АСТРОФИЗИКА

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STABILITY OF ACCELERATING UNIVERSE WITH LINEAR EQUATION OF STATE IN $f(T)$ GRAVITY USING HYBRID EXPANSION LAW

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We investigate the dynamics of locally rotationally symmetric and spatially homogeneous Bianchi type-I (LRS) space-time with linear equation of state filled with perfect fluid in the framework of $f(T)$ gravity. We apply the gravitational field equations for the linear and quadratic form of $f(T)$ gravity. We determine the aspects of Bianchi type-I space-time by considering hybrid expansion law (HEL) for the average scale factor that yields power-law and exponential-law cosmologies, in its special cases. It is observed that initially the model is unstable and then it is stable in both linear and quadratic case. Also, the universe exhibits transition from deceleration to acceleration phase.

Key words: *Bianchi type-I space-time: linear equation of state: $f(T)$ gravity: hybrid expansion law*

1. *Introduction*. The recent cosmological observations [1-5] indicate that our universe is undergoing in an accelerating expansion phase due to an exotic energy which has a component with negative pressure so called dark energy (DE) (whose nature and cosmological origin still remains enigmatic at present). Many candidates of DE have been proposed such as Quintessence, Phantom, Quintom, Chaplygin gas and so on. The simplest candidate of the DE is the cosmological constant Λ, but there are serious theoretical problems associated with it (such as the fine-tuning problem, the coincidence problem) [6].

Another alternative approach dealing with the acceleration problem of the universe is changing the gravity law through the modification (the modification of gravitational action may resolve cosmological problems, dark matter, paradigm DE issues.) of action in general relativity (GR). This modification gives the Scalar tensor theory, Brans-Dick theory, string theory, Gauss-Bonnet theory, $f(R)$ theory, $f(R,T)$ theory and $f(T)$ theory. The modified theories have recently gained a lot of interest during the last decade. In the present investigation, we focus towards $f(T)$ theory as it is best to account for the present accelerating expansion. In $f(T)$ theory, Weitzenbock connection is used instead of the curvature defined via the Levi-Civita connection in GR. If one chooses to use the Weitzenbock connection, the geometry is flat in the sense that the affine connection has zero Rieman curvature and the field equations are completely described in terms of the torsion tensor. In order to explain the present cosmic accelerating expansion, Linder [7] investigated that the power and exponential law models depending upon torsion might give the de-Sitter fate of the universe by proposing two new $f(T)$ models. Wu and Yu [8] analyzed the dynamical property of this theory and showed that the universe could go forward from radiation dominated era to matter dominated era and finally enter in an exponential expansion era. Considering two new form of $f(T)$ models Wu and Yu [9] showed how the crossing of phantom divide line takes place to these models. Karami and Abdolmaleki [10] obtained equations of the state parameter of polytropic, standard, generalized and modified Chaplygin gas in this modified scenario. Bamba et al. [11] studied the cosmological equations of state in exponential, logarithmic and their combined $f(T)$ models. Myrzakulov [12] discussed different $f(T)$ models with scalar fields and gave systematic solutions for scale factors and scalar fields. Sharif and Rani explored Bianchi type-1 universe using different gravity $f(T)$ models [13]. Recently, some interesting $f(T)$ models have been explored by different authors, Chirde and Shekh [14] deliberated a spatially homogeneous and isotropic model in the context of $f(T)$ gravity with thermodynamic aspects. Jamil Amir and Yussouf [15] construct of $f(T)$ models within the Kantowski-Sachs universe using the conservation equation and equation of state parameter, which represents the different phases of the universe. Gamal and Nashed [16] investigated anisotropic models with two fluids in linear and quadratic forms of $f(T)$ gravitational models.

The draw round of this paper is as follows: In Sect. 2, the preliminary review of the $f(T)$ theory is presented. In Sect. 3, Field equation and their solution are provided. Stability of linear and quadratic form of the $f(T)$ model is given in Sect. 4. In Sect. 5 we discuss some kinematical parameters and finally Sect. 6 contains concluding remarks.

2. $f(T)$ *Gravity formalism*. In this section we give a brief description of the $f(T)$ model and a detailed derivation of its field equations.

Let us define the notations of the Latin subscript as these related to the tetrad field and the Greek one related to the space-time coordinates. For a general space-time metric, we can define the line element as

$$dS^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu . \qquad (1)$$

This line element can be converted to the Minkowski's description of the transformation called tetrad, as follows

$$dS^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij}\theta^i\theta^j , \qquad (2)$$

$$dx^{\mu}=e_i^{\mu}\theta^i,\quad \theta^i=e_{\mu}^i dx^{\mu},\tag{3}$$

where η_{ij} is a metric on Minkowski space-time and $\eta_{ij}=\mathrm{diag}[1,-1,-1,-1]$ and $e_i^{\mu}e_{\nu}^i=\delta_{\nu}^{\mu}$ or $e_i^{\mu}e_{\mu}^j=\delta_i^j$. The root of metric determinant is given by $\sqrt{-g}=$ $=\det[e_{\mu}^i]=e$. For a manifold in which the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) and only the non-zero torsion terms exist, the Weitzenbocks connection components are defined as

$$\Gamma_{\mu\nu}^{\alpha}=e_i^{\alpha}\partial_{\nu}e_{\mu}^i=-e_{\mu}^i\partial_{\nu}e_i^{\alpha},\tag{4}$$

which has a zero curvature but nonzero torsion. Through the connection, we can define the components of the torsion tensors as

$$T_{\mu\nu}^{\alpha}=\Gamma_{\mu\nu}^{\alpha}-\Gamma_{\nu\mu}^{\alpha}=e_i^{\alpha}\left(\partial_{\mu}e_{\nu}^i-\partial_{\mu}e_{\mu}^i\right).\tag{5}$$

