АСТРОФИЗИКА

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EFFECT OF THE KERR METRIC ON PHOTOSPERIC RADIUS EXPANSION IN X RAY BURST

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Main objective of this paper is to study general relativistic effects on the photospheric radius expansion during X-ray burst. We examine how the Kerr metric causes a shift in the effective temperature and radiation flux with respect to the Schwarzschild values during mass accretion onto a neutron star or a black hole resulting in the X-ray burst. Spin of the compact object is used up to the maximal Kerr limit $\chi = 0.99$ with different latitude of accretion emission. The amplitude of temperature shift relative to the Schwarzschild case is found to be $\delta T/T \approx -(10^{-3} - 10^{-4})$ for the range $\chi = 0.1 - 0.99$ at latitudes, $\theta = 0^{\circ}$, 30°, 45° and 88°. The ratio of emission flux in Kerr metric to that in the Schwarzschild metric, $F(K)/F(S)$ is found to be less than unity. It goes up to maximum of 0.9 for the lowest non-zero value of the spin parameter (i.e. 0.1). For the maximal Kerr limit, $\chi = 0.99$, it saturates near 0.8. This effect is more prominent towards the pole. This reduction in temperature and flux is found to be consistent with the absence of photospheric radius expansion in the X Ray burst LMXB 4U 1608-52, observed by NuSTAR. Although this is not uniquely ascribed to the metric, it is believed that spacetime metric effect in the burst phenomena can be used as a probe for testing general relativity. Also, the shift in temperature or the radiation flux might have observable signature in the element synthesis processes in such environments.

Key words: *general relativity: metric: spin: x-ray burst*

1. *Introduction*. Photospheric radius expansion (PRE) burst is a well-studied phenomenon associated with X-ray binaries hosting a gravitationally collapsed object such as a black hole or a neutron star (henceforth called as compact primary). Near Eddington luminosity during accretion emission causes the photospheric layers to expand and cool [1,2]. Recent developments in this field have been fuelled by data obtained from X-ray satellites such as RXTE, XMM Newton and NuSTAR. Observational constraints on the luminosity indicate the possibility of occurrence of the PRE. There are few cases in the data released by *XMM Newton* [3] and *NuSTAR* [2] of having no PRE. X-ray bursters are, however, extensively studied [2-7] for understanding the nature of the sources. They are also potential sites for synthesis of heavy nuclei [8,9] where accreted matter rich in H or He accumulates near the compact primary and undergo thermonuclear runaway. However, a concordant picture of the burst recurrence time, efficiency of burning of accumulated fuel and change in burst duration, observed softening of the accretion emission [2] is yet to come.

It is well understood lesson from general relativity that spacetime metric gets coupled to all non-gravitational phenomena. Sensitivity of the process of synthesis of nuclei in the primordial universe to the cosmological metric parameters (scale factor $a(t)$, curvature parameter Ω_k and expansion rate $H(t)$ in FRW metric) is an interesting problem for cosmology. The primordial nucleosynthesis shows how the cosmological metric affects the nucleosynthesis processes. On local scales, general relativity predicts the dependence of rate of a physical process on the position in the gravitational potential well - thanks to the curvature of time. The processes mentioned in the previous paragraph occur not only much deeper in the gravitational potential well ($\phi_{Compact} \sim 10^3 \phi_{Sun}$ for a compact primary of size $100\,\mathrm{Km}$), but also in a rotating spacetime metric. Thus a more general metrical structure of general relativity has to be considered on these phenomena. It reflects on the luminosity and hence on the temperature and flux of the accretion emission during the burst phenomena near a compact object.

Strohmayer and Bildsten [6] considered the non-rotating (Schwarzschild type) metric factor $g_{00}^{-1/2} = \left(1 - 2GM/c^2R\right)^{-1/2}$ in the Eddington luminosity of X-ray burst (it is to be noted that g_{00} is the time curvature factor which varies from metric to metric). Luminosity of the order of Eddington limit ($L_{Edd} \approx 2\times 10^{38}$ erg/sec) is typical of bright X-ray bursts. In this work, we take the Kerr metric correction, keeping in mind the intrinsic rotation of the primary, and calculate the shift in the black-body (a reasonable assumption for coarse binning of energy spectra) temperature, T_{eff} of the burst and that in the flux of accretion emission relative to the Schwarzschild case. These changes are examined up to the maximal Kerr limit of the spin parameter, $\chi = 0.99$. Although high values of the spin parameter may be in tension with neutron stars, and considered for a matter of principle, these are consistent with black hole systems such as the one with $\chi > 0.97$ at the centre of the spiral galaxy NGC 1365 found by NuSTAR and XMM Newton [10].

