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DYNAMICAL BEHAVIOUR OF COUPLED MAGNETIZED DARK ENERGY IN LYRA'S GEOMETRY

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The axially symmetric space-time with magnetized anisotropic generalized ghost pilgrim dark energy has been investigated in Lyra's geometry. To get a determinate solution, we considered the expansion scalar θ in the model is proportional to shear scalar σ. We found that the equation of state parameter of generalized ghost pilgrim dark energy behaves like a phantom dark energy. By stability analysis our model found to be stable. We have studied the correspondence between the models of generalized ghost pilgrim dark energy and polytropic gas dark energy. Accordingly, the potential and dynamics of scalar field of polytropic gas is reconstructed. Moreover, we have calculated various physical and kinematical parameters of the model and observed that these are compatible with recent observations.

Keywords: *axially symmetric space-time: generalized ghost pilgrim dark energy: Lyra's geometry*

1. *Introduction*. Recent observations of two teams Supernova Cosmology Project and the High-Z Supernova Team indicate that expansion of the universe is accelerating [1-7]. An exotic component with large negative pressure is termed as dark energy (DE) that produces repulsive force which gives rise to the current accelerating expansion of the universe. One of the simplest candidate for the dark energy is the cosmological constant Λ [8-11] having equation of state (EoS) parameter $\omega = -1$, but it has coincidence and fine tuning problems [12]. In order to avoid such problems, many authors [13-15] have been taken dynamical EoS parameter $\omega = p/\rho$ (<0). The examples of such dark energy candidates are the quintessence [16,17], phantom [18-20], K-essence [21-23], tachyon [24,25], quintom [26,27], anisotropic DE [28,29] and so on.

Due to presence of phantom DE in the universe will force it towards big rip singularity. This indicates that the phantom-like universe possesses ability to prevent the black hole (BH) formation. Wei [30] has introduced cosmological parameters for pilgrim dark energy (PDE) model with Hubble horizon and provided different possibilities for avoiding the BH formation through PDE parameter. To make the BH free phantom universe some authors suggested different possible ways. The interacting PDE in flat as well as non-flat universe with different infrared (IR) cutoffs have investigated by Sharif and Jawad [31,32].

Sharif and Rani [33], Jawad [34] and Jawad and Debnath [35] have investigated the PDE cosmological models in various modified theories of gravitation. Jawad and Majeed [36] have investigated the correspondence of PDE with scalar field models. The aspects of some new versions of PDE in DGP braneworld have studied by Jawad et al. [37]. Sheykhi and Movahed [38], Feng et al. [39], Zubair and Abbas [40], Sadeghi and Khurshudhan [41], Honarvaryan et al. [42], Hosseinkhani et al. [43,44] have studied the various aspects of ghost and generalized ghost dark energy (GGDE) models. The GGDE density in terms of PDE is called as generalized ghost pilgrim dark energy (GGPDE). Jawad [45], Sharif and Jawad [46], Sharif and Nazir [47], Rao and Prasanti [48], Santhi et al. [49,50] have investigated the GGPDE model in different contexts.

The Einstein [51] in 1916 introduced his theory of general relativity (GR) which provides a geometrical description of gravitation. Many physicists attempted to generalize the geometrical behaviour of the gravitation to include a geometrical description of electromagnetism. Based on this Weyl [52] proposed a more general theory by formulating a new kind of gauge theory involving metric tensor to geometrize gravitation and electromagnetism. Due to non-integrability of length of vector under parallel displacement, Weyl theory was criticized.

To remove the non-integrability condition Lyra [53] developed a modification of Riemannian geometry by introducing a gauge function into the structure less manifold which is known as Lyra's Geometry. Halford [54] pointed out that the constant displacement vector field β in Lyra geometry behaves like a cosmological constant Λ in general relativity. He has also observed that the scalar-tensor treatment based in Lyra geometry predicts the same effects, within observational limits, as in Einstein's theory (Halford [55]). Many authors investigated the cosmological models in Lyra's geometry. Adhav [56] has studied the Bianchi type-I metric with anisotropic DE for exponential volumetric expansion. Samanta [57] has investigated the Bianchi type-III cosmological model with anisotropic DE in the presence of a single imperfect fluid with a dynamical anisotropic equation of state parameter. Singh et al. [58] discussed the anisotropic Bianchi type-II DE cosmological model with constant deceleration parameter. Pawar et al. [59] have investigated a magnetized dark energy of Bianchi type-VI_0 space time with time dependent cosmological term for uniform and time varying displacement field. Katore et al. [60] investigated the Kaluza Klein universe with magnetized anisotropic dark energy in general relativity and Lyra manifold. A Kantowski-Sachs cosmological model in the presence of an anisotropic dark energy is investigated by Shri Ram et al. [61]. Katore et al. [62] have studied the Bianchi type-VI_0 holographic dark energy models in general relativity and Lyra's geometry. Shri Ram et al. [63] discussed the Kantowski-Sachs cosmological model with anisotropic DE.

