АСТРОФИЗИКА

ТОМ 64 НОЯБРЬ, 2021 ВЫПУСК 4

MODIFIED RENYI HOLOGRAPHIC DARK ENERGY (MRHDE) IN $f(R,T)$ THEORY OF GRAVITY

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Received 25 June 2021
Accepted 20 October 2021

In this work, we investigate the dynamics of Bianchi type VI_0 space-time in the framework of $f(R, T)$ theory of gravity where R stands for Ricci scalar and T stands for the trace of the stress energy-momentum tensor with Modified Renyi Holographic Dark Energy (MRHDE). With the specific choice of the functional $f(R,T) = f_1(R) + f_2(T) = \mu_1 R + \mu_2 T$ where $f_1(R)$ and $f_2(T)$ are arbitrary functions of R and T respectively and μ_1 and μ_2 are two parameters, we have obtained the exact solutions of the model by considering the energy density of MRHDE and by using a law of variation for the Hubble parameter H. It is found that our model leads to the accelerated expansion of the Universe. The EOS parameter $\omega_{DE} > -1$ indicates that our cosmological model behaves like a quintessence dark energy model which is consistent with the recent observations. A correspondence between MRHDE and quintessence dark energy is established. The quintessence dynamics of the potential and scalar field are reconstructed, which illustrates the late-time cosmic acceleration. All physical parameters are calculated and discussed graphically.

Keywords: *MRHDE: $f(R,T)$ theory of gravity: quintessence: EOS parameter: statefinder parameter*

1. *Introduction.* From the observational astrophysical results [1-5] it is confirmed that our Universe is accelerating. The source driving this acceleration is known as dark energy (DE), an unknown form of energy with negative pressure, whose origin is still a mystery in modern cosmology. Recent experiments indicates that DE constitutes about 70% of present total cosmic energy. However, so far, the nature of DE is still unknown. The most familiar candidate for dark energy is the cosmological constant Λ which is characterized by the equation of state $p = \omega\rho$ with $\omega = -1$ where p is the pressure and ρ is the energy density of DE. But from theoretical point of view, it fails to resolve the hurdle of fine-tuning and cosmic coincidence problem [6,7]. There are several candidates to play the position of dark energy (DE), which is the dominant part of the Universe. Some of them are quintessence [8], phantom [9], k-essence [10], tachyon [11] and so on.

In recent studies, to understand the nature of the Universe, a new DE model has been constructed based on holographic principle named as holographic dark energy (HDE) was first put forward by Hooft [12] and Susskind [13]. According to this principle, the entropy of the system scales not with its volume, but also its surface area (L^2) and arrived at a conclusion that in quantum field theory a short distance cut-off is related to a long-distance cut-off due to the limit set by the black hole formation [14]. By taking ρ_{HDE} as the quantum zero-point energy density caused by a short distance cut-off in a region of size L, the total energy density should not exceed the black hole mass of the same size, giving $L^3\rho_{HDE} \le LM_p^2$. The maximal value L allowed is the one saturating this inequality, giving the HDE density as $\rho_{HDE} = 3c^2M_p^2L^{-2}$, M_p is the reduced Planck mass with $M_p^{-2} = 8\pi G$ and $3c^2$ is the numerical constant [15].

The late-time acceleration of the Universe can be studied by two ways- by introducing DE in our Universe and secondly by modifying General Relativity (GR). There are various modifications of Einstein theory. The presence of a late-time cosmic acceleration of the Universe can be explained by $f(R)$ gravity [16]. Harko et al. [17] have proposed a new generalized theory known as $f(R,T)$ gravity. According to this theory, gravitational Lagrangian involves an arbitrary function of the scalar curvature R and trace of the energy-momentum tensor T. The $f(R,T)$ gravity model depends on a source term, representing the variation of the matter stress-energy tensor with respect to the metric. Mishra and Sahoo [18] have studied Bianchi type cosmological models assuming $f(R,T) = R + 2f(T)$. In that work, Mishra and Sahoo have obtained exact solutions to the modified field equations by assuming a specific anisotropic relation. Adhav [19] obtained exact solutions of the field equations for LRS Bianchi type-I space-time with perfect fluid in the framework of $f(R,T)$ theory of gravity. The $f(R,T)$ gravity models can explain the late time cosmic accelerated expansion of the Universe.

Recently, several entropy formalisms have been used to construct and investigate the cosmological models. Some new HDE models are constructed such as Tsallis HDE (THDE) [20,21], Sharma-Mittal HDE (SMHDE) [22] and RHDE model [23]. Among these models, a new dark energy model proposed by Moradpour et al. [23] named the Rényi holographic dark energy (RHDE) model for the cosmological and gravitational investigations shows more stability by itself. Several researchers have discussed RHDE in different theories of gravity. Using the Rényi entropy, the modified Friedmann equations are obtained [24-26]. The inflation may be found in the Rényi formalism suggested by Ghaffari et al. [27]. Dubey et al. [28] have studied interacting RHDE in Brans-Dicke theory of gravity. Saha et al. [29] have investigated RHDE in the framework of Kaluza-Klein space-time. Prasanthi & Aditya [30] have explored RHDE in General Relativity (GR) in Bianchi type VI_0 metric. Dubey et al. [31] have worked out RHDE in a flat Universe.