The Kerr metric is found to be associated with decline of temperature and flux for all possible values of the spin parameter. A decline in coronal temperature during accretion emission has been held responsible for absence of PRE in the X-ray burst LMXB 4U 1608-52, observed by NuSTAR [2]. Therefore, for a comprehensive consensus of the X-ray bursts and related processes, the rotating metric may take an important role.

The paper is organised as follows. Section 2 contains calculations and results for the changes in temperature and ratio of the radiation fluxes for the two metrics (Schwarzschild and Kerr) for various accretion emission direction and spin parameters. Section 3 concludes.

2. *Flux and temperature shift with Kerr metric*. The time part of the Kerr metric in Boyer-Lindquist coordinates (t, r, θ) is given by,

$$g_{00}(K)=\left(1-\frac{2GMr}{c^2r^2+\frac{J^2}{M^2}\cos^2\theta}\right), \tag{1}$$

where M and J are mass and angular momentum of the compact primary and θ is the latitude angle measured from the pole. The Schwarzschild case is simply that of $J=0$. Let us see how the metric correction $g_{00}^{-1/2}$ appears in the expression of the Eddington luminosity. In general relativity theory, the time curvature is expressed as $dt=dt_{rest}g_{00}^{-1/2}$, where dt_{rest} is the time interval between any two events in a frame with local absence of gravity and dt is the time interval between same two events measured in any other frame for which the events are occurring in a metric field. The luminosity is, then written as $L=dE/dt_{rest}=(dE/dt)g_{00}^{-1/2}$, where $dE/dt=L(0)$ is the luminosity in absence of the metric correction of space. The quantity $L_{EDD}(0)=dE/dt=4\pi cGM/\kappa$, is the standard Eddington luminosity without metric correction which is ubiquitous in radiative equilibrium theory of non-compact stars (κ being the opacity of the stellar matter).

The Eddington luminosity at a point r in the Kerr spacetime can, therefore, be expressed as $L_{EDD}(K)=L_{EDD}(0)g_{00}(K)^{-1/2}$ which is

$$L_{EDD}(K)=\frac{4\pi cGM}{\kappa}\left(1-\frac{2GMr}{c^2r^2+\frac{J^2}{M^2}\cos^2\theta}\right)^{-1/2}. \tag{2}$$

In the Schwarzschild case ($J=0$), this formula takes the form $L_{EDD}(S)=(4\pi cGM/\kappa)\times$ $\times\left(1-2GM/c^2r\right)^{-1/2}$, which was used by Strohmayer and Bildsten [6] in calculating the Eddington luminosity of X-ray burst.

In the present work, the Compton scattering value $\kappa\approx 0.2\ \mathrm{cm^2\,g^{-1}}$, for the opacity factor κ is taken whenever required. The existence of Kerr horizon (cosmic censorship hypothesis [11,12]) demands that the spin parameter $\chi=Jc/GM^2$ is bounded above as $\chi\leq 1$, with $\chi=1$ being the extreme Kerr case. The time curvature parameter in Kerr case is such that the factor $g_{00}(K)^{-1/2}$ is always lying below the corresponding Schwarzschild factor $g_{00}(S)^{-1/2}$ as shown in Fig.1, 2 and 3 for the latitude angles $\theta=0°$, $\theta=45°$ and $\theta=88°$ respectively, with spin parameter lying in the range $\chi=0.1-0.99$. Therefore, at any distance r from the compact primary, the radiation flux resulting from the accretion is reduced in the Kerr case with respect to the Schwarzschild case. Of course, at higher latitude the Kerr group of curves is densely packed (see Fig.2 and 3) so that it merges with Schwarzschild at latitude $\theta=90°$. Thus the effect must be prominent towards the pole. Fig.4, 5 and 6 show the reduction in flux ratio $F(K)/F(S)$ (i.e. the ratio

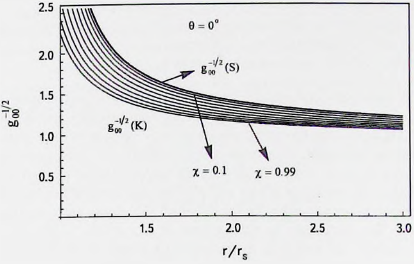

Fig.1. Variation of the metric factor $g_{00}^{-1/2}$ for both the Schwarzschild (topmost curve) and Kerr case at latitude $\theta = 0°$. In the latter case the spin parameters range from $\chi = 0.1$ to $\chi = 0.99$.