The difference between the Levi-Civita and Weitzenbock connections is a space-time tensor, and is known as the contorsion tensor:

$$K_{\alpha}^{\mu\nu}=\left(-\frac{1}{2}\right)\left(T_{\alpha}^{\mu\nu}+T_{\alpha}^{\nu\mu}-T_{\alpha}^{\mu\nu}\right).\tag{6}$$

For facilitating the description of the Lagrangian and the equations of motion, we can define another tensor $S_{\alpha}^{\mu\nu}$ from the components of the torsion and contorsion tensors, as

$$S_{\alpha}^{\mu\nu}=\left(\frac{1}{2}\right)\left(K_{\alpha}^{\mu\nu}+\delta_{\alpha}^{\mu}T_{\beta}^{\beta\nu}-\delta_{\alpha}^{\nu}T_{\beta}^{\beta\mu}\right).\tag{7}$$

The torsion scalar T is

$$T=T_{\mu\nu}^{\alpha}S_{\alpha}^{\mu\nu}.\tag{8}$$

Now, we define the action by generalizing the TG i.e. $f(T)$ theory as

$$S=\int\left[T+f(T)+L_{matter}\right]e\,d^4x.\tag{9}$$

Here, $f(T)$ denotes an algebraic function of the torsion scalar T. Making the functional variation of the action (9) with respect to the tetrads, we get the following equations of motion

$$S_{\mu}^{\nu\rho}\partial_{\rho}Tf_{TT}+\left[e^{-1}e_{\mu}^i\partial_{\rho}\left(ee_i^{\alpha}S_{\alpha}^{\nu\rho}\right)+T_{\lambda\mu}^{\alpha}S_{\alpha}^{\nu\lambda}\right](1+f_T)+\frac{1}{4}\delta_{\mu}^{\nu}(T+f)=4\pi T_{\mu}^{\nu}.\tag{10}$$

The field equation (10) is written in terms of the tetrad and partial derivatives and appears very different from Einstein's equation.
where T_{μ}^{ν} is the energy momentum tensor, $f_T=df(T)/dT$ and by setting $f(T)=a_0=\text{constant}$ this is dynamically equivalent to the GR.

3. *Solution of the field equations*. In this section we find exact solutions for Bianchi type-I space-time in $f(T)$ gravity and some physical quantities.

Spatially homogeneous and anisotropic Bianchi type-I (LRS) space-time is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)\left[dy^2 + dz^2\right], \tag{11}$$

where A and B be the metric potential which is the functions of cosmic time t only.

The corresponding Torsion scalar is given by

$$T = -2\left(2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2}\right). \tag{12}$$

Let us assume that the matter content is a perfect fluid such that the energy momentum tensor T_μ^ν is

$$T_\mu^\nu = (p+\rho)u^\nu u_\mu - pg_\mu^\nu, \tag{13}$$

and satisfies the equation of state

$$p = \varepsilon\rho - \gamma, \tag{14}$$

where ε and γ are constants.

The commoving coordinates are

$$u^\nu = (0,0,0,1) \quad \text{and} \quad u^\nu u_\nu = 1, \tag{15}$$

where u^ν is the four-velocity vector of the fluid, p and ρ be the pressure and energy density of the fluid, respectively.

From the equation of motion (10), Bianchi type-I space-time (11) for the fluid of stress energy tensor (13) can be written as

$$(T+f) + 4(1+f_T)\left\{\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} + 4\frac{\dot{B}}{B}\dot{T}f_{TT} = k^2(\gamma - \varepsilon\rho), \tag{16}$$

$$(T+f) + 2(1+f_T)\left\{\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 3\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} + 2\left\{\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right\}\dot{T}f_{TT} = k^2(\gamma - \varepsilon\rho), \tag{17}$$

$$(T+f) + 4(1+f_T)\left\{\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} = k^2\left(\frac{p+\gamma}{\varepsilon}\right), \tag{18}$$

where the dot $(\cdot)$ denotes the derivative with respect to time t.

Finally, here we have three differential equations with five unknowns namely A, B, f, p, ρ. The solution of these equations is discussed in next section. In the following we define some kinematical quantities of the space-time.

We define average scale factor a and volume V as

$$V = a^3 = \sqrt{AB^2}. \tag{19}$$

Another important dimensionless kinematical quantity is the deceleration parameter (DP) q, which shows whether the universe exhibits accelerating volumetric expansion or not

$$q=-1+\frac{d}{dt}\left(\frac{1}{H}\right). \tag{20}$$

For $-1\leq q<0$, $q>0$ and $q=0$ the universe exhibit correspondingly accelerating volumetric expansion, decelerating volumetric expansion and expansion with constant-rate respectively.