The form of the Bekenstein entropy of a system is $S = \bar{A}/4$ where $\bar{A} = 4\pi L^2$ and L is the IR cut-off. Rényi entropy [23] can be written as $S = \frac{1}{\delta}\log\left(\frac{\delta\bar{A}}{4}+1\right) =$

$= \frac{1}{\delta}\log\left(\pi\delta L^2 + 1\right)$. By considering $\rho_{DE}\, dV \propto TdS$ where V and T denote the volume and temperature of the system, the expression of RHDE assumes the form $\rho_{DE} = \frac{3c^2}{8\pi L^2}\left(\pi\delta L^2 + 1\right)^{-1}$. By considering Hubble horizon as a candidate for IR cut-off i.e. $L = H^{-1}$, the energy density of RHDE is obtained as $\rho_{DE} = \frac{3c^2 H^2}{8\pi(\pi\delta/H^2 + 1)}$ where c^2 is a numerical constant. Since the DE occupies almost 70% of the content of the Universe today, it is rational to assume that the density of DE is a function of the Hubble parameter H and its derivative w.r. to time [32]. In cosmology anisotropic and spatially homogeneous universes have obtained much interest. The major observational evidence from CMBR [33] has been considered to support the existence of a transition from an anisotropic phase of the universe to an isotropic phase [34]. It is believed that at the early stages of evolution, the Universe is, in general spatially homogeneous and anisotropic in nature. Bianchi type spaces are usually used for studying spatially homogeneous and anisotropic cosmological models. Recently, many researchers have presented interesting cosmological models in the presence of DE within the background of anisotropic Bianchi space-times. Chaubey and Shukla [35] obtained a new class of Bianchi cosmological models in $f(R,T)$ gravity by using a special law of variation.

Motivated by the above investigations we present in this paper Modified Rényi Holographic Dark Energy (MRHDE) in $f(R,T)$ theory of gravity by considering the energy density in the framework of Bianchi type VI_0 Universe. Here we modify the energy density of RHDE as $\rho_{DE} = \frac{3c^2H^2 + \beta \dot{H}}{8\pi\left(\pi\delta/H^2 + 1\right)}$ where β is an arbitrary dimensionless parameter.

The outline of the paper is as follows: Section 2 describes the $f(R,T)$ gravity formalism. The metric and field equations are discussed in Section 3. In Section 4, we derive the solutions of field equations. Sections 5.1 and 5.2 deals with Statefinder parameters and anisotropy parameter respectively. The stability analysis and energy conditions are described in Section 6. Various parameters are discussed graphically in Section 7. Section 8 deals with correspondence between MRHDE and quintessence scalar field. The paper ends with concluding remarks in Section 9.

2. *Gravitational field equations of $f(R,T)$ gravity*. The gravitational field equations of $f(R,T)$ theory are derived from the Hilbert-Einstein variation principle. The action for the modified $f(R,T)$ gravity is

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\, d^4x + \int L_m \sqrt{-g}\, d^4x, \tag{1}$$

where L_m is the matter Lagrangian density.

The stress-energy tensor of matter [36] is

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta}{\delta g^{ij}} \left(\sqrt{-g}\, L_m \right), \tag{2}$$

where trace $T = g^{ij} T_{ij}$.

By assuming that L_m of matter depends only on metric tensor components g_{ij} and not on its derivatives, we have obtained the field equations of $f(R,T)$ gravity as

$$f_R(R,T) R_{ij} - \frac{1}{2} f(R,T) g_{ij} + \left(g_{ij} h - \nabla_i \nabla_j \right) f_R(R,T) = 8\pi T_{ij} - f_T(R,T) T_{ij} - f_T(R,T) \theta_{ij}, \tag{3}$$

where

$$\begin{gathered} h = \nabla^k \nabla_k, \quad f_R = \frac{\partial f}{\partial R}, \quad f_T = \frac{\partial f}{\partial T}, \\ \theta_{ij} = -2 T_{ij} + g_{ij} L_m - 2 g^{\alpha\beta} \frac{\partial^2 L_m}{\partial g^{ij} \partial g^{\alpha\beta}} \end{gathered} \tag{4}$$

and ∇^i is the covariant derivative.

The problem of the perfect fluids described by an energy density ρ, pressure p_{DE} and four velocity u^i is more complicated because there is no unique definition of L_m. Here we have assumed T_{ij} is of the form

$$T_{ij} = (\rho + p_{DE}) u_i u_j - p_{DE} g_{ij}, \tag{5}$$

where

$$L_m = -p_{DE}, \quad u_i u^i = 1, \quad u^i \nabla_j u_i = 0. \tag{6}$$

Using Eqs. (5) and (6) in Eq. (4), we get

$$\theta_{ij} = -2 T_{ij} - p_{DE} g_{ij}. \tag{7}$$

In general, the field equations also depend through the tensor θ_{ij} on the physical nature of the matter field. Hence in the case of $f(R,T)$ gravity, depending on the nature of the matter source, various theoretical models corresponding to matter contributions for $f(R,T)$ gravity are obtained. Harko et al. [17] derived three classes of these models:

$$f(R,T) = \begin{cases} R + 2 f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T). \end{cases} \tag{8}$$

Here we have focused in the second case i.e. $f(R,T) = f_1(R) + f_2(T) = \mu_1 R + \mu_2 T$ where $f_1(R)$ and $f_2(T)$ are arbitrary functions of R and T respectively and μ_1 and μ_2 are two parameters.

