АСТРОФИЗИКА

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PHOTOMETRIC AND KINEMATIC PROPERTIES OF THE NEARBY OPEN STAR CLUSTER NGC 2112

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In this paper, the photometric parameters (including the limited core and tidal radii, distances, luminosity and mass functions, total mass, and relaxation time) of the intermediate-aged open star cluster, NGC 2112, are estimated based on data collected from the PPMXL catalog. The kinematic properties of the cluster are also estimated. NGC 2112 lies nearby, at a distance of 898 ± 41 pc from the Sun, with an age of $\log(\mathrm{age}) = 9.25 \pm 0.05$ assuming solar metallicity ($Z = 0.019$). The estimated relaxation time of the cluster is shorter than its estimated age, which indicates that the cluster may be dynamically relaxed. The position of the cluster convergent point or apex is computed using the AD diagram method, and is found to have the equatorial coordinates 59°.430, -19°.398. Finally, the parameters of the cluster ellipsoidal velocity are also computed.

Key words: *NGC 2112: open clusters: photometric analysis: kinematical studies: Oort constant: apex: convergent point*

1. *Introduction*. A nearby cluster with a large diameter, such as NGC 2112, can provide us with a good opportunity to accurately determine its convergent point. Its large angular size in the sky and the assumption of parallel spatial motion for its stars means that the proper motions of its members can be determined, and then used to define a unique convergent (or divergent) point.

The poorly-studied open cluster NGC 2112 is located in the eastern section of the Orion arm ($l = 205°.872$, $b = -12°.615$) in the direction of the famous HII region known as Barnard's loop. Fig.1 presents an image of the cluster taken from the LEDAS Digitized Sky Survey (DSS). [1] first studied the BV photometry of about 80 stars in the cluster down to 15 mag. This study gave the distance of the cluster from the Sun as 800 pc, and estimated a reddening value in the cluster area of 0.5 mag.

Results from BV CCD photometry for about 500 stars in a field of 200 arcmin^2 around the cluster were presented by [2]. The cluster age was derived to be about 3 to 5 Gyr and its distance was determined to be about 700-800 pc from the Sun. An $E(B-V)$ color excess equal to 0.6 mag was measured.

NGC 2112 is a very metal-poor cluster with $[\mathrm{Fe/H}] = -1.3$, as demonstrated by [3] and [4]. The BVI photometry of the cluster down to $V = 20$ mag was studied by [5], and they gave the following photometric parameters: reddening

$E(B-V) = 0.63 \pm 0.14$, distance $850\,\text{pc} \pm 100\,\text{pc}$, and age 2.0 ± 0.3 Gyr.

Furthermore, [6] carried out multicolor UBVRI CCD photometry for a field of $20\,\text{arcmin}^2$ around NGC 2112 as well as medium resolution spectroscopy for 35 red giants and turn-off stars in that region. They determined the cluster age to be about 1.8 Gyr, its reddening to be 0.6, its distance to be 940 pc, and its mean radial velocity to be $30.9 \pm 0.4\,\text{km s}^{-1}$.

The present paper comprises two main sections. One is devoted to photometric parameter analysis (e.g., age, reddening, and distance), and the other to the kinematic study of the cluster (e.g., the convergent point, spatial velocity, and velocity ellipsoid parameters). We use the cluster radial velocity data from [6], while the photometric and proper motion data in the region of $20 \times 20\,\text{arcmin}^2$ around NGC 2112 are extracted from the PPMXL catalogue via the Vizier web page.

2. *Photometric Analysis.*

2.1. *Cluster Center Determination.* We began our data analysis by re-calculating the location of the cluster center using the common procedure previously presented by many authors, e.g., [7-9]. Following this procedure, two perpendicular strips were cut along the declination and right ascension at the approximate center of the cluster, and histograms of the star counts were then built along each strip.

The histograms of both coordinates were fitted using a Gaussian distribution function. The maximum value gives the position of the new cluster center, as illustrated in Fig.1 and listed in Table 1. The PPMXL data were then extracted again, now centered on the new re-determined cluster center with an extraction circle of radius 20 arcminutes.