To understand the early stages of evolution of the universe the study of anisotropic models are very important. Recently, many authors have investigated dark energy cosmological models with anisotropic background. Das et al. [64] have investigated the magnetized anisotropic ghost dark energy cosmological model in general relativity and it is fond that the anisotropy of the universe and that of ghost dark energy tends to zero at late times and the universe becomes spatially homogeneous, isotropic and flat. Santhi et al. [65] have studied the anisotropic GGPDE model in general relativity and observed that the investigated work favors the PDE phenomenon.

Motivated by the above investigations, here we take up the study of the anisotropic GGPDE in presence of magnetic field for axially symmetric space time within the framework of Lyra's geometry. The format of the paper is as follows. In section 2, the metric and field equations are described. Section 3 is devoted to the solution of the field equations with physical and geometrical properties of the model. In section 4, the statefinder parameters are adopted to characterize different phases of the universe. Section 5 deals with the stability analysis. We have established a correspondence between GGPDE model and polytropic gas dark energy model in section 6. In last section we have summarized the results.

2. *Metric and field equations*. We consider the anisotropic and axially symmetric space-time (Bhattacharya and Karade [66]) as

$$ds^2 = dt^2 - A^2(t)\left[d\chi^2 + f^2(\chi)d\Phi^2\right] - B^2(t)dz^2 , \tag{1}$$

with the convention $x^1 = \chi$, $x^2 = \Phi$, $x^3 = z$, $x^4 = t$ and A, B are functions of time t only while f is a function of the coordinate χ alone.

The Einstein's modified field equation in normal gauge for Lyra's manifold obtained by Sen [67] is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\,\phi_k\phi^k = -\left(T_{ij} + \bar{T}_{ij}\right) - E_{ij} . \tag{2}$$

Here $\phi_i = (0, 0, 0, \beta(t))$ is the displacement vector, where $\beta(t)$ being time-dependent gauge function. T_{ij} and $\bar{T}_{ij}$ are the energy momentum tensor for matter (cold dark matter) and GGPDE which are respectively given by

$$T_{ij} = \mathrm{diag}[1, 0, 0, 0]\rho_m \tag{3}$$

and

$$\bar{T}_{ij} = \mathrm{diag}[1, -\omega_G, -\omega_G, -(\omega_G + \delta)]\rho_G , \tag{4}$$

where ρ_m and ρ_G are the energy densities of matter and GGPDE while ω_G is the EoS parameter of GGPDE. The skewness parameter δ is the deviations from ω_G in the direction of z axis.

The energy momentum tensor for magnetic field is given by

$$E_{ij} = \frac{1}{4\pi}\left(F_{ik}F_{jl}g^{kl} - \frac{1}{4}g_{ij}F^{kl}F_{kl}\right), \tag{5}$$

where F_{ij} is the electromagnetic field tensor which satisfied the Maxwell's equation

$$F_{[i,j,k]} = 0, \left(F^{ij}\sqrt{-g}\right)_{;j} = 0. \tag{6}$$

In this article we consider the electromagnetic field along the x-direction. The only non-vanishing component of electromagnetic field tensor F_{ij} is

$$F_{23} = I\,(\text{constant}). \tag{7}$$

In a co-moving co-ordinate system, the field equations for axially symmetric space-time are obtain as

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2}\left(\frac{f''}{f}\right) - \frac{3}{4}\beta^2 = \rho_m + \rho_G + \frac{I}{A^2B^2}, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = -\omega_G\rho_G + \frac{I}{A^2B^2}, \tag{9}$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 - \frac{1}{A^2}\left(\frac{f''}{f}\right) + \frac{3}{4}\beta^2 = -(\omega_G + \delta)\rho_G - \frac{I}{A^2B^2}, \tag{10}$$

$$\frac{3}{2}\beta\dot{\beta} + \frac{3}{2}\beta^2\left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0. \tag{11}$$

Here over head dot and dash denotes the ordinary differentiation with respect to t and χ respectively.

Using Eqs. (8) and (9), the function dependence of the metric is given by

$$\frac{f''}{f} = k_1^2, \tag{12}$$

where k_1^2 is a constant.

If $k_1 = 0$, then

$$f(\chi) = k_2\chi + k_3, \tag{13}$$

where k_2 and k_3 are integrating constants.