The directional Hubble parameter in the direction of x, y and z-axis, respectively, are

$$H_1=\frac{\dot{A}}{A},\quad H_2=\frac{\dot{B}}{B},\quad H_3=\frac{\dot{B}}{B}.$$

The mean Hubble parameter, which expresses the volumetric expansion rate of the universe, is given as

$$H=\frac{1}{3}\left(H_1+H_2+H_3\right). \tag{21}$$

Using equations (19) and (21), we obtain

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{1}{3}\left(H_1+H_2+H_3\right)=\frac{\dot{a}}{a}. \tag{22}$$

To discuss whether the universe either approach isotropy or not, we define an anisotropy parameter A_m as

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2. \tag{23}$$

The expansion scalar θ and shear scalar σ^2 are defined as follows

$$\theta=u^{\mu}_{;\mu}=\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}, \tag{24}$$

$$\sigma^2=\frac{3}{2}H^2A_m. \tag{25}$$

To solve the field equation, as it was mentioned, we have used the hybrid expansion law (HEL). The power-law and exponential law cosmologies can be used only to describe epoch based evolution of the Universe, because of the constancy of deceleration parameter. These cosmologies do not exhibit the transition of the universe from deceleration to acceleration. In order to explain such transition Kumar [17], Akarsu et al. [18] consider the following form of scale factor of the universe

$$a(t)=a_1t^{\alpha}e^{\beta t}, \tag{26}$$

where $a_1>0$, $\alpha\geq 0$ and $\beta\geq 0$ are constants.

This generalized form of scale factor is called as the HEL which leads to the power-law cosmology for $\beta = 0$ and exponential-law cosmology for $\alpha = 0$ and the case $\alpha > 0$ and $\beta > 0$ leads to a new cosmology arising from the HEL. Thus, the power-law and exponential law cosmologies are the special cases of the HEL cosmology. This choice of scale factor yields a time-dependent DP (see eq. 28 & Fig.1) such that before the DE era, the corresponding solution gives the inflation and radiation/matter dominated era with subsequent transition from deceleration to acceleration. Thus, our choice of scale factor is physically acceptable. Using HEL very recently, Bhoyar et at. [19] investigated some features of non-static plane symmetric universe filled with magnetized anisotropic dark energy in $f(R,T)$.

Also, the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic within ≈ 30 percent [20,21]. To put more precisely, red-shift studies place the limit $(\sigma/H) \leq 0.3$ on the ratio of shear σ to Hubble constant H in the neighborhood of our galaxy. Collin et al. [22] have pointed out that for spatially homogeneous metric; the normal congruence to the homogeneous expansion satisfies the condition that (σ/θ) is constant i.e. the expansion scalar is proportional to the shear scalar. This gives the relation between metric potentials as

$$A = B^m . \tag{27}$$

For the scale factor (26), we get time dependant values of the DP as

$$q = -1 + \frac{\alpha}{t(\alpha + \beta t)}. \tag{28}$$

Initially when the universe starts to expand the sign of q becomes positive which