Table 1

THE NEW CENTER AND RDP PARAMETERS OF THE NGC 2112 OPEN CLUSTER

	Value
α (hh mm ss)	05 53 45.83 ± 1.2504
δ (hh mm ss)	00 24 14.9 ± 2.03
l°	205°.8794
b°	-12°.6143
r_{lim} (arcmin)	13.871 ± 1.3
r_{core} (arcmin)	5.0885 ± 0.51
c	0.44

2.2. *Radial Density Profile (RDP).* The radial density profile (RDP) is the distribution of the mean surface density in concentric rings as a function of radius from the cluster center outward. We calculated the density $\rho(r)$ of each

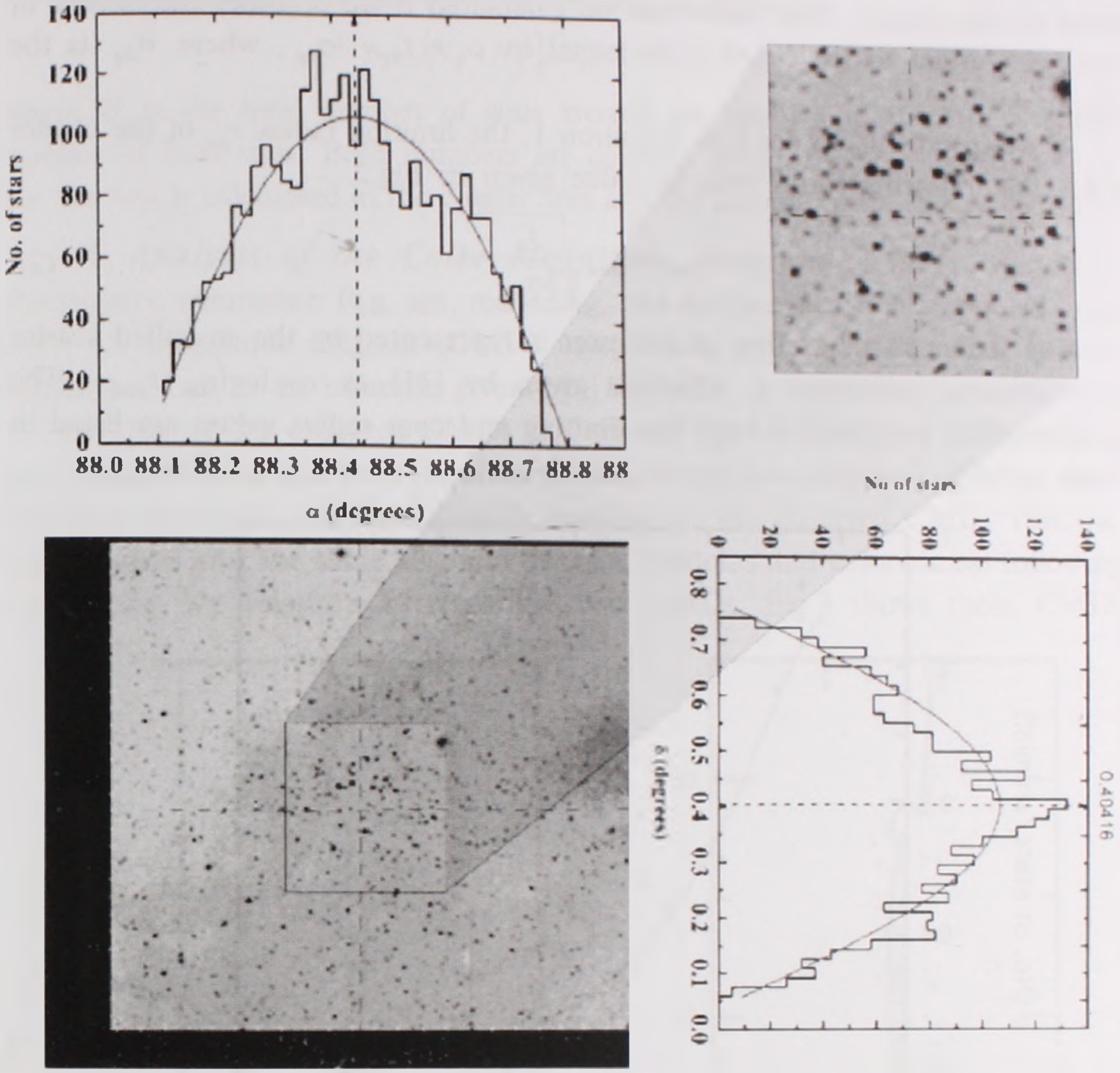

Fig.1. The Gaussian fit provides the coordinates of highest density areas in α and δ for NGC 2112 open cluster, and an image with size (20×20 arcmin^2) in the field of NGC 2112 by LEDAS Digitized Sky Survey DSS.

ring by dividing the number of stars in the ring by its area (i.e. N_i / A_i). We then applied the empirical relation from [10] to parameterize the density function $\rho(r)$ as:

$$\rho(r) = f_{bg} + \frac{f_0}{\left(r / r_{core}\right)^2}, \tag{1}$$

where r_{core}, f_0 and f_{bg} are the core radius