The gravitational field equations obtained from Eq. (3) with the use of Eqs. (7) and (8) and the aforesaid choice of $f(R,T)$ is

$$R_{ij}-\frac{1}{2}g_{ij}R-\left(p_{DE}+\frac{T}{2}\right)\frac{\mu_2}{\mu_1}g_{ij}=\left(\frac{8\pi+\mu_2}{\mu_1}\right)T_{ij}\,. \tag{9}$$

3. *Metric and field equations*. The Bianchi type VI_0 metric is given by

$$ds^2=dt^2-A^2dx^2-B^2e^{2x}dy^2-C^2e^{-2x}dz^2\,, \tag{10}$$

where A, B and C are cosmic scale factors and functions of cosmic time t only.

The energy momentum tensor for dark matter (pressure-less) is

$$\overline{T}^i_j=\mathrm{diag}\left[\rho_m,0,0,0\right], \tag{11}$$

where ρ_m is the energy density of dark matter (DM).

The energy momentum tensor for MRHDE is

$$\overset{'}{T}{}^i_j=\mathrm{diag}\left[\rho_{DE},-\omega_{DE}\rho_{DE},-\omega_{DE}\rho_{DE},-\omega_{DE}\rho_{DE}\right], \tag{12}$$

where ρ_{DE} is the energy density and $\omega_{DE}=p_{DE}/\rho_{DE}$ is the EOS parameter of MRHDE.

$$T^i_j=\overline{T}^i_j+\overset{'}{T}{}^i_j \tag{13}$$

Using Eqs. (11)-(13), the field equations (9) for the metric (10) are obtained as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\frac{1}{A^2}=\frac{16\pi+3\mu_2}{2\mu_1}p_{DE}-\frac{\mu_2}{2\mu_1}\left(\rho_m+\rho_{DE}\right) \tag{14}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{A^2}=\frac{16\pi+3\mu_2}{2\mu_1}p_{DE}-\frac{\mu_2}{2\mu_1}\left(\rho_m+\rho_{DE}\right) \tag{15}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=\frac{16\pi+3\mu_2}{2\mu_1}p_{DE}-\frac{\mu_2}{2\mu_1}\left(\rho_m+\rho_{DE}\right) \tag{16}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{1}{A^2}=-\frac{16\pi+3\mu_2}{2\mu_1}\left(\rho_m+\rho_{DE}\right)+\frac{\mu_2}{2\mu_1}p_{DE} \tag{17}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0\,. \tag{18}$$

Integrating Eq. (18) and assuming integrating constant to be unity, we get

$$B=C\,. \tag{19}$$

Using Eq. (19) in Eqs. (14)-(17), we get

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{1}{A^2}=\frac{16\pi+3\mu_2}{2\mu_1}p_{DE}-\frac{\mu_2}{2\mu_1}\left(\rho_m+\rho_{DE}\right) \tag{20}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=\frac{16\pi+3\mu_2}{2\mu_1}p_{DE}-\frac{\mu_2}{2\mu_1}\left(\rho_m+\rho_{DE}\right) \tag{21}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}-\frac{1}{A^2}=-\frac{16\pi+3\mu_2}{2\mu_1}\left(\rho_m+\rho_{DE}\right)+\frac{\mu_2}{2\mu_1}p_{DE}. \tag{22}$$

The energy conservation equation is

$$\dot{\rho}_m+\dot{\rho}_{DE}+3H\left(\rho_m+\rho_{DE}+p_{DE}\right)=0, \tag{23}$$

where overhead (.) denotes differentiation w.r. to cosmic time t.

Throughout the study, we have considered that there is no interaction between DM and MRHDE.

4. *Solutions of field equations*. The spatial volume V is given by

$$V=AB^2=a^3, \tag{24}$$

where a is the average scale factor.

The Hubble's parameter H is defined by

$$H=\frac{\dot{a}}{a}=\frac{\dot{V}}{3V}=\frac{1}{3}\left(H_x+H_y+H_z\right)=\frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right), \tag{25}$$

where $H_x=\dot{A}/A$ and $H_y=H_z=\dot{B}/B$ are the directional Hubble parameters in the directions of x, y and z axes respectively.

The deceleration parameter q is defined as

$$q=-\frac{a\ddot{a}}{\dot{a}^2}. \tag{26}$$

Eqs. (20)-(22) are three field equations with five unknowns A, B, ρ_m, ρ_{DE} and p_{DE}. So, we are in search of two extra conditions:

(i) The MRHDE density is defined as

$$\rho_{DE}=\frac{3c^2H^2+\beta\dot{H}}{8\pi\left(\frac{\pi\delta}{H^2}+1\right)}. \tag{27}$$

(ii) The relation between average Hubble parameter H and average scale factor a as proposed by Berman [37]

$$H=na^{-1/n}\quad\left(n>0\right). \tag{28}$$

From Eqs. (25) and (28), we get the average scale factor a as

$$a=\left(t+k_1\right)^n \tag{29}$$

k_1 is a constant of integration.