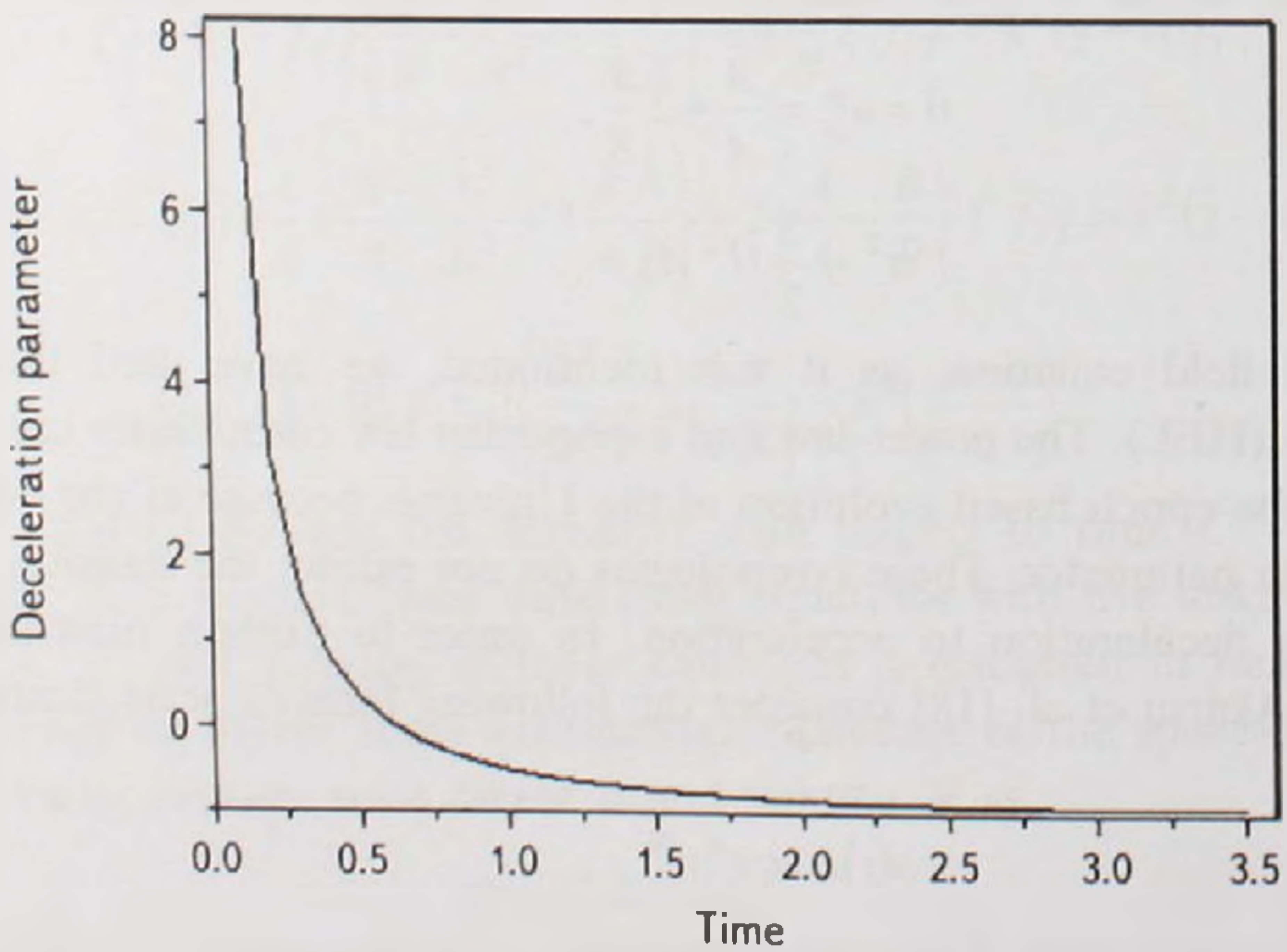

Fig.1. Deceleration parameter verses time t.

correspond to the standard decelerating behavior, which is consistent with the recent observations [1-3], as well as with the high red shifts of type Ia supernova, whereas with the expansion of the universe the sign of q become negative which correspond to the standard accelerating behavior of the universe, This scenario is also consistent with recent observations. This value is very near to the observed value of DP i.e. $-1 \le q \le 0$. This implies that the Bianchi type-I space-time shows flipping, decelerating to accelerating phase. This behavior depicts in Fig.1.

For this model, the corresponding metric coefficients A and B comes out to be

$$A = a_1^{3m/(m+2)} (t)^{3m\alpha/(m+2)} e^{3m\beta t/(m+2)}, \tag{29}$$

$$B = a_1^{3/(m+2)} (t)^{3\alpha/(m+2)} e^{3\beta t/(m+2)}. \tag{30}$$

From equation (12), the torsion scalar T becomes

$$T = -2\left(\frac{9+18m}{(m+2)^2}\right)\left(\beta + \frac{\alpha}{t}\right)^2. \tag{31}$$

Using equations (29) and (30), spatially homogeneous and anisotropic Bianchi type-I space-time with linear EoS filled with perfect fluid within the framework of $f(T)$ gravity becomes

$$\begin{aligned} ds^2 &= dt^2 - a_1^{6m/(m+2)} (t)^{6m\alpha/(m+2)} e^{6m\beta t/(m+2)} dx^2 - \\ &- a_1^{6/(m+2)} (t)^{6\alpha/(m+2)} e^{6\beta t/(m+2)} \left(dy^2 + dz^2\right). \end{aligned} \tag{32}$$

The metric (32) with $A(t)$ and $B(t)$ given by the equations (29) and (30) represents an exact accelerating Bianchi type-I space-time with the specific form of the scale factor. At the initial time $t = 0$, when the universe start to expand, the directional scale factors $A(t)$ and $B(t)$ vanish. At a special case of $m = 1$ our derived universe approaches isotropy.

4. *In the following section we inspect some well recognized $f(T)$ models and their stability.*

Case-I: Linear $f(T)$ gravity

In this case we substitute $f(T) = T$ in equations (16)-(18), then the field equations take the form

$$T + 4\left\{\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} = \frac{k^2}{2}(\gamma - \varepsilon\rho), \tag{33}$$

$$T + 2\left\{\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 3\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} = \frac{k^2}{2}(\gamma - \varepsilon\rho), \tag{34}$$

$$T + 4\left\{\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\} = \frac{k^2}{2}\left(\frac{p+\gamma}{\varepsilon}\right). \tag{35}$$

Using the values of metric coefficients $A(t)$ and $B(t)$ which are given in equations (29) and (30), energy density, pressure and stability function of the universe comes out to be energy density,

$$\rho = \frac{2}{k^2}\left\{\frac{18(1+2m)}{(m+2)^2}\right\}\left(\beta + \frac{\alpha}{t}\right)^2. \tag{36}$$

In linear case, it is seen that the energy density of the universe is a function of time and decreases with the expansion and at $t \to \infty$, $\rho \to 0$. Thus the universe approaches towards a flat universe at late time. As a result, our model is in good agreement with the recent observation. So that we have for the pressure

$$p = \frac{2}{k^2}\left\{\frac{12\alpha}{t^2(m+2)} - 54\left(\beta + \frac{\alpha}{t}\right)^2\right\}, \tag{37}$$

and the stability function,

$$\frac{\partial p}{\partial \rho} = \frac{2(m+2)}{3(1+2m)(\beta t+\alpha)} - \frac{3\alpha(m+2)^2}{(1+2m)} \tag{38}$$

We consider the stability by using the function $c^2 = dP/d\rho$. The stability of the model occurs when $c^2 \geq 0$. It is observed that in our model it depends on the value of constant m. For $0 < m < 1.5$, at the time of big-bang the model is unstable $c^2 < 0$. For small interval of t and for $m \geq 1.5$ the model is stable. It is clearly depicted in Fig.2.