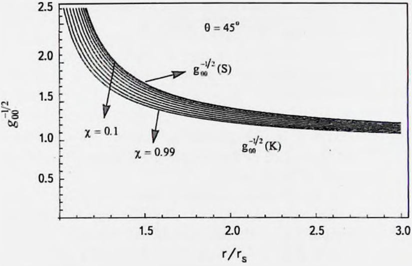

Fig.2. Variation of the metric factor $g_{00}^{-1/2}$ for both the Schwarzschild and Kerr case at latitude $\theta = 45°$. The spin parameter's range in the latter case is same as that in Fig.1.

falling below unity which is the case for $\chi = 0$) for spin parameter values up to near extreme Kerr $\chi \approx 0.99$, for the three latitudes $\theta = 0°$, $45°$ and $88°$. The distance is measured in the unit of the Schwarzschild horizon $r_S = 2GM/c^2$ and varied up to the radius of innermost stable circular orbit $r \sim 3r_S$. It is seen that at a given point the higher value of the primary's spin causes more reduction in flux than the lower value.

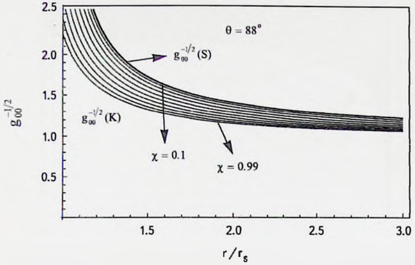

Fig.3. Variation of the metric factor $g_{00}^{-1/2}$ for both the Schwarzschild and Kerr case at latitude $\theta = 45°$. The range of the spin parameter is same as that in Fig.1 and 2.

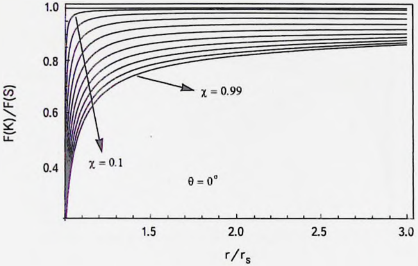

Fig.4. Reduction of the flux with distance from the primary for spin parameter in the range $\chi = 0.1 - 0.99$, at emission latitude $\theta = 0°$. The horizontal line is for the Schwarzschild case, $\chi = 0$.

High quality data of the X-ray burst source LMXB 4U 1608-52 has given the accretion geometry [2]. One of the geometric parameters is the innermost radius of the accretion disk. It is found to be $r_{in} = (3.5 - 5)r_S$. The flux reduction $F(K)/F(S)$ at such radii (see Fig.4, 5 and 6) is consistent with the report of absence of the PRE for this source.

The X-ray bursts result from thermonuclear burning during mass accretion [7]

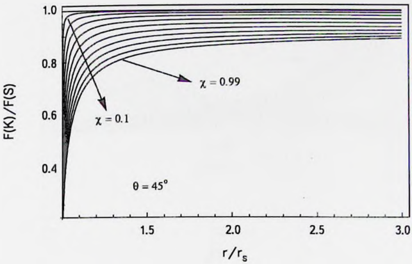

Fig.5. Reduction of the flux with distance from the primary for spin parameter in the range $\chi = 0.1 - 0.99$, at emission latitude $\theta = 45^{\circ}$. The horizontal line is for the Schwarzschild case, $\chi = 0$.