The spatial volume V is obtained as

$$V=AB^2=a^3=\left(t+k_1\right)^{3n}. \tag{30}$$

From Eq. (30), we get

$$\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}=\frac{3n}{t+k_1}. \tag{31}$$

From Eqs. (20) and (21), we obtain

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_2}{V}\exp\left(\int\frac{-\frac{2}{A^2}}{\frac{\dot{B}}{B}-\frac{\dot{A}}{A}}dt\right) \tag{32}$$

k_2 is a constant of integration.

Following Adhav [38], we assume

$$\frac{\dot{B}}{B}-\frac{\dot{A}}{A}=\frac{2}{A^2}. \tag{33}$$

Using Eq. (33) in Eq. (32), we get

$$\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{k_2}{V}e^{-t}. \tag{34}$$

By the use of Eqs. (31) and (34) and then integrating, we obtained the scale factors as

$$A=k_3^2(t+k_1)^n\exp\left[\frac{2k_2}{3}\int\frac{e^{-t}}{(t+k_1)^{3n}}dt\right] \tag{35}$$

$$B=k_3^{-1}(t+k_1)^n\exp\left[-\frac{k_2}{3}\int\frac{e^{-t}}{(t+k_1)^{3n}}dt\right], \tag{36}$$

where k_3 is a constant of integration.

The Hubble parameter H is obtained as

$$H=\frac{n}{t+k_1}. \tag{37}$$

The Hubble parameter H decreases as cosmic time t evolves and approaches to a small value at the later stage of the Universe.

The deceleration parameter q is obtained as

$$q=-1+\frac{1}{n}. \tag{38}$$

From Eq. (38), we can arrive at a conclusion that for $q>0$ $(0<n<1)$ our Universe is in decelerating phase and for $q<0$ $(n>1)$ our Universe is in accelerating phase.

The MRHDE density ρ_{DE} is obtained as

$$\rho_{DE}=\frac{(3c^2n-\beta)n^3}{8\pi(t+k_1)^2\left[\pi\delta(t+k_1)^2+n^2\right]}. \tag{39}$$

The energy conservation equation for matter obtained from Eq. (23) is

$$\dot{\rho}_m+3H\rho_m=0. \tag{40}$$

Putting the value of H in Eq. (40) and then integrating, we get

$$\rho_m = \rho_0 (t + k_1)^{-3n}, \tag{41}$$

where ρ_0 is a constant of integration.

The energy conservation equation for MRHDE obtained from Eq. (23) is

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0, \quad p_{DE} = \omega_{DE}\rho_{DE}. \tag{42}$$

Putting the values of ρ_{DE} and H in Eq. (42), we get

$$\omega_{DE} = -1 + \left(\frac{t + k_1}{3n}\right)\left[\frac{2}{t + k_1} + \frac{2\pi\delta(t + k_1)}{\pi\delta(t + k_1)^2 + n^2}\right] \tag{43}$$

$$p_{DE} = \rho_{DE}\omega_{DE} = \frac{(3c^2 n - \beta)n^3}{8\pi(t + k_1)^2\left[\pi\delta(t + k_1)^2 + n^2\right]} \times$$
$$\times \left\{-1 + \left(\frac{t + k_1}{3n}\right)\left[\frac{2}{t + k_1} + \frac{2\pi\delta(t + k_1)}{\pi\delta(t + k_1)^2 + n^2}\right]\right\}. \tag{44}$$

5. *Statefinder and anisotropy parameters.*

5.1. *Statefinder parameters*. Sahni et al. [39] proposed a cosmological diagnostic pair $\{r, s\}$ called statefinder parameters, which is defined as

$$r = \frac{\dddot{a}}{aH^3}, \quad S = \frac{r - 1}{3\left(q - \frac{1}{2}\right)} \tag{45}$$

to differentiate among different forms of dark energy. The Statefinder is dimensionless and, like the Hubble and deceleration parameters H and q, is constructed from the scale factor of the Universe and its derivatives only. For ΛCDM (cosmological constant cold dark matter) models, the statefinder parameters have the value $\{r, s\} = \{1, 0\}$.

For our model, the $\{r, s\}$ parameters take the form

$$r = 1 - \frac{9s}{2} + \frac{9s^2}{2}. \tag{46}$$

5.2. *Anisotropy parameter*. The anisotropy parameter A_p is defined as

$$A_p = \frac{1}{3H^2}\sum_{i=1}^{3}(H_i - H)^2 = \frac{2k_2^2 e^{-2t}}{9n^2 (t + k_1)^{6n-2}}. \tag{47}$$

6. *Stability analysis*. In this section we have examined the stability of our model. The square speed of sound is defined as $v_s^2 = \dot{p}_{DE}/\dot{\rho}_{DE}$. The sign of v_s^2 plays a vital role for stability analysis of a background evolution of cosmic models. The model is stable if $v_s^2 > 0$ and if $v_s^2 < 0$ the model is classically unstable [40]. Also, the casualty condition must be satisfied. It means that the sound speed is

less than the speed of light.