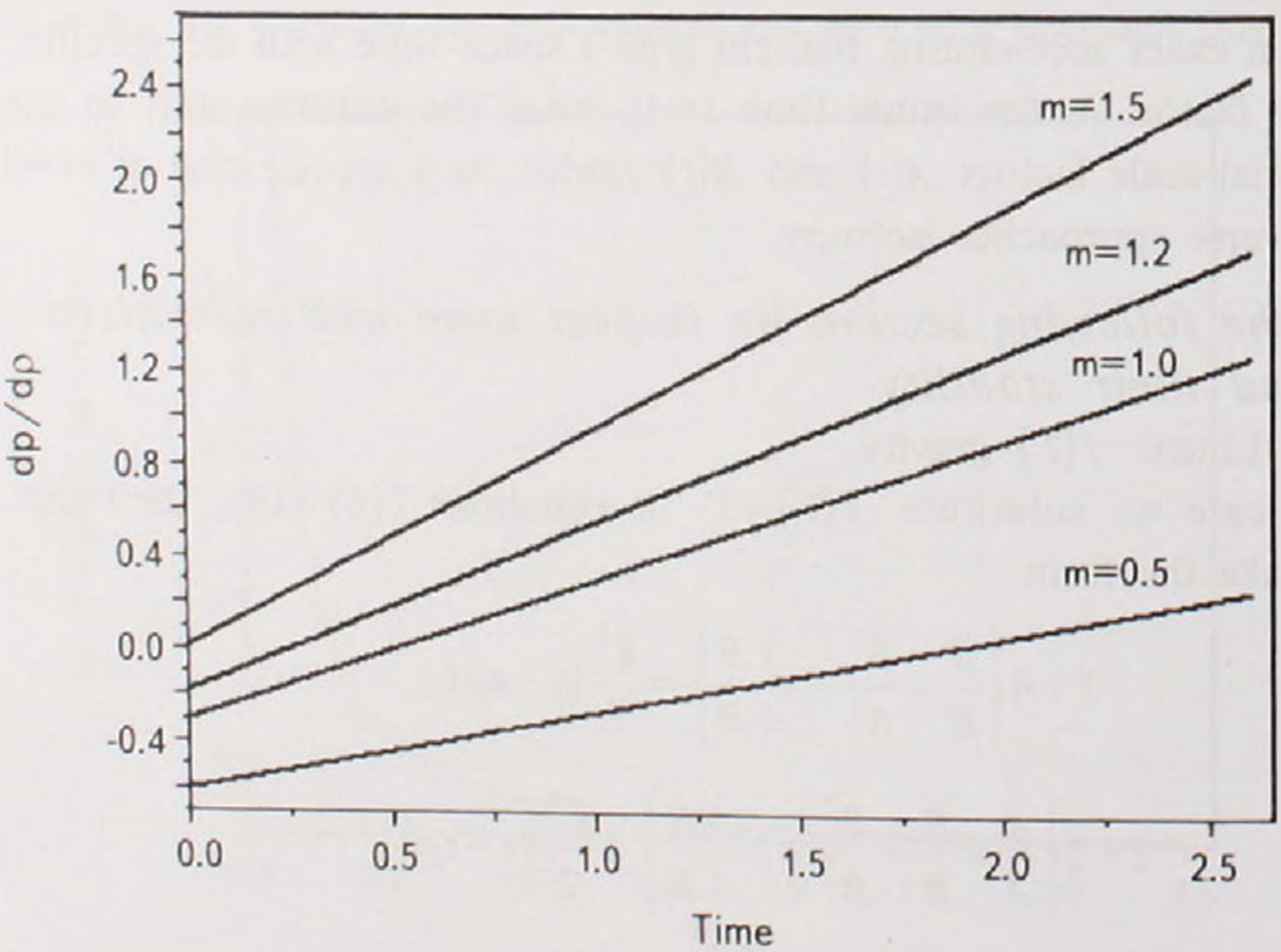

Fig.2. Behavior of $dP/d\rho$ versus cosmic time for linear case.

Case-II: Non-linear $f(T)$ gravity

In this case we substitute $f(T)=T+nT^2$ in equations (16)-(18), then the field equations take the form

$$\left(2T+nT^2\right)+8(1+nT)\left\{\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\}+8n\frac{\dot{B}}{B}\dot{T}=k^2(\gamma-\varepsilon\rho), \tag{39}$$

$$\left(2T+nT^2\right)+4(1+nT)\left\{\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+3\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\}+8n\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{T}=k^2(\gamma-\varepsilon\rho), \tag{40}$$

$$\left(2T+nT^2\right)+8(1+nT)\left\{\frac{\dot{B}^2}{B^2}+2\frac{\dot{A}}{A}\frac{\dot{B}}{B}\right\}=k^2\left(\frac{p+\gamma}{\varepsilon}\right). \tag{41}$$

With use of the values of metric coefficients given by equations (29) and (30), energy density, pressure and stability function of the universe acquire the form as follows:

energy density,

$$\rho=\frac{1}{k^2}\left\{\frac{4(n-4)(9+18m)^2(\beta t+\alpha)^4}{t^4(m+2)^4}+\frac{4(9+18m)(\beta t+\alpha)^2}{t^2(m+2)^2}\right\}. \tag{42}$$

It is seen that in a non-linear case the energy density shows the same behavior as that in linear case, i.e., $t\to 0$, $\rho\to\infty$ and at $t\to\infty$, $\rho\to 0$, which is in good agreement with the recent observation.