Without loss of generality, by taking $k_2 = 1$ and $k_3 = 0$ in Eq. (13), i.e. $f(\chi) = \chi$, the field Eqs. (8) to (11) reduces to

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} - \frac{3}{4}\beta^2 = \rho_m + \rho_G + \frac{I}{A^2B^2}, \tag{14}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = -\omega_G\rho_G + \frac{I}{A^2B^2}, \tag{15}$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 + \frac{3}{4}\beta^2 = -(\omega_G + \delta)\rho_G - \frac{I}{A^2B^2}, \tag{16}$$

$$\beta\dot{\beta}+\beta^2\left(\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}\right)=0. \tag{17}$$

Some physical parameters are defined as follows:

For axially symmetric space-time the directional and mean Hubble parameters are defined as

$$H_x=H_y=\frac{\dot{A}}{A}, \quad H_z=\frac{\dot{B}}{B}, \tag{18}$$

$$H=\frac{1}{3}\left(\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}\right). \tag{19}$$

The spatial volume V is given by

$$V=A^2B. \tag{20}$$

The scalar expansion θ, shear scalar σ, anisotropy parameter Δ and deceleration parameter q are defined as

$$\theta=3H=\left(\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}\right), \tag{21}$$

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-3H^2\right), \tag{20}$$

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2, \tag{22}$$

$$q=-1+\frac{d}{dt}\left(\frac{1}{H}\right). \tag{23}$$

3. *Solutions of field equations*. Here we have four linearly independent equations (14)-(17) and seven variables ($A, B, \beta, \rho_m, \rho_G, \omega_G, \delta$), thus the system is initially undetermined and hence we need three additional conditions to solve it. In order to solve the field equations completely, we assume the expansion scalar θ in the model is proportional to shear scalar σ (Thorne [68], Katore et al. [69]), which leads to

$$A=B^n, \tag{24}$$

where $n\neq 1$ is a positive constant.

The generalized ghost pilgrim dark energy (Sharif and Nazir [70]) is defined as

$$\rho_G=\left(\alpha_1H+\alpha_2H^2\right)^\gamma, \tag{25}$$

where γ is the dimensionless constant.

Subtracting Eq. (16) from Eq. (15), we obtain

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}+\frac{\dot{A}}{A}\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)=\delta\rho_G+\frac{2I}{A^2B^2}. \tag{26}$$

On integrating, we get

$$\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)=\frac{c_0}{V}e^{\int\left(\left(\delta\rho_G+\frac{2I}{A^2B^2}\right)\Big/\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)\right)dt}. \tag{27}$$

Following Das et al. [71], we choose integral of the right hand side of Eq. (27) in such a way that Eq. (27) is integrable. So without loss of generality, we use the condition

$$\delta\rho_G+\frac{2I}{A^2B^2}=\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right). \tag{28}$$

Using Eqs. (27) and (28), we obtain the expression for metric coefficients as

$$B=\left(c_2e^t+c_3\right)^{1/(2n+1)}, \tag{29}$$

$$A=\left(c_2e^t+c_3\right)^{n/(2n+1)}, \tag{30}$$

where $c_2=c_0(2n+1)/(1-n)$, $c_3=c_1(2n+1)$ and c_0, c_1 are constants of integration.

Using Eq. (17), the displacement vector β is

$$\beta=\frac{\beta_0}{\left(c_2e^t+c_3\right)}, \tag{31}$$

where β_0 is constant of integration.

The expression for the generalized mean Hubble parameter, spatial volume, anisotropy parameter, shear scalar and deceleration parameter for the model (1) are respectively given by

$$H=\frac{c_2e^t}{3\left(c_2e^t+c_3\right)}, \tag{32}$$

$$V=\left(c_2\,e^t+c_3\right), \tag{33}$$

$$\Delta=\frac{2(n-1)^2}{(2n+1)^2}, \tag{34}$$

$$\sigma^2=\frac{(n-1)^2\,c_2^2\,e^{2t}}{3(2n+1)^2\left(c_2\,e^t+c_3\right)^2}, \tag{35}$$

$$q=-\left(1+\frac{3c_3}{c_2\,e^t}\right). \tag{36}$$

From Fig.1 it is observed that the displacement vector β is decreasing function of time and tends to a small positive value at late time, which similar with the result of Halford [54] and with the recent observations [3-6,72] leading to the conclusion that Λ is decreasing function of *t*. The anisotropy parameter Δ is constant throughout the evolution of the universe and hence given model is

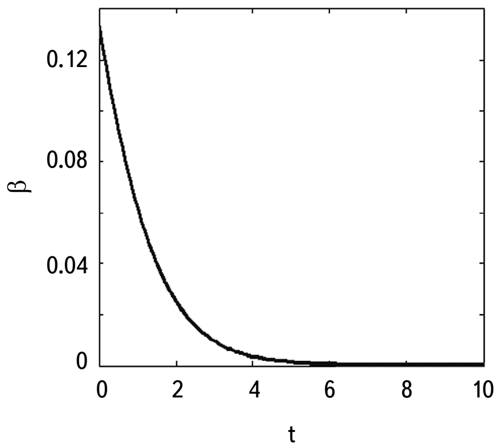

Fig.1. The plot of displacement vector β versus time t with $c_2 = 5$, $c_3 = 2.5$, $\beta_0 = 1$.

