АСТРОФИЗИКА

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AN UNNOTICED SIGNIFICANCE OF THE CHANDRASEKHAR MASS LIMIT

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Simultaneous occurrence of the three fundamental constants G, $\hbar$ and c and hence the role of Planck mass in the Chandrasekhar mass limit is critically examined in the light of cosmology, by incorporating the cosmological constant through vacuum fluctuation at Planck scale and the holographic principle. A new interpretation of the cosmological constant problem is also put forward.

Key words: *Chandrasekhar mass limit: Planck mass: cosmological Constant*

1. *Introduction*. Simultaneous occurrence of the three fundamental constants G, $\hbar$ and c in the expression of the Chandrasekhar mass limit is an aesthetics of the astrophysical theory of white dwarfs. It is, however, not a mere appearance of numbers in astrophysical phenomena involving gravity, quantum theory (statistics, particularly in cold degenerate stars) and relativity but also possesses important information related to cosmology and possibly future fundamental theories of space-time. One admits that such a combination is typical to quantum gravity where the classical gravitational action, $A_G = \left(-c^4/16\pi G\right)\int d^4 X\sqrt{-g}\left(R+2\Lambda\right)$ becomes comparable to the quantum action $\hbar$. Incorporating these constants together in the equations of astrophysics and cosmology is, at first look, not so surprising. This happens naturally when we include quantum fields in gravitational phenomena. In absence of a quantum gravity theory, this approach is still semiclassical in nature in the sense that the matter is treated quantum mechanically whereas gravity is the classical metric field or the Newtonian potential. Davies [1] has listed a plethora of cosmic phenomena where these constants appear together. Black hole thermodynamics [2-4] involves these constants in determining the black hole entropy and temperature. In the early universe, the classical FRW metric and hence the thermal evolution is governed by the Dirac (electrons, neutrinos and their antiparticles), Maxwell (photons), scalar fields and possibly supersymmetric fields (-ons and -inos) which contribute to the active mass density. The result is appearance of these three constants in the time temperature relation. They also find their joint roles in holographic principle [5,6] in cosmology which states that the vacuum energy density $\varepsilon_{VAC} \sim E_0^4 c^{-3}\hbar^{-3}$ (E_0 being some UV cut off) in a given

region of space should not exceed the mass energy of a black hole of size same as that of the region i.e. $\varepsilon_{VAC} \sim E_0^4 c^{-3} \hbar^{-3} \leq M_{BH} c^2 / l_{BH}^3$, where $l_{BH} \sim GM_{BH}/c^2$. The holographic principle is found to be useful for understanding the recent cosmic acceleration [7,8]. Therefore, the simultaneous appearance of these three numbers must be considered deeply for a complete theory of matter and space-time.

This work reports the unnoticed meaning of the appearance of the Planck mass $m_{Pl} = \sqrt{\hbar c / G}$ in the Chandrasekhar mass limit of white dwarf which is gravitationally classical but non-gravitationally a quantum gas system. Although there is report on the role of the Planck mass in the Chandrasekhar limit [9], it has not been seriously examined till a very recent study of quantum gravity correction to the Chandrasekhar mass limit due to modification of Heisenberg algebra of the position and momentum operators resulting in a so called generalized uncertainty principle: $[x_i, p_j] = i\hbar(\delta_{ij} + f(p, p_i p_j))$ [10]. This gives rise to modified Lane-Emden equation. The present work focuses on how the Chandrasekhar mass encapsulates the signature of a much deeper future theory of gravity and some cosmological issues.

The paper is organized as follows. Section 2 contains the mass limit and its indication for a fundamental theory. To make the paper self-contained, the way to obtain Chandrasekhar limit is briefly reviewed. Certain distinctive remarks on the cosmological constant are then made from the mass limit by the use of holographic cosmological principle. Section 3 concludes.

2. *The mass limit and its other side*. The mass limit for white dwarfs originates in the polytropic model for the pressure and density of degenerate electron gas, $P = K\rho^{1+1/n}$, where is the polytropic index and K is the microscopic parameter of the gas. Such an equation of state is also found to be preserved in quantum gravity induced modification of the Heisenberg algebra as advocated in [10]. The condition of gravitational stability gives the mass-radius relation (see Chandrasekhar [11], Prialnik [12]) of the type,

$$M = M(R, n, K, G) = \left(\frac{1}{4\pi}\right)^{1/(n-1)} G^{n/1-n} M_n \left(\frac{R}{\xi_1}\right)^{(n-3)/(n-1)} \{(n+1)K\}^{n/n-1}. \quad (1)$$

This relation is usually obtained from the following mass formula for a poytrope in the Lane-Emden theory,

$$M = \int_0^R dr\, 4\pi r^2 \rho(r), \quad (2)$$

where $r = \alpha\xi$ with $\alpha = \sqrt{(n+1)K / 4\pi G \rho_c^{1-1/n}}$ being a length parameter and $\rho = \rho_c \theta^n$ with ρ_c being the central density of the polytrope. The dimensionless Lane-Emden functions θ and ξ representing density and radius respectively, satisfy the