Pressure,

$$p=\frac{1}{k^2}\left\{\frac{(108n+72mn)(9+18m)(\beta t+\alpha)^4}{t^4(m+2)^4}+\frac{27\alpha}{t^2(m+2)^2}-\frac{108(\beta t+\alpha)^2}{(m+2)^2t^2}+\frac{96\alpha(9+18m)(\beta t+\alpha)^2}{t^4(m+2)^3}\right\}. \tag{43}$$

Stability function,

$$\frac{\partial p}{\partial\rho}=\frac{1}{4}\left\{\frac{2(108n+72mn)(9+18m)(\beta t+\alpha)^3}{t^2}-\frac{108(\beta t+\alpha)}{(m+2)^2}+\frac{96(9+18m)(\beta t+\alpha)}{(m+2)^3t}-\frac{192(9+18m)(\beta t+\alpha)^2}{(m+2)^3t^2}\right\}\Bigg/\left\{\frac{2(n-4)(9+18m)^2(\beta t+\alpha)^3}{(m+2)^2t^2}+\frac{(9+18m)(\beta t+\alpha)}{(m+2)^2}\right\}. \tag{44}$$

We consider the stability by using the function $c^2=dP/d\rho$. The stability of the model occurs when $c^2\geq 0$. It is observed that the model is unstable ($c^2\leq 0$) for small interval of initial time whereas for whole expansion it is stable (see Fig.3).

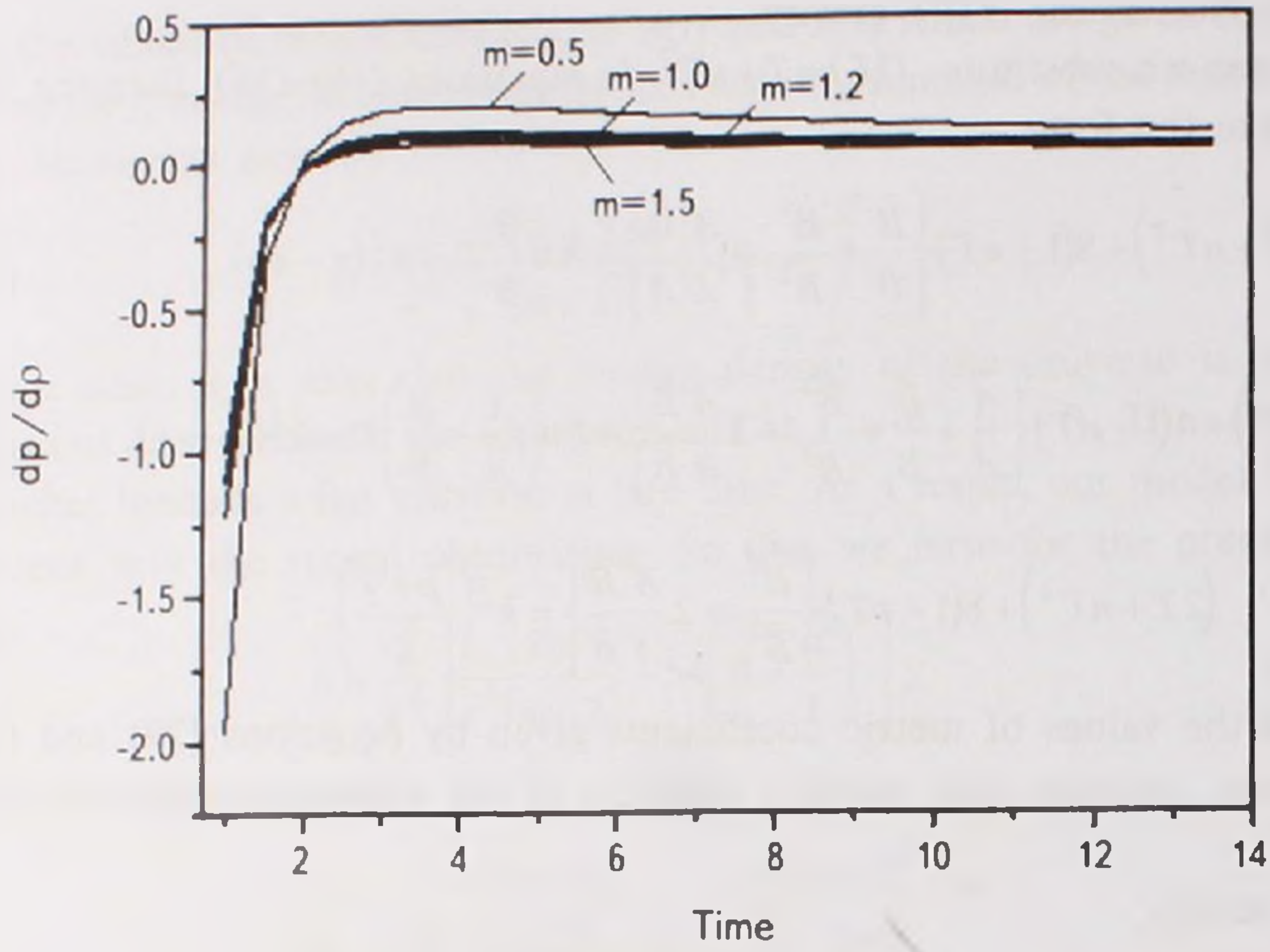

Fig.3. Behavior of $dP/d\rho$ versus cosmic time for non-linear case.