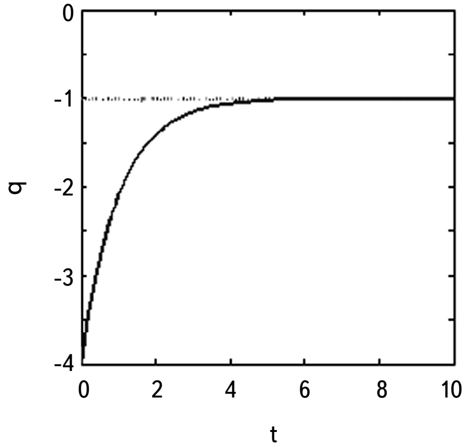

Fig.2. The plot of deceleration parameter q versus time t.

anisotropic. The sign of q indicates whether the model inflates or not. The positive sign indicates decelerating universe whereas negative sign indicates accelerating universe. From Fig.2 it is found that at initial epoch the deceleration parameter q is negative and after some finite time it tends to a constant value -1. This indicates that our universe is accelerating throughout the evolution of the universe. In our model the value of deceleration parameter is consistent with the observations of type Ia Supernovae [5,72-74].

Using Eqs. (25) and (32), we obtain the energy density of GGPDE as

$$\rho_G = \left(\frac{\alpha_1 c_2 e^t}{3\left(c_2 e^t + c_3\right)} + \frac{\alpha_2 c_2^2 e^{2t}}{9\left(c_2 e^t + c_3\right)^2} \right)^{\gamma}. \tag{37}$$

From Eqs. (14) and (37), the energy density of matter is

$$\begin{aligned}\rho_m = &\frac{n(n+2) c_2^2 e^{2t}}{(2n+1)^2 \left(c_2 e^t + c_3\right)^2} - \frac{3\beta_0^2}{4\left(c_2 e^t + c_3\right)^2} - \\ &- \left(\frac{\alpha_1 c_2 e^t}{3\left(c_2 e^t + c_3\right)} + \frac{\alpha_2 c_2^2 e^{2t}}{9\left(c_2 e^t + c_3\right)^2} \right)^{\gamma} - \frac{I}{\left(c_2 e^t + c_3\right)^{2(n+1)/(2n+1)}}.\end{aligned} \tag{38}$$

From Eqs. (15) and (37), the EoS parameter of GGPDE is

$$\omega_G = \frac{\dfrac{n(n+2)c_2^2 e^{2t}}{(2n+1)^2 (c_2 e^t + c_3)^2} - \dfrac{(n+1)c_2 e^t}{(2n+1)(c_2 e^t + c_3)} - \dfrac{3\beta_0^2}{4(c_2 e^t + c_3)^2} + \dfrac{I}{(c_2 e^t + c_3)^{2(n+1)/(2n+1)}}}{\left(\dfrac{\alpha_1 c_2 e^t}{3(c_2 e^t + c_3)} + \dfrac{\alpha_2 c_2^2 e^{2t}}{9(c_2 e^t + c_3)^2}\right)^{\gamma}}. \tag{39}$$

The behaviour of EoS parameter of GGPDE is shown in Fig.3 for different values of PDE parameter γ. It is observed that, the EoS parameter is decreasing function of time and later on it tends to some constant value. The EoS parameter is always negative and less than -1. Thus the EoS parameter behaves like phantom

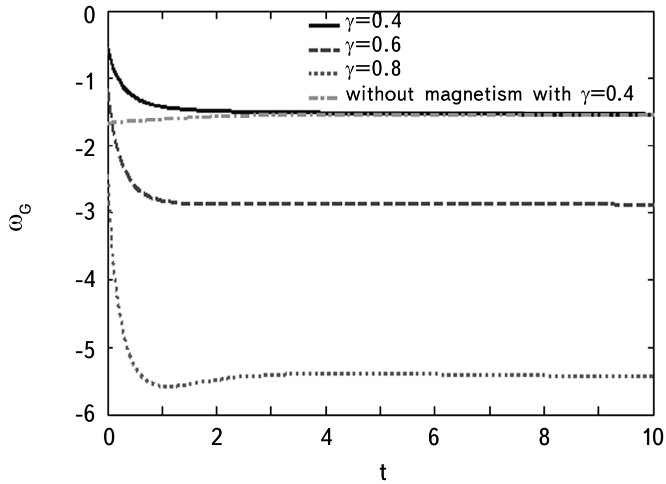

Fig.3. The plot of EoS parameter ω_G versus time t with $n = 0.5$, $c_2 = 5$, $c_3 = 2.5$, $I = \beta_0 = 1$.

DE throughout the evolution of universe and goes toward the aggressive phantom region ($\omega_G << -1$) as PDE parameter γ increases. Hence EoS parameter of GGPDE satisfy PDE phenomenon. From figure it is also observed that the EoS parameter of GGPDE is affected due to magnetism.