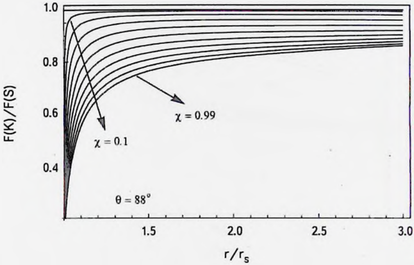

Fig.6. Reduction of the flux with distance from the primary for spin parameter in the range $\chi = 0.1 - 0.99$ at emission latitude $\theta = 88^{\circ}$. The horizontal line is for the Schwarzschild case, $\chi = 0$.

and the spectra, at least for coarsely binned energy, are observed to be roughly of black body. If the burst is bright enough the effective black body temperature is estimated from the relation, $L_{Edd}(K) \sim (4\pi cGM/\kappa) g_{00}^{-1/2}(K)$. Using the spin parameter $\chi = Jc/GM^2$ and the Schwarzschild horizon length $r_S = 2GM/c^2$ in equation (1), the Kerr metric factor, $g_{00}(K)^{-1/2}$ can be written as,

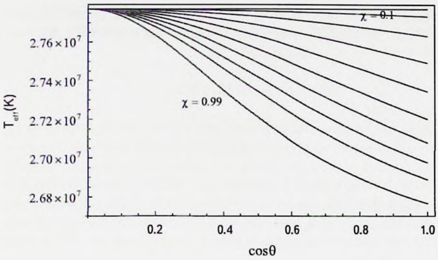

Fig.7. The decrease of effective black-body temperature of the accretion with latitude ($\theta = 0^\circ - 90^\circ$) for spin parameters $\chi = 0.1$, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.99 (coming down from the top). The horizontal line ($\chi = 0$) is for the Schwarzschild case.

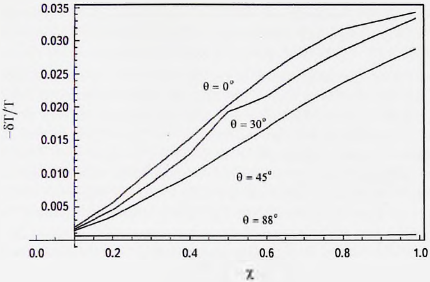

Fig.8. Fractional decrease in temperature of accretion at various latitudes with respect to the spin parameter running from $\chi = 0.1$ to $\chi = 0.99$.

$$g_{00}(K)^{-1/2} = \left(1 - \frac{\frac{r_S}{r}}{1 + \frac{\chi^2}{4}\left(\frac{r_S}{r}\right)^2 \cos^2\theta}\right)^{-1/2} \tag{3.a}$$

Using equation (3.a) in the expression for $L_{EDD}(K)$ and equating this to the black body luminosity, $L = 4\pi r^2 \sigma T_{eff}^4$, we obtain the following relation for the temperature of the burst.

$$T_{eff} = \left(\frac{4\pi cGM}{\kappa}\right)^{1/4} \left(\frac{1}{4\pi r^2 \sigma}\right)^{1/4} \left(1 - \frac{r_S/r}{1 + \frac{\chi^2}{4}\left(\frac{r}{r_S}\right)^2 \cos^2\theta}\right)^{-1/8} \tag{3.b}$$

The reduction in temperature with latitude for different values of the spin parameter has been shown in Fig.7. Calculation is done at the distance $r = 3r_S$ with a $1M_{\odot}$ primary. It is again seen that for a given spin, the cooling becomes effective towards the pole and at given latitude it is prominent for higher spins. This ensures that the Kerr metric cools down the accretion.

The change in the temperature, $\delta T/T = \left(T_{eff}(K) - T_{eff}(S)\right)/T_{eff}(S)$ at four different emission latitudes, $\theta = 0^\circ$, 30°, 45° and 88° is plotted with respect to the spin parameter in Fig.8. The absolute value of the fractional decrease in temperature is found to be about 10^{-3}-10^{-4}. At the highest latitude (i.e. $\theta = 88^\circ$, the shift in the temperature is found to be smaller than that in the lowest latitude ($\theta = 0^\circ$) by an order of 1 or 2 depending upon the primary's spin.