For our model, v_s^2 takes the form

$$v_s^2\left\{\frac{2\pi\delta(t+k_1)^2+n^2}{(t+k_1)^3\left[\pi\delta(t+k_1)^2+n^2\right]^2}\right\}=\left\{\frac{2}{3n}+\frac{2\pi\delta(t+k_1)^2}{3n\left[\pi\delta(t+k_1)^2+n^2\right]}-1\right\}\times$$
$$\times\left\{\frac{(t+k_1)\left[2\pi\delta(t+k_1)^2+n^2\right]}{\left[\pi\delta(t+k_1)^4+n^2(t+k_1)^2\right]^2}\right\}-\frac{2\pi n\delta}{3(t+k_1)\left[\pi\delta(t+k_1)^2+n^2\right]^3}. \tag{48}$$

The Energy Conditions namely, Weak Energy Conditions (WEC), Dominant Energy Conditions (DEC) and Strong Energy Conditions (SEC) are respectively given by

(I) $\rho_{DE}\geq 0$

(II) $\rho_{DE}+p_{DE}\geq 0$

(III) $\rho_{DE}+3p_{DE}\geq 0$

The left-hand sides of (I), (II) and (III) based on Eqs. (43) and (44) have been plotted in Fig.2a and found that (I) $\rho_{DE}\geq 0$, (II) $\rho_{DE}+\rho_{DE}\geq 0$ and (III)

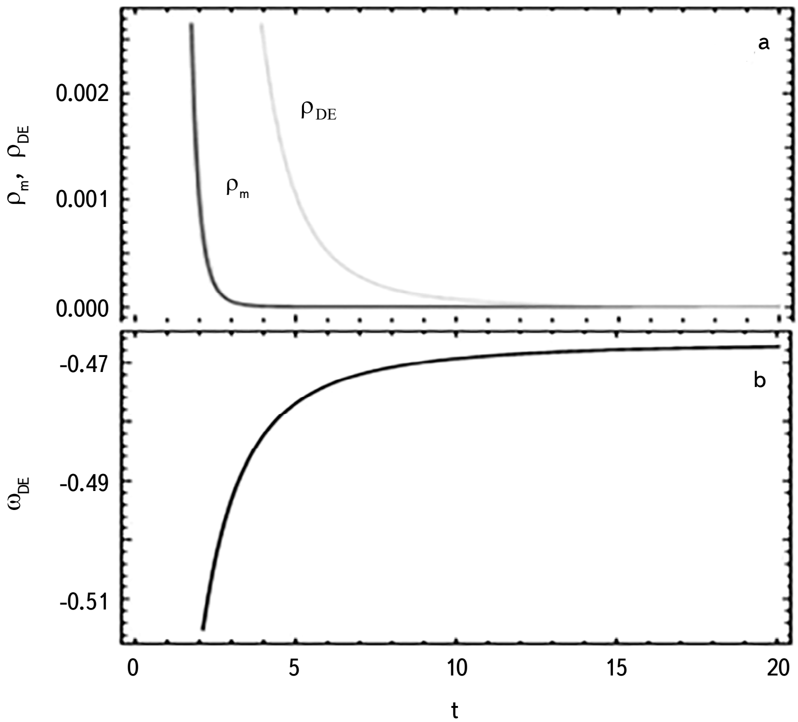

Fig.1. a) The variations of ρ_m and ρ_{DE} versus *t*. $\rho_m\to 0$ and $\rho_{DE}\to 0$ as $t\to\infty$ as seen from the above figure. b) The graph of ω_{DE} versus *t*. $\omega_{DE}>-1$ as observed from the above figure. Thus, our cosmological model corresponds to quintessence DE.

$\rho_{DE}+3p_{DE}\leq 0$. So, WEC and DEC are satisfied whereas SEC is violated. The violation of SEC gives anti-gravitational effect for which Universe gets jerk and thus our model exhibits transition from early decelerating to present accelerating Universe. So, our model is in harmony with recent cosmological observations.