From Eqs. (16) and (37), the skewness parameter δ is

$$\delta = \frac{\dfrac{-n(n-1)c_2 e^t}{(2n+1)(c_2 e^t + c_3)} - \dfrac{2I}{(c_2 e^t + c_3)^{2(n+1)/(2n+1)}}}{\left(\dfrac{\alpha_1 c_2 e^t}{3(c_2 e^t + c_3)} + \dfrac{\alpha_2 c_2^2 e^{2t}}{9(c_2 e^t + c_3)^2}\right)^{\gamma}}. \tag{40}$$

The evolution of the skewness parameter δ versus cosmic time t with and without magnetism is shown in Fig.4. The skewness parameter increases as time increases, after some finite time it attains maximum value and then becomes constant throughout the evolution of the universe. Also it is observed that the

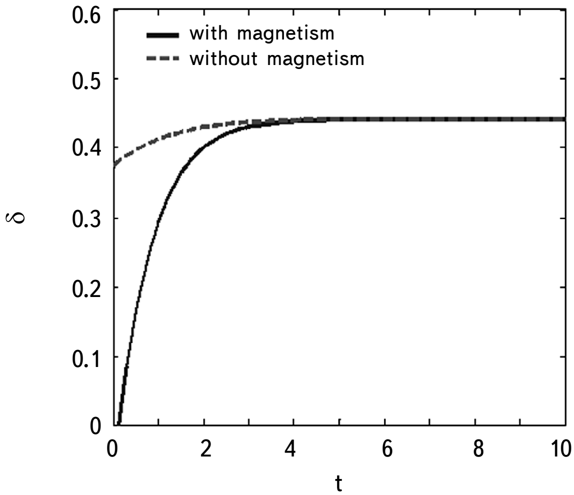

Fig.4. The plot of skewness parameter δ versus time t with and without magnetism for $n = 0.5$, $c_2 = 5$, $c_3 = 2.5$, $\beta_0 = 1$, $\gamma = 0.4$.

skewness parameter is affected due to magnetic field.

Let $\bar{r}$ be the coincidence parameter and it is defined as $\bar{r} = \rho_G/\rho_m$ i.e. the ratio of the DE density parameter and matter energy density parameter. Using Eqs. (37) and (38), the coincidence parameter becomes

$$\bar{r} = \frac{\left(\frac{\alpha_1 c_2 e^t}{3\left(c_2 e^t + c_3\right)} + \frac{\alpha_2 c_2^2 e^{2t}}{9\left(c_2 e^t + c_3\right)^2}\right)^{\gamma}}{\frac{n(n+2)c_2^2 e^{2t}}{(2n+1)^2\left(c_2 e^t + c_3\right)^2} - \frac{3\beta_0^2}{4\left(c_2 e^t + c_3\right)^2} - \left(\frac{\alpha_1 c_2 e^t}{3\left(c_2 e^t + c_3\right)} + \frac{\alpha_2 c_2^2 e^{2t}}{9\left(c_2 e^t + c_3\right)^2}\right)^{\gamma} - \frac{I}{\left(c_2 e^t + c_3\right)^{\frac{2(n+1)}{(2n+1)}}}}. \tag{41}$$

Fig.5 shows the variation of coincidence parameter $\bar{r}$ with respect to cosmic

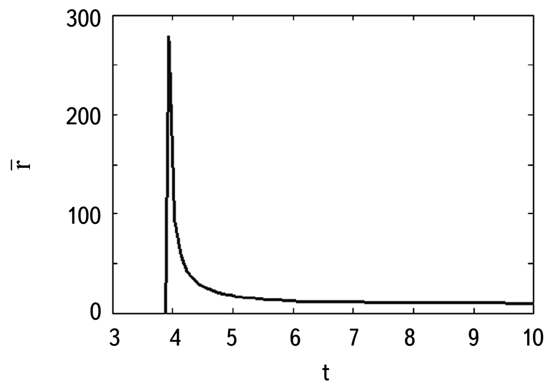

Fig.5. The plot of coincidence parameter $\bar{r}$ versus time t with $n = 0.5$, $c_2 = 5$, $c_3 = 2.5$, $I = \beta_0 = 1$.

time t. The coincidence parameter $\bar{r}$ varies at early stage of evolution, but after some finite time it converges to a constant value and remains constant throughout the evolution of the universe, thereby avoiding the coincidence problem (unlike ΛCDM).

4. *Statefinder parameters {r, s}*. In order to get an accurate analysis of different DE models Sahni et al. [75] has introduced a new geometrical diagnostic named as statefinder pair $\{r, s\}$ which is constructed from scale factor a and its derivative upto third order. The statefinder parameters are defined as follows

$$r = \frac{\dddot{a}}{aH^3}, \quad s = \frac{r-1}{3(q-1/2)}. \tag{42}$$

These parameters allow us to characterize the properties of dark energy. Using these parameters one can describe the well-known region as $(r, s) = (1, 0)$ indicates ΛCDM limit and $(r, s) = (1, 1)$ indicates CDM limit, while $s > 0$ and $r < 1$ corresponds to region of phantom and quintessence dark energy era.