Lane-Emden equation,

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right)=-\theta^n \tag{3}$$

In equation (1), $\xi_1 = R/\alpha$ is the value of ξ at which the density θ vanishes (the surface of the polytrope) and M_n is a model parameter given as [12],

$$M_n = -\xi_1^2\left(\frac{d\theta}{d\xi}\right)_{\xi_1} . \tag{4}$$

The detailed way of obtaining equation (1) from (2), (3) and (4) is referred to literature [11-14]. For the relativistic electron gas the microscopic parameter K is evaluated from the relativistic degeneracy pressure equation,

$$P=\frac{1}{3}\int_0^{p_F} dp(pc)\frac{8\pi p^2}{h^3} , \tag{5}$$

where, $p_F=\left(3n_e h^3/8\pi\right)^{1/3}$ is the electron Fermi momentum and $n_e=\rho/\mu_e m_H$ is the number density of electrons given by the density of the star and the mean molecular weight of the free electrons - μ_e. The value of K is found as,

$$K=\left(\frac{2\pi hc}{3}\right)\left(\frac{3}{8\pi}\right)^{4/3}\left(\frac{1}{\mu_e m_H}\right)^{4/3} . \tag{6}$$

For $n=3$, the constant mass resulting from equation (1) is the Chandrasekhar mass limit for white dwarfs. Up to the model parameter M_3 (nearly 2.02 as found by numerical solution of the Lane-Emden equation for $n=3$) and the electron mean molecular weight μ_e (which is 2 for a hydrogen depleted environment) the Chandrasekhar limit is expressed as $M_{Ch} \approx m_{Pl}^3/m_H^2$, where m_H is the proton mass. Although a dimensionless number is realized as quirk in Nature, m_H^2 can be replaced by the gravitational fine structure constant $\alpha_G = Gm_H^2/hc$. This gives the following rewriting of the mass limit.

$$M_{Ch} \approx \alpha_G^{-1} m_{Pl} . \tag{7}$$

The smallness of the value of α_G (10^{-39}!) has itself been a deep puzzle for cosmology and once gave rise to mysterious ideas such as the Large Number Hypothesis of Dirac [15,16]. Except the anthropic principle, we still do not have any better guidance for understanding the number α_G . A formula of the type $M \propto m_{Pl}$, was obtained by Bisnovatyi-Kogan and Novikov [17] for a configuration of neutrinos as constituents of missing halo mass for galaxies. Markov [18] derived similar relation for limiting mass of a degenerate neutrino star. However, the problem of the Chandrasekhar limit deepens through the Planck mass m_{Pl}. Being ubiquitous in the quantum gravity regime of spacetime the Planck mass can be realized in the following manner.

In the regime of Planck scale, vacuum fluctuations produce particles of mass m_{Pl} separated by the Compton length $\lambda = \hbar / m_{Pl} c$ and having number density $(m_{Pl} c / \hbar)^3$. The gravitational interaction between these particles defines a vacuum energy density $\varepsilon_{VAC} = (G m_{Pl}^2 / \lambda)(1 / \lambda^3)$. Zeldovich [19] first interpreted the cosmological constant as the energy density of vacuum which appears in the theory of elementary particles. In his work, the cosmological constant was connected to the proton mass as [19] $\Lambda \propto m_p^6$ which pointed towards a mechanism for a decreasing Λ. In the gravitation theory, Λ presents an energy density in empty space $\varepsilon_\Lambda = \Lambda c^4 / 8\pi G$. Identifying this with the vacuum energy density, the Planck mass can be expressed as $m_{Pl} = (\Lambda \hbar^4 / 8\pi G^2)^{1/6}$. The connection $\Lambda \propto m_{Pl}^6$ is identical to the original suggestion of Zeldovich but with a much larger mass scale, $m_{Pl} \approx 10^{19} m_p$. This is the scale of vacuum fluctuations corresponding to the limit of the theory of elementary particles [20]. What follows here explains why such a connection is necessary to understand existence of cold degenerate astrophysical objects like the white dwarf and thereby to provide with a new meaning of the cosmological constant. Equation (7) then takes the form,

$$M_{Ch} = \alpha_G^{-1} \left(\frac{\Lambda \hbar^4}{8\pi G^2} \right)^{1/6}. \tag{8}$$