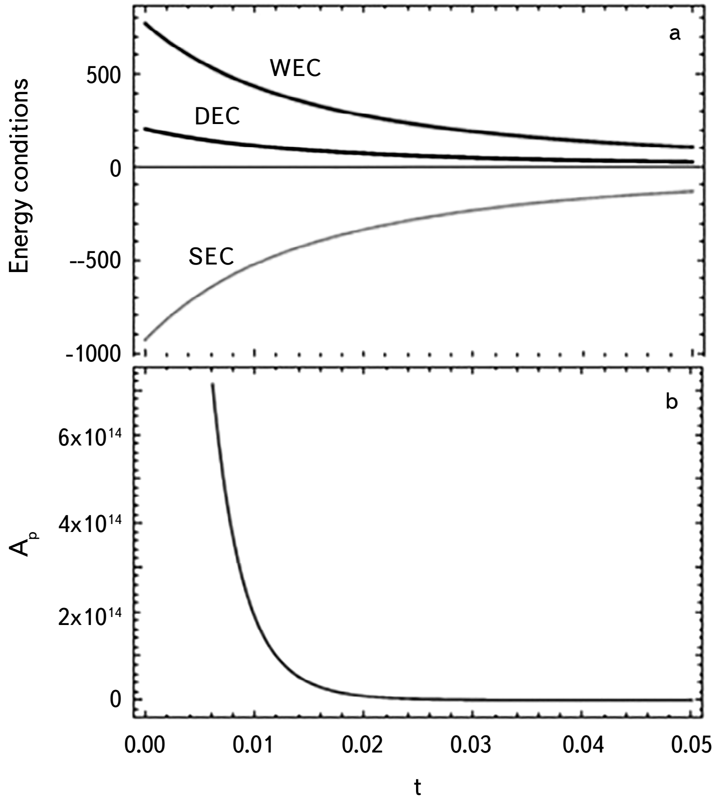

Fig.2. a) The graph of Energy Conditions versus cosmic time *t*. WEC and DEC are satisfied whereas SEC is violated. The violation of SEC gives anti-gravitational effect for which Universe gets jerk and thus our model exhibits transition from the early deceleration to present cosmic acceleration. So, our model is in good agreement with recent cosmological observations. b) The graph of A_p versus *t*. $A_p \to 0$ as $t \to \infty$ as observed from the figure. Thus, our Universe approaches isotropy at the later epoch.

7. *Correspondence between MRHDE and quintessence scalar field.* The pressure and energy density for quintessence scalar field [41] are given by

$$p_{\varphi}=\frac{\dot{\varphi}^2}{2}-V(\varphi) \tag{49}$$

$$\rho_{\varphi}=\frac{\dot{\varphi}^2}{2}+V(\varphi), \tag{50}$$

where φ denotes the scalar field and $V(\varphi)$ is the scalar field potential.

The EOS parameter ω_φ is defined as

$$\omega_\varphi = \frac{p_\varphi}{\rho_\varphi} = \frac{\dot{\varphi}^2 - 2V(\varphi)}{\dot{\varphi}^2 + 2V(\varphi)}. \tag{51}$$

Eqs. (39) and (50) together implies

$$\frac{\left(3c^2 n - \beta\right)n^3}{8\pi(t+k_1)^2\left[\pi\delta(t+k_1)^2 + n^2\right]} = \frac{\dot{\varphi}^2}{2} + V(\varphi). \tag{52}$$

Eqs. (43) and (51) together implies

$$\frac{\dot{\varphi}^2}{2} = \left(\frac{1+\omega_{DE}}{1-\omega_{DE}}\right)V(\varphi). \tag{53}$$

Eqs. (52) and (53) together implies

$$V(\varphi) = \left(\frac{1-\omega_{DE}}{2}\right)\left\{\frac{\left(3c^2 n - \beta\right)n^3}{8\pi(t+k_1)^2\left[\pi\delta(t+k_1)^2 + n^2\right]}\right\}. \tag{54}$$

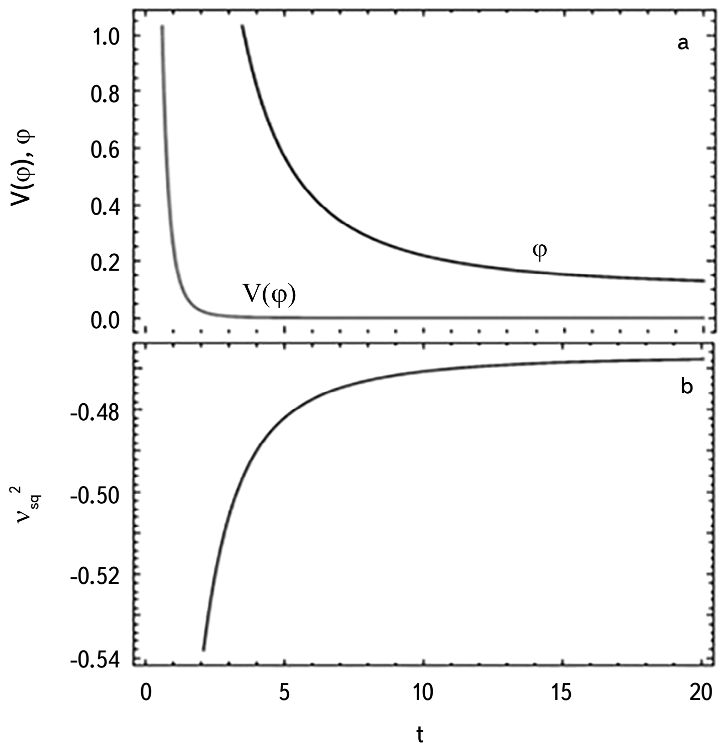

Fig.3. a) The plots of scalar field potential $V(\varphi)$ and scalar field φ versus cosmic time t. It is clear from the above figure that $V(\varphi)$ decreases and ultimately tends to zero whereas φ decreases and approaches to small value at the later stage of the Universe. b) The plot of v_s^2 versus cosmic time t. v_s^2 is negative throughout the evolution of the Universe. It clearly manifests the unstable nature of the Universe.