The statefinder parameters r and s are given by

$$r = 1 + \frac{9c_3^2}{c_2^2 e^{2t}}, \quad s = \frac{-2c_3^2}{c_2 e^t \left(c_2 e^t + c_3\right)}. \tag{43}$$

The relation between r and s is given by

$$s = -\frac{r-1}{9/2 + 3(r-1)^{1/2}}. \tag{44}$$

From Fig.6, it is observed that the curve passes through a phase ΛCDM at the point $(r = 1,\ s = 0)$, this implies that at late time cosmic evolution the DE dominates and drives the cosmic acceleration.

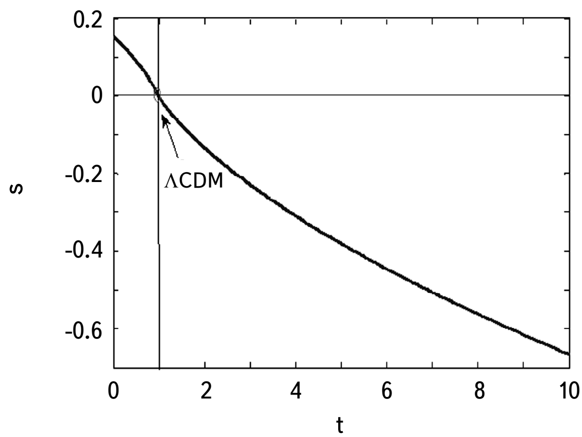

Fig.6. The plot of statefinder parameter s versus r.

5. *Stability analysis*. To find the stability condition of many DE cosmological models, we use squared speed of sound υ_s^2. A positive value of squared speed of sound represents a stable model while the negative value indicates the instability of model. A squared speed of sound υ_s^2 is defined as

$$\upsilon_s^2 = \frac{\dot{p}_G}{\dot{\rho}_G}. \tag{45}$$

The squared speed of sound for axially symmetric space-time is given by

$$\upsilon_s^2 = \frac{\left\{\begin{matrix} \dfrac{-2n(n+2)c_2^3e^{3t}}{(2n+1)^2\left(c_2e^t+c_3\right)^3} + \dfrac{3\beta_0^2 c_2e^t}{2\left(c_2e^t+c_3\right)^3} + \dfrac{\left(4n^2+7n+1\right)c_2^2e^{2t}}{(2n+1)^2\left(c_2e^t+c_3\right)^2} \\ -\dfrac{(n+1)c_2e^t}{(2n+1)\left(c_2e^t+c_3\right)} - \dfrac{2I(n+1)c_2e^t}{(2n+1)\left(c_2e^t+c_3\right)^{(2(n+1)/(2n+1))+1}} \end{matrix}\right\}}{\gamma\left(\dfrac{\alpha_1 c_2e^t}{3\left(c_2e^t+c_3\right)} + \dfrac{\alpha_2 c_2^2e^{2t}}{9\left(c_2e^t+c_3\right)^2}\right)^{\gamma-1}\left(\dfrac{-2\alpha_2 c_2^3e^{3t}}{9\left(c_2e^t+c_3\right)^3} + \dfrac{\left(2\alpha_2-3\alpha_1\right)c_2^2e^{2t}}{9\left(c_2e^t+c_3\right)^2} + \dfrac{\alpha_1 c_2e^t}{3\left(c_2e^t+c_3\right)}\right)}. \tag{46}$$

The behaviour of squared speed of sound υ_s^2 with respect to time for different values of PDE parameter is shown in Fig.7. It is observed that in our model the squared speed of sound remains positive (i.e. $\upsilon_s^2 > 0$), hence our model is stable in accelerated expanding universe.

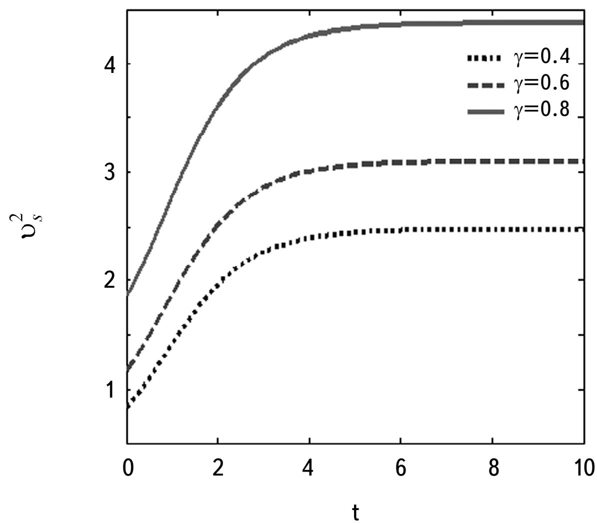

Fig.7. The plot of squared speed of sound υ_s^2 versus time t with and without magnetism for $n = 0.5$, $c_2 = 5$, $c_3 = 2.5$, $I = \beta_0 = 1$.

6. *Correspondence between the GGPDE and polytropic gas DE model*. The polytropic gas equation of state parameter [76] is defined as

$$p_{pg} = K \rho_{pg}^{1+1/\varepsilon}, \tag{47}$$

where K is the positive constant and ε is the polytropic index.