Equations (7) and (8) are deep expressions for the Chandrasekhar limit and are more than mere collection of the fundamental constants. Most significantly the microscopic cosmological constant (vacuum energy density) appears in the stellar mass. Equation (8) can be further deepened through de Sitter fluctuation to gravitational interaction. For any mass m, the term Gm^2 / λ is typical to the Schwarzschild metric. A Schwarzschild-de Sitter metric with a geometric cosmological constant, however, adds a term $\Lambda mc^2 \lambda^2 / 6$ to the interaction energy. This gives an effective vacuum energy density,

$$\varepsilon_{VAC}(eff) = \left(\frac{Gm^2}{\lambda} + \frac{\Lambda mc^2 \lambda^2}{6} \right) \frac{1}{\lambda^3}. \tag{9}$$

Defining a mass $m_\Lambda = (\Lambda c^2 / 8\pi G)(4\pi\lambda^3 / 3)$ associated with the cosmological constant, the effective vacuum energy density is expressed as,

$$\varepsilon_{VAC}(eff) = \frac{\Lambda_{eff} c^4}{8\pi G} = \frac{Gm^6 c^4}{\hbar^4} \left(1 + \frac{m_\Lambda}{m} \right) = \varepsilon_{VAC}(\Lambda = 0) \left(1 + \frac{m_\Lambda}{m} \right). \tag{10}$$

The factor $(1 + m_\Lambda / m)$ can be supplied by considering a Planck mass black hole, $m_{BH} = m_{Pl}$ and applying the holographic principle that the maximum permissible energy in the empty space corresponds to the rest energy of a black hole. This gives $m_\Lambda = m = m_{Pl}$ and hence the following.

$$m_{pl} = \left(\frac{\Lambda_{eff} \hbar^4}{16\pi G^2} \right)^{1/6} \tag{11}$$

This gives the Chandrasekhar limit in similar footing to equation (8),

$$M_{Ch} = \alpha_G^{-1} \left(\frac{\Lambda_{eff} \hbar^4}{16\pi G^2} \right)^{1/6} \tag{12}$$

It is seen from equations (8) and (12) that whether the metric fluctuations are of pure Schwarzschild or Schwarzschild-de Sitter type, the Chandrasekhar limit is governed by the quantity $\left(\Lambda\hbar^4/\pi G^2\right)^{1/6}$. Clearly, a cosmological constant of magnitude $\Lambda \approx 10^{62}$ cm^{-2} gives a mass limit nearly of the size $M_{Ch} \sim 1 M_\odot$. This magnitude of the cosmological constant is about 120 orders of magnitude larger than the astronomically inferred value 10^{-54} cm^{-2}. Interpretation of this hierarchy has led to the well-known cosmological constant (CC) problem - "why the present value of the cosmological constant is so small?". It is, however, the most satisfactory candidate for the dark energy behind the recent cosmic acceleration as shown by the measurements on the cosmic microwave background [21].

Here comes a note for the cosmological constant which emanates from the expressions (8) or (12). The interpretation of the cosmological constant problem can be reversed. Instead of "why small?" it could be "why so large?". Had it been somewhat small, we would not have found a white dwarf we know today! This is new and distinct from a general belief that quantum gravity will somehow provide with a cancellation mechanism so that $|\Lambda + 8\pi G \rho_{VAC}| \approx 0$ [22]. Therefore, irrespective of the CC problem, a future theory of quantum gravity has to predict such a large value of the cosmological constant. It might be possible that the cosmological constant does not gravitate (to avoid unnatural cancellation or fine tuning) as advocated by degravitation formalisms [23-25]. If this is so, there is no necessity for explaining the smallness of the cosmological constant. This opens the door to the possibility that dark energy may be caused by either of (i) new form of matter, (ii) new gravitational physics and (iii) different background topology of spacetime which somehow mimics a cosmological constant in current epoch. Although appreciated in modern cosmology, this is a philosophical consequence of the Chandrasekhar limit.

3. ***Conclusion***. The role of quantum gravity phenomena, the cosmological constant and some unconventional ideas in cosmology (holographic principle is one) are encapsulated in the Chandrasekhar mass limit. The universal nature of the Chandrasekhar mass limit resulting from the three fundamental constants of Nature boils down to a deep cosmological issue - "why the cosmological constant is so large?". If the cosmological constant does not gravitate, the "why small?"

problem is likely to evaporate. This new meaning of the Chandrasekhar limit opens other possibilities for the dark energy phenomenon. Such previously unnoticed richness of the mass limit provides with new interpretation of a cosmological problem. It may hold the clue for a future deep understanding of gravity and cosmology.

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НЕЗАМЕЧЕННЫЙ СМЫСЛ ЧАНДРАСЕКАРОВСКОГО ПРЕДЕЛА

С.КАЛИТА

В рамках космологии критически рассматриваются одновременное существование трех фундаментальных констант G, $\hbar$ и c и роль планковской массы в предельной массе Чандрасекара путем введения космологической константы через флюктуации вакуума в планковских масштабах и использования голографического принципа. Предлагается новая интерпретация проблемы космологической константы.

Ключевые слова: *чандрасекаровский предел массы: планковская масса космологическая постоянная*

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