5. *Some Kinematical Parameters*:

The average scale factor α and the spatial volume V become

$$a = a_1 t^{\alpha} e^{\beta t}, \tag{45}$$

$$V = a_1^3 \left(t^{\alpha} e^{\beta t}\right)^3. \tag{46}$$

The mean Hubble parameter H and the expansion scalar θ take the form

$$H = \left(\beta + \frac{\alpha}{t}\right), \tag{47}$$

$$\theta = 3\left(\beta + \frac{\alpha}{t}\right). \tag{48}$$

The anisotropy parameter A_m and shear scalar σ^2 are given by

$$A_m = -1 + \frac{3(m^2+3)}{(m+2)^2}, \tag{49}$$

$$\sigma^2 = \frac{3}{2}\left(\beta + \frac{\alpha}{t}\right)^2 \left(-1 + \frac{3(m^2+3)}{(m+2)^2}\right). \tag{50}$$

In a hybrid expansion model, we observed that the torsion of the universe is time dependant. Spatial volume of the universe starts with big bang (at $t \to 0$) and with the increase of time it always expands. Thus, inflation is possible in this model. This shows that the universe starts evolving with zero volume and expands with time t.

The Hubble parameter H, scalar expansion θ and shear σ are the functions of time and decrease as t increases and approaches to small constant value at later time. This suggests that at initial stage of the universe the expansion of the model is much more faster and then slow down for later time. This shows that the evolution of the universe starts with infinite rate and with the expansion it declines. The anisotropic parameter is constant and independent of time. It depends only on the values of constant m and is equal to zero for $m = -2$. Thus the nature of an anisotropic parameter is constant in the evaluation of the universe and in particular for $m = 1$, the universe is isotropic in both phases (acceleration & deceleration), which is observed by observational data of WMAP. So that our anisotropic model is the generalized model and it goes over to particular isotropic model, for particular value of $m = 1$. It is also seen that the model has shear that disappear in the model. The behavior of Hubble parameter and expansion scalar versus time t is shown in Fig.4.