The energy density of polytropic gas is given by

$$\rho_{pg} = \left(Da^{3/\varepsilon} - K\right)^{-\varepsilon}, \tag{48}$$

where $D > 0$ is constant of integration and a is the average scale factor.

Using Eqs. (47) and (48), the EoS parameter of polytropic gas is obtained as

$$\omega_{pg} = \frac{p_{pg}}{\rho_{pg}} = -1 - \frac{Da^{3/\varepsilon}}{K - Da^{3/\varepsilon}}. \tag{49}$$

Following Karami [76], if the polytropic gas treating as an ordinary scalar field then the energy density and pressure of the scalar field are defined as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{50}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{51}$$

Using Eqs. (47)-(51), the scalar potential and the kinetic energy terms for the polytropic gas are given by

$$V(\phi) = \frac{\frac{1}{2}Da^{3/\varepsilon} - K}{\left(Da^{3/\varepsilon} - K\right)^{\varepsilon+1}} \tag{52}$$

and

$$\dot{\phi}^2 = \frac{Da^{3/\varepsilon}}{\left(Da^{3/\varepsilon} - K\right)^{\varepsilon+1}}. \tag{53}$$

We consider that the GGPDE density is equivalent to the energy density of polytropic gas. Hence using Eqs. (37) and (48), we obtain

$$\rho_G = \left(\frac{\alpha_1 c_2 e^t}{3\left(c_2 e^t + c_3\right)} + \frac{\alpha_2 c_2^2 e^{2t}}{9\left(c_2 e^t + c_3\right)^2}\right)^\gamma = \left(Da^{3/\varepsilon} - K\right)^{-\varepsilon}. \tag{54}$$

Comparing Eqs (39) and (49), the EoS parameter is obtain as

$$\omega_G = \frac{\left\{\begin{array}{l} \dfrac{n(n+2)c_2^2 e^{2t}}{(2n+1)^2\left(c_2 e^t + c_3\right)^2} - \dfrac{(n+1)c_2 e^t}{(2n+1)\left(c_2 e^t + c_3\right)} \\ - \dfrac{3\beta_0^2}{4\left(c_2 e^t + c_3\right)^2} + \dfrac{I}{\left(c_2 e^t + c_3\right)^{2(n+1)/(2n+1)}} \end{array}\right\}}{\left(\dfrac{\alpha_1 c_2 e^t}{3\left(c_2 e^t + c_3\right)} + \dfrac{\alpha_2 c_2^2 e^{2t}}{9\left(c_2 e^t + c_3\right)^2}\right)^\gamma} = -1 - \frac{Da^{3/\varepsilon}}{K - Da^{3/\varepsilon}}. \tag{55}$$

Solving Eqs. (54) and (55), we get

$$D=\frac{\left\{\begin{aligned}&\frac{n(n+2)c_2^2e^{2t}}{(2n+1)^2\left(c_2e^t+c_3\right)^2}-\frac{(n+1)c_2e^t}{(2n+1)\left(c_2e^t+c_3\right)}-\frac{3\beta_0^2}{4\left(c_2e^t+c_3\right)^2}\\&+\frac{I}{\left(c_2e^t+c_3\right)^{2(n+1)/(2n+1)}}+\left(\frac{\alpha_1c_2e^t}{3\left(c_2e^t+c_3\right)}+\frac{\alpha_2c_2^2e^{2t}}{9\left(c_2e^t+c_3\right)^2}\right)^{\gamma}\end{aligned}\right\}}{\left(c_2e^t+c_3\right)^{1/\varepsilon}\left(\frac{\alpha_1c_2e^t}{3\left(c_2e^t+c_3\right)}+\frac{\alpha_2c_2^2e^{2t}}{9\left(c_2e^t+c_3\right)^2}\right)^{\gamma+\gamma/\varepsilon}} \tag{56}$$

and

$$K=\frac{\frac{n(n+2)c_2^2e^{2t}}{(2n+1)^2\left(c_2e^t+c_3\right)^2}-\frac{(n+1)c_2e^t}{(2n+1)\left(c_2e^t+c_3\right)}-\frac{3\beta_0^2}{4\left(c_2e^t+c_3\right)^2}+\frac{I}{\left(c_2e^t+c_3\right)^{2(n+1)/(2n+1)}}}{\left(\frac{\alpha_1c_2e^t}{3\left(c_2e^t+c_3\right)}+\frac{\alpha_2c_2^2e^{2t}}{9\left(c_2e^t+c_3\right)^2}\right)^{\gamma+\gamma/\varepsilon}}. \tag{57}$$