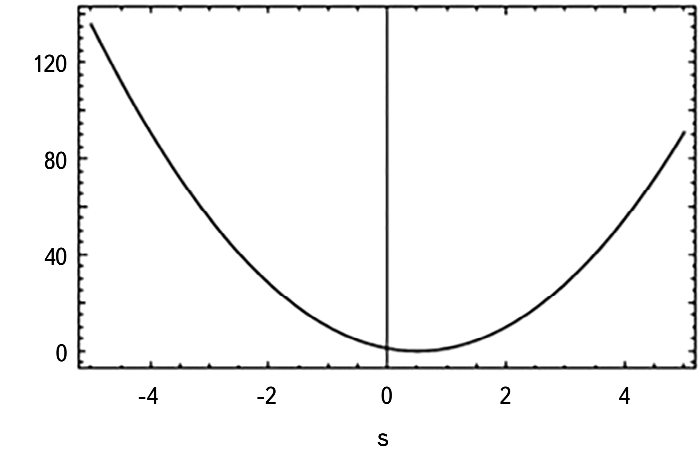

Fig.4. The plot of r versus s. From the above figure it is observed that the curve (r, s) passes through the point (1, 0). Thus, it indicates that the model corresponds to ΛCDM model at the later stage of the Universe.

Using Eq. (54) in Eq. (53) and then integrating, we get

$$\varphi = \varphi_0 + \int \left[(1+\omega_{DE}) \left\{ \frac{(3c^2 n - \beta) n^3}{8\pi (t+k_1)^2 \left[\pi\delta (t+k_1)^2 + n^2 \right]} \right\} \right]^{1/2} dt, \tag{55}$$

where φ_0 is a constant of integration.

8. *Graphical discussions*. The graphical representations of various parameters are discussed here. The numerical values used in the graphs are

$$\begin{gathered} c = 1, \quad n = 2.5, \quad \beta = 0.5, \quad k_1 = 0.03, \quad k_2 = 0.06, \\ \rho_0 = 0.2, \quad \varphi_0 = 0.1 \quad \text{and} \quad \delta = 2. \end{gathered} \tag{56}$$

9. *Conclusions*. In this paper, we have constructed MRHDE in $f(R,T)$ theory of gravity in the framework of Bianchi type VI_0 Universe. To obtain the exact solutions of the field equations, we have considered the law of variation for the Hubble parameter H as proposed by Berman [37]. We have studied the isotropy and the expansion of the universe. It is seen that the anisotropic parameter $A_p \to 0$ as $t \to \infty$ i.e., our model becomes isotropic at later age of the universe. And hence our results are in favour of the recent observational data which suggests the present-day isotropic behaviour of the universe. Also, it is observed that the deceleration parameter, $q > 0$ $(0 < n < 1)$ which implies that our universe is in a decelerating phase and $q < 0$ $(n > 1)$ indicates that our universe is in accelerating phase. In this paper, we have seen that $\omega_{DE} > -1$, which depicts that our model behaves like quintessence DE. From the study of statefinder parameters, we can

conclude that our model corresponds to ΛCDM model as cosmic time evolves. Again, it is observed that v_s^2 is negative at late times, which indicates that our investigated model is unstable. It is found from Fig.2a that WEC and DEC are satisfied whereas SEC is violated. The violation of SEC gives anti-gravitational effect for which universe gets jerk and thus our model exhibits transition from the early deceleration to present cosmic acceleration. From Fig.3a we can arrive at a conclusion that the scalar field potential $V(\varphi)$ decreases and ultimately tends to zero whereas the scalar field φ decreases and approaches to small value at the later stage of the Universe.

Thus, the physical properties of the model obtained by using Berman's law provides a very nice description of the transition from the early deceleration to present cosmic acceleration, which is an essential feature for evolution of the Universe. Moreover, the correspondence between MRHDE and quintessence scalar field is constructed in our model. The quintessence dynamics of the potential and scalar field are reconstructed which describes the current accelerating stage of the Universe. This shows that our model strongly agrees the present-day observations.

Acknowledgements. The authors are thankful to the reviewer for giving valuable suggestions to improve the work and are also thankful to the Department of Mathematics, Gauhati University for providing facilities to carry out the research work.

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МОДИФИЦИРОВАННАЯ ГОЛОГРАФИЧЕСКАЯ ТЕМНАЯ ЭНЕРГИЯ РЕНЬИ (MRHDE) В ТЕОРИИ ГРАВИТАЦИИ $f(R,T)$

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В работе исследована динамика пространства-времени типа Бьянки VI_0 в рамках теории гравитации $f(R,T)$, где R - скаляр Риччи, а T - след тензора энергии-импульса напряжения с Модифицированной голографической темной энергией Реньи (MRHDE). При конкретном выборе функционала $f(R,T)=$ $= f_1(R)+f_2(T)=\mu_1 R+\mu_2 T$, где $f_1(R)$ и $f_2(T)$ - произвольные функции от R и T - соответственно, а μ_1 и μ_2 - два параметра, мы получили точные решения модели, учитывая плотность энергии MRHDE и используя закон изменения параметра Хаббла *H*. Обнаружено, что наша модель приводит к ускоренному расширению Вселенной. Параметр EOS $\omega_{DE} > -1$ указывает, что наша космологическая модель ведет себя как модель темной энергии квинт-эссенции, которая согласуется с недавними наблюдениями. Установлено соответствие между MRHDE и квинтэссенцией темной энергии. Реконструируется квинтэссенция динамики потенциала и скалярного поля, которая иллюстрирует космическое ускорение в позднем времени. Все физические параметры рассчитываются и обсуждаются в графическом виде.