3. *Conclusion*. The lower values of the flux ratio (<1) is consistent with the observation that there is no sign of photospheric radius expansion for the X-ray burst LMXB 4U 1608-52. This softening of accretion emission has been ascribed to cooling of the corona. At emission latitudes $\theta = 0^\circ$, 30°, 45° and 88°, we find that the amplitude of such cooling lies in the range $\delta T/T = -\left(10^{-3} - 10^{-4}\right)$ for the range of the spin parameter, $\chi = 0.1 - 0.99$. Although some values of the spin parameter may be high for a neutron star and considered for a matter of principle, it is applicable readily to black hole systems e.g. rapidly rotating supermassive black holes such as the one at the centre of the spiral galaxy NGC 1365 found by NuSTAR and XMM Newton. Such cooling can appreciably affect the thermonuclear reaction rates near the primary for those reactions which are quite sensitive to temperature. One such process is the hot CNO cycle that undergoes in rapid proton capture events during accretion. We, however, do not strongly ascertain that the reduction in temperature and emission flux is uniquely due to the Kerr spacetime. But it is firmly believed that the metrical effect is buried in the observed processes of the burst phenomena. Therefore, the burst phenomena can also be used as probes for testing general relativity and its different metric forms.

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ВЛИЯНИЕ МЕТРИКИ КЕРРА НА УВЕЛИЧЕНИЕ РАДИУСА ФОТОСФЕРЫ ПРИ РЕНТГЕНОВСКОЙ ВСПЫШКЕ

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Основной целью настоящей работы является изучение эффектов общей теории относительности на увеличение радиуса фотосферы при рентгеновской вспышке. Рассматривается вопрос, каким образом метрика Керра приводит к изменению эффективной температуры и потока излучения по сравнению с шварцшильдовскими значениями, принимаемыми при аккреции вещества на нейтронную звезду или черную дыру при рентгеновской вспышке. Спин компактного объекта в зависимости от широты эмиссии доходит до максимального предела Керра $\chi = 0.99$. Для амплитуды отклонения температуры по сравнению с шварцшильдовским случаем получено $\delta T/T \approx -\left(10^{-3} \div 10^{-4}\right)$ в области $\chi = 0.1 \div 0.99$ и широт $\theta = 0^{\circ}$, 30°, 45° и 88°. Найдено, что отношение потока эмиссии в метрике Керра к такому потоку при шварцшильдовской метрике $F(K)/F(S)$ меньше единицы. Оно увеличивается до 0.9 для наименьшего отличного от нуля параметра спина 0.1. При максимальном пределе Керра $\chi = 0.99$ оно насыщается, примерно к значению 0.8. Указанный эффект особенно значителен вблизи полюса. Такая редукция температуры и потока согласуется с отсутствием увеличения радиуса фотосферы при вспышках LMXB 4U1608-52, наблюденных NuSTAR-ом. Хотя это нельзя однозначно приписать к метрике, тем не менее думается, что влияние пространственно-временной метрики на явление вспышки может быть использовано как пробный тест для общей теории относительности. Отклонения температуры и потока излучения может иметь наблюдаемые проявления в процессах синтеза элементов в такого рода средах.

Ключевые слова: *общая теория относительности: метрика: спин: рентгеновская вспышка*

REFERENCES

1. *R.N.Boyd*, An Introduction to Nuclear Astrophysics (The University of Chicago Press), 2008.
2. *N.Degenaar et al.*, Mon. Not. Roy. Astron. Soc., **456**, 4256, 2010.

3. *A.Papitto et al.*, Mon. Not. Roy. Astron. Soc., **407**, 2577, 2010.
4. *J.H.Swank et al.*, Astrophys. J. Lett., **212**, 73, 1977.
5. *J.A.Hoffman, W.H.G.Lewin, J.Doty*, Astrophys. J. Lett., **201**, 23, 1997.
6. *T.Strohmayer, L.Bildsten*, in Compact Stellar X-ray Sources, eds. W.H.G.Lewin, M. van der Klis, (Cambridge University Press), 2005.
7. *A.L.Watts*, Annu. Rev. Astron. Astrophys., **50**, 609, 2012.
8. *W.Lewin, J. van Paradijs, R.Taam*, in W.H.G.Lewin, J. van Paradijs and E.P.J. van den Heuvel, ed., X-ray Binaries (Cambridge University Press), 1995.
9. *H.Schatz*, K.Rehm, Nuc. Phys., A **777**, 601, 2006.
10. *D.J.Walton et al.*, arXiv: 1404.5620.
11. *R.Penrose*, Riv. Nuovo. Cimento, **1**, 252, 1969.
12. *S.Hawking*, Phys. Rev., D **14**, 2460, 1976.