Using the value of D and K in Eqs. (52) and (53), we get the kinetic term and scalar potential as

$$\begin{aligned}V(\phi)=&\frac{-n(n+2)c_2^2e^{2t}}{(2n+1)^2\left(c_2e^t+c_3\right)^2}+\frac{(n+1)c_2e^t}{(2n+1)\left(c_2e^t+c_3\right)}+\frac{3\beta_0^2}{4\left(c_2e^t+c_3\right)^2}\\&-\frac{I}{\left(c_2e^t+c_3\right)^{2(n+1)/(2n+1)}}+\left(\frac{\alpha_1c_2e^t}{3\left(c_2e^t+c_3\right)}+\frac{\alpha_2c_2^2e^{2t}}{9\left(c_2e^t+c_3\right)^2}\right)^{\gamma}.\end{aligned} \tag{58}$$

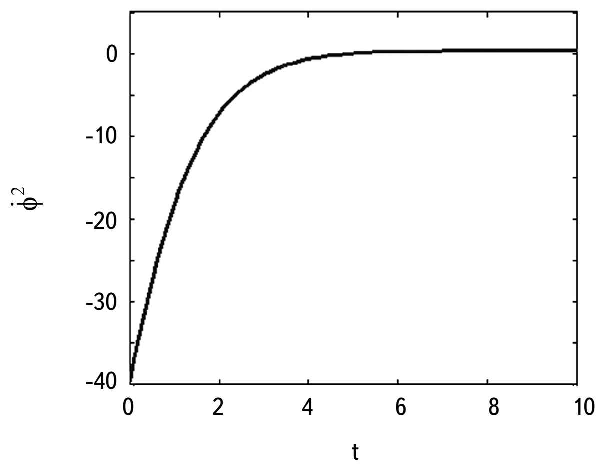

Fig.8. The plot of squared speed of sound $\dot{\phi}^2$ versus time t with $n=0.5$, $c_2=5$, $c_3=2.5$, $I=1$, $\beta_0=10$, $\gamma=0.8$.

and

$$\phi=\int\left\{\begin{array}{l}\dfrac{n(n+2)c_2^2\,e^{2t}}{(2n+1)^2\left(c_2\,e^t+c_3\right)^2}-\dfrac{(n+1)c_2\,e^t}{(2n+1)\left(c_2\,e^t+c_3\right)}-\dfrac{3\beta_0^2}{4\left(c_2\,e^t+c_3\right)^2}\\ +\dfrac{I}{\left(c_2\,e^t+c_3\right)^{2(n+1)/(2n+1)}}+\left(\dfrac{\alpha_1\,c_2\,e^t}{3\left(c_2\,e^t+c_3\right)}+\dfrac{\alpha_2\,c_2^2\,e^{2t}}{9\left(c_2\,e^t+c_3\right)^2}\right)^{\gamma}\end{array}\right\}^{1/2}dt. \tag{59}$$

From Fig.8 we observed that $\dot{\phi}^2<0$ for particular values of β_0 and γ, which implies that the scalar field ϕ has a phantomic behaviour. This type of potential can produce an accelerated expansion of the universe. Hence we can establish a correspondence between the GGPDE and polytropic gas and describe GGPDE by making use of polytropic gas.

7. *Conclusion*. In the present work, we have studied the physical and geometrical behaviour of magnetized and anisotropic GGPDE within the framework of Lyra's geometry. We have studied some physical aspect of the model in presence and absence of magnetic field. The deceleration parameter q describes that the present universe is undergoing an accelerated expansion. It is observed that the EoS parameter of GGPDE behaves like phantom dark energy throughout the evolution of universe which satisfies the PDE phenomenon hence it possesses the ability of prevention of black hole formation [30]. Statefinder diagnostic pair $\{r, s\}$ is applied to the model in order to distinguish our DE model with other existing DE model. For stability analysis, we calculate the squared speed of sound and found that our model is stable. A correspondence between the GGPDE model and polytropic gas DE model is established. We have also reconstructed the potential and dynamics of the polytropic scalar field which shows the accelerated expansion of the universe.

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ДИНАМИЧЕСКОЕ ПОВЕДЕНИЕ СВЯЗАННОЙ МАГНЕТИЗИРОВАННОЙ ТЕМНОЙ ЭНЕРГИИ В ГЕОМЕТРИИ ЛИРЫ

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В рамках геометрии Лиры исследовано осесимметричное пространство-время с намагниченной анизотропной обобщенной странствующей призрачной темной энергией (ghost pilgrim). Чтобы получить определенное решение, мы считали, что скаляр расширения θ в модели пропорционален скалярному сдвигу σ. Мы обнаружили, что уравнение параметра состояния обобщенной темной энергии странствующего призрака (ghost pilgrim) ведет себя как призрачная (phantom) темная энергия. Анализ стабильности показал, что наша модель стабильна. Мы изучили соответствие между моделями обобщенной темной энергии странного призрака (ghost pilgrim) и темной энергии политропного газа. Соответственно, реконструируются потенциал и динамика скалярного поля политропного газа. Кроме того, мы рассчитали различные физические и кинематические параметры модели и обнаружили, что они совместимы с недавними наблюдениями.

Ключевые слова: *осесимметричное пространство-время: обобщенная призрачная странствующая темная энергия: геометрия Лиры*

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