Ключевые слова: *MRHDE: $f(R,T)$ теория гравитации: квинтэссенция: параметр EOS: параметр Statefinder*

REFERENCES

1. *A.G.Riess et al.*, Astron. J., **116**, 1009, 1998.
2. *S.Perlmutter et al.*, Nature, **391**, 51, 1998.
3. *C.L.Bennett et al.*, Astrophys. J. Suppl. Ser., **148**, 1, 2003.
4. *D.N.Spergel et al.*, Astrophys. J. Suppl. Ser., **148**, 175, 2003.
5. *M.Tegmark et al.*, Phys. Rev. D, **69**, 103501, 2004.
6. *S.Weinberg*, Rev. Mod. Phys., **61**, 1, 1989.
7. *J.M.Overduin*, *F.I.Cooperstock*, Phys. Rev. D, **58**, 043506, 1998.
8. *T.Barreiro*, *E.J.Copeland*, *N.J.Nunes*, Phys. Rev. D, **61**, 127301, 2000.
9. *R.R.Caldwell*, *M.Kamionkowski*, *N.N.Weinberg*, Phys. Rev. Lett., **91**, 071301, 2003.
10. *C.Armendariz-Picon*, *V.Mukhanov*, *P.J.Steinhardt*, Phys. Rev. D, **63**, 103510, 2001.
11. *J.S.Bagla*, *H.K.Jassal*, *T.Padmanabhan*, Phys. Rev. D, **67**, 063504, 2003.
12. *t'G.Hooft*, arXiv: gr-qc/9310026, 1993.
13. *L.Susskind*, J. Math. Phys., **36,** 6377, 1995.
14. *A.G.Cohen*, *D.B.Kaplan*, *A.E.Nelson*, Phys. Rev. Lett., **82**(25), 4971, 1999.
15. *M.Li*, Phys. Lett. B, **603**, 1, 2004.
16. *S.M.Carroll*, *V.Duvvuri*, *M.Trodden et al.*, Phys. Rev. D, **70**, 043528, 2004.
17. *T.Harko*, *F.S.N.Lobo*, *S.Nojiri et al.*, Phys. Rev. D, **84**, 024020, 2011.
18. *B.Mishra*, *P.K.Sahoo*, Astrophys. Space Sci., **352**(1), 331, 2014.
19. *K.S.Adhav*, Astrophys. Space Sci., **339**, 365, 2012.
20. *M.Tavayef*, *A.Sheykhi*, *K.Bamba et al.*, Phys. Lett. B, **781**, 195, 2018.
21. *C.Tsallis*, *L.J.L.Cirto*, Eur. Phys. J. C, **73**, 2487, 2013.
22. *A.S.Jahromi et al.*, Phys. Lett. B, **780**, 21, 2018.
23. *H.Moradpour*, *S.A.Moosavi*, *I.P.Lobo et al.*, Eur. Phys. J. C, **78**(10), 829, 2018.
24. *H.Moradpour*, *A.Bonilla*, *E.M.C.Abreu et al.*, Phys. Rev. D, **96**(12), 123504, 2017.

25. *H.Moradpour*, Int. J. Theor. Phys., **55**(9), 4176, 2016.
26. *N.Komatsu*, Eur. Phys. J. C, **77**(4), 229, 2017.
27. *S.Ghaffari*, *A.H.Ziaie*, *V.B.Bezerra et al.*, Mod. Phys. Lett. A, **35**(1), 1950341, 2020.
28. *V.C.Dubey*, *U.K.Sharma*, *A.A.Mamon*, Adv. High Energy Phys., 2021, doi: 10.1155/2021/6658862.
29. *A.Saha*, *S.Ghose*, *A.Chanda et al.*, arXiv: 2101.04060v1 [gr-qc], 2021.
30. *U.Y.Divya Prasanthi*, *Y.Aditya*, Results Phys., **17**, 103101, 2020.
31. *V.C.Dubey*, *A.K.Mishra*, *U.K.Sharma*, Astrophys. Space Sci., **365**, 129, 2020.
32. *S.Chen*, *J.Jing*, Phys. Lett. B, **679**, 144, 2009.
33. *C.L.Bennett et al.*, Astrophys. J. Suppl. Ser, **148**, 1, 2003.
34. *Ö.Akarsu*, *C.B.Kilinc*, Astrophys. Space Sci., **326**, 315, 2010.
35. *R.Chaubey*, *A.K.Shukla*, Astrophys. Space Sci., **343**, 415, 2013.
36. *L.D.Landau*, *E.M.Lifshitz*, The Classical theory of Fields (Butterworth Heinemann, Oxford), 1998.
37. *M.S.Berman*, Nuovo Cimento B, **74**, 182, 1983.
38. *K.S.Adhav*, Int. J. Astron. Astrophys., **1**, 204, 2011.
39. *V.Sahni*, *T.D.Saini*, *A.A.Starobinsky et al.*, JETP Lett., **77**, 201, 2003.
40. *Y.S.Myung*, Phys. Lett. B, **652**, 223, 2007.
41. *A.Sangwan*, *A.Mukherjee*, *H.K.Jassal*, JCAP, 01, 018, 2018.