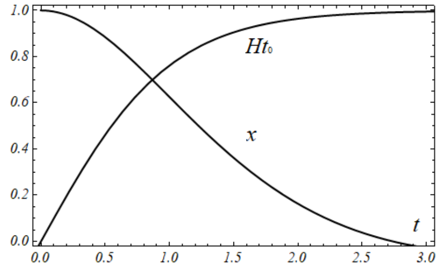

Fig. 1. Time dependence of the Hubble parameter $Ht_0 = const$ corresponds to the exponential growth of the scale parameter $a(t)$. Also depicted is the time dependence x(t), which indicates the decay of the scalar field.

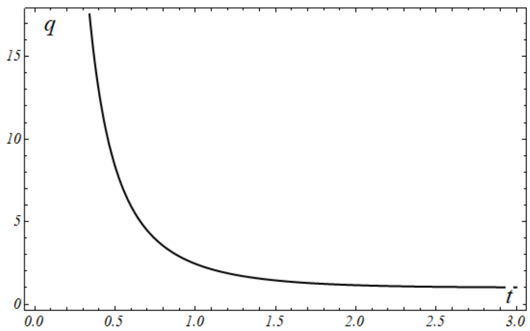

Fig. 2. Time dependence of the dimensionless parameter *q*.

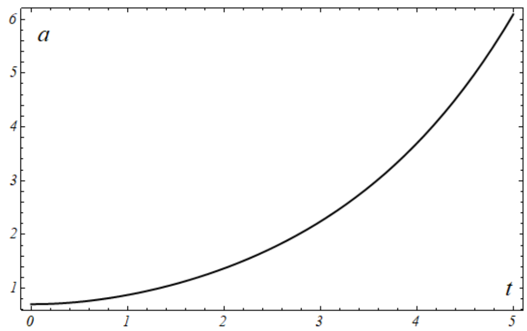

Fig. 3. Exponential growth of the scale factor *a*.

The evolution of the Universe at an early stage of development is considered in the framework of a conformal analogue of the modified Jordan theory in the case of taking into account the interaction of the scalar field ψ with the gravitational field and the presence of the cosmological constant Λ is considered. As a result, it was shown that the accelerated expansion of the Universe also takes place at the early stage of evolution. The presence of a scalar field along with the vacuum energy also leads to expansion with an exponential law, as in the case of the presence of vacuum energy, the density of which remains unchanged during the expansion of the Universe. With the inflationary expansion, in addition to what has been said, a mechanism is also needed to stop inflation, as a result of which the Universe heats up, passing into a hot stage. The decay of a scalar field can serve as such a mechanism. Regarding the evaluation of e-folds [14]

$$N = \int_0^{t_{\inf}} Hdt = \int_0^{\tau_{\max}} Ht_0 d\tau = \frac{1}{2}\ln\frac{ch(2\tau_{\max}+\delta)}{ch\delta}, \tag{31}$$

where $t_{\inf}$ -inflationary time

$$t_{\inf} = \frac{\tau_{\max}}{2H_0\sqrt{\overset{0}{\Omega}_\Lambda}} \tag{32}$$

If we take into account the generally accepted assessment $H_0 \sim 10^{-4} M_{pl}$, then the value $t_{\inf}$ coincides with the generally accepted one.

6. Conclusion

In the frame of this paper, a cosmological model was investigated and accelerated expansion of the Universe at the early stage of evolution was shown. We studied the inflationary regime taking into account the conformal transformations of JBD modified theory.

Acknowledgements

The authors are immensely grateful to Prof. A.A. Saharyan, Prof. R.M. Avagyan, and Prof. A.V. Hovsepyan for their valuable comments that greatly improved the manuscript.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization: G.H. Harutunyan; Methodology G.H. Harutunyan, H.K. Teryan; Investigation and data curation G.H. Harutunyan, A.S. Kotanjyan and H.K. Teryan, Writing-original draft preparation: G.H. Harutunyan; Writing-review and editing H.K. Teryan; Software and visualization: A.S. Kotanjyan. All authors have read and agreed to the published version of the manuscript.

References

[1] A.A. Starobinsky, Phys. Lett. **30** (1979) 682.
[2] A.A. Starobinsky, Phys. Lett. B **91**(1980) 99.
[3] V.Ch. Gurevich, A.A. Starobinsky, JETP, **77** (1979) 1699.
[4] A.A. Starobinski, Phys. Lett. B **117** (1982) 175.
[5] R.M. Avagyan, G.H. Harutunyan, V.V. Papoyan, Astrophysics **48** (2005) 455.
[6] R.M. Avagyan, G.H. Harutunyan, Astrophysics **48** (2005) 633.
[7] . R.M. Avagyan, G.H. Harutunyan, A.V. Hovsepyan, A.S. Kotanjyan, Astrophysics **57** (2014) 327.
[8] I.D. Bekenshtein. Ann. Phys. (N.V.) **82** (1974) 535.
[9] N.A. Chernikov, PEPAN, **18** (1987) 1011.
[10] O.I. Bogoyavlensky, Methods in the Qualitative Theory of Dynamical Systems (Springer, Berlin,1985).
[11] I. Wainwright, G.F.R. Ellis, Dynamical Systems in Cosmology (Cambridge University Press., Cambridge, 1977).
[12] . R.M. Avagyan, E.V. Chubaryan, G.H. Harutunyan, A.A. Saharyan, Gen. Relative Gravity **48** (2016) 21.
[13] R.M. Avagyan, G.H. Harutunyan, S.V. Sushkov, Astrophysics **60** (2017) 159.
[14] D.S. Gorbunov, V.A. Rubanov, Introduction to the Theory of the Early Universe, (URSS, M., 2008).