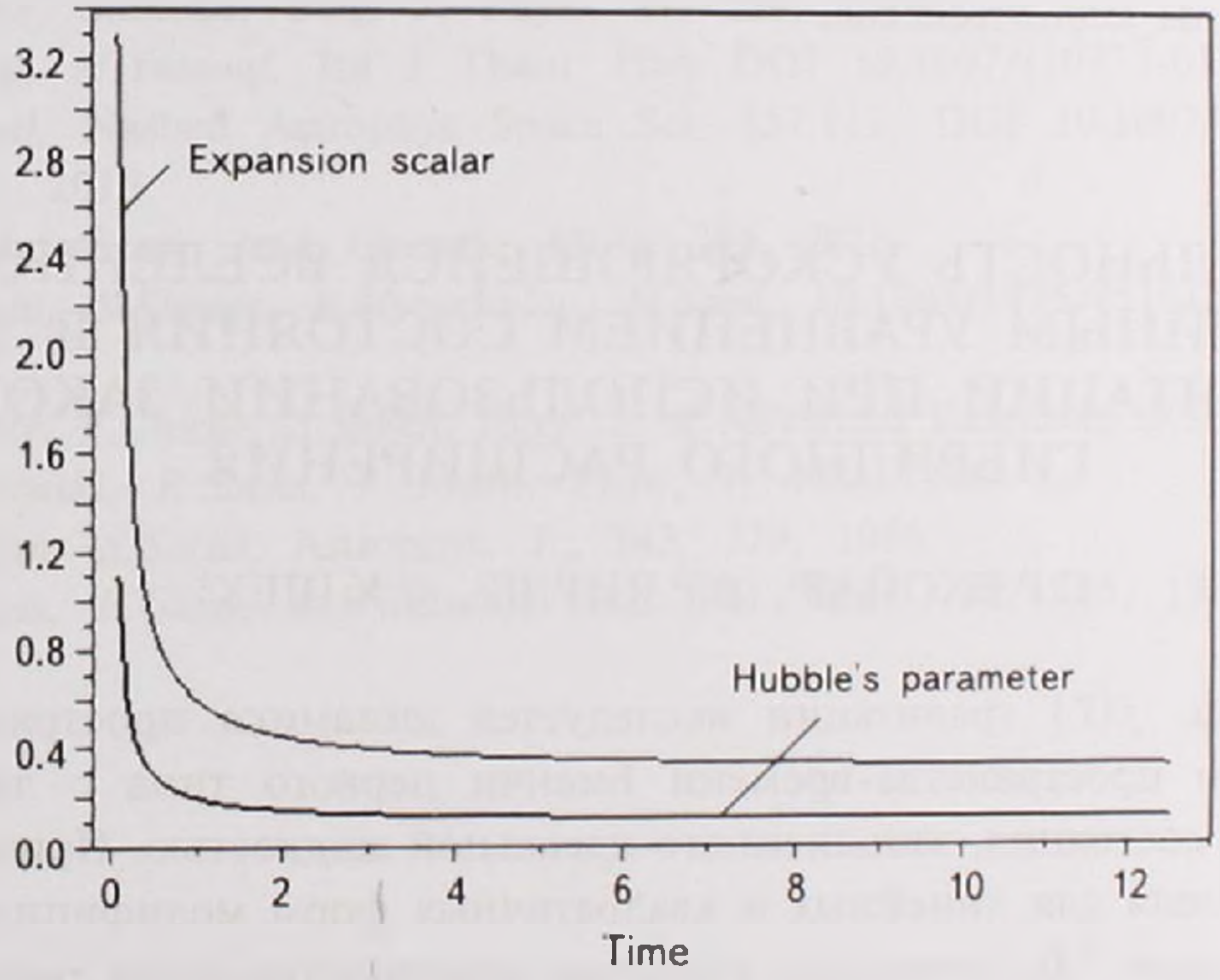

Fig.4. Behavior of Hubble parameter and Expansion scalar versus time t.

6. *Conclusion*. In this paper, we have studied spatially homogeneous Bianchi type-I space-time with linear equation of state filled with perfect fluid within the framework of $f(T)$ (linear and quadratic form) gravity. We choose a kinematical ansatz called hybrid expansion law which yields power-law and exponential law cosmologies in special cases to figure out the exact solutions of the field equations. Also we have discussed some geometrical and physical properties of the model with following observations.

The present model exhibits a singularity and it evolves with zero volume at time $t \to 0$. We see that the deceleration parameter decreases rapidly and approaches to -1 asymptotically which shows de-Sitter like expansion at late time. For this model, the deceleration parameter gives a transition from a decelerating expansion phase to the present accelerating phase. The Hubble parameter, the scalar expansion and shear are the functions of t. The universe starts with infinite rate and with the expansion it declines. It is also seen that the model has shear that it disappear with expansion. In both cases the model initially is unstable and with the expansion it becomes stable.

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СТАБИЛЬНОСТЬ УСКОРЯЮЩЕЙСЯ ВСЕЛЕННОЙ С ЛИНЕЙНЫМ УРАВНЕНИЕМ СОСТОЯНИЯ В $f(T)$ ГРАВИТАЦИИ ПРИ ИСПОЛЬЗОВАНИИ ЗАКОНА ГИБРИДНОГО РАСШИРЕНИЯ

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В рамках $f(T)$ гравитации исследуется динамика пространственно однородного пространства-времени Бианчи первого типа с линейным уравнением состояния, заполненного идеальной жидкостью. Применяются уравнения поля для линейных и квадратичных форм модифицированной $f(T)$ гравитации. Мы определяем указанное пространство-время, рассматривая закон гибридного расширения для усредненного масштабного множителя, который в частных случаях приводит к космологиям с показательными и экспоненциальными законами. Оказывается, что в обоих случаях - линейном и квадратичном, модель вначале нестабильна, но затем стабилизируется. Вселенная показывает также переход от фазы замедления к фазе ускорения.

Ключевые слова: *пространство-время Бианчи первого типа: линейное уравнение состояния: $f(T)$ гравитация: закон гибридного расширения*

REFERENCES

1. *A.Riess et al.*, Astron. J., **116**, 1009, 1998.
2. *S.Perlmutter et al.*, Bull. Am. Astron. Soc., **29**, 1351, 1997.
3. *S.Perlmutter et al.*, Astrophys. J., **517**, 565, 1999.
4. *D.Spergel et al.*, Astrophys. J., **148**, 175, 2003.
5. *M.Tegmark et al.*, Phys. Rev. D, **69**, 26, 2004.
6. *E.Komatsu et al.*, Astrophys. J. Suppl. Ser., **180**, 330, 2009.
7. *E.Linder*, Phys. Rev. D, **81**, 127301, 2010.
8. *P.Wu*, *H.Yu*, Phys. Lett. B, **692**, 176, 2010.
9. *U.Wu*, *H.Yu*, Eur. Phys. J. C, **71**, 1552, 2011.
10. *K.Karami*, *A.Abdolmaleki*, J. Phys. Conf. Ser., **375**, 032009, 2012.
11. *K.Bamba et al.*, JCAP, **1101**, 021, 2011.
12. *R.Myrzakulov*, arXiv:1006.3879.
13. *M.Sharif*, *S.Rani*, Mod. Phys. Lett. A, **26**, 1657, 2011.
14. *V.Chirde*, *S.Shekh*, Bulg. J. Phys., **41**, 258, 2014.
15. *M.Jamil*, *M.Yussouf*, Int J Theor Phys DOI 10.1007/s10773-015-2517-2.
16. *G.Gamal*, Nashed Astrophys Space Sci, 357:111, DOI 10.1007/s10509-015-2339-y, 2015.
17. *S.Kumar*, Gravi. And Cosmo., **19**(4), 284, 2013.
18. *O.Akarsu*, *S.Kumar*, *R.Myrzakulov*, *M.Sami*, 10.1088/1475-7516/2014/01/022, 2012.
19. *S.Bhoyar*, *V.Chirde*, *S.Shekh*, Inter. J. of Advanced Research, **3**(9), 492, 2015.
20. *R.Kantowski*, *R.Sachs*, J. Math. Phys., **7**, 433, 1966.
21. *J.Kristian*, *S.Sachs*, Astrophys. J., **143**, 379, 1966.
22. *C.Collins*, *E.Glass*, *D.Wilkisons*, Gen. Rel. Grav., **12**, 805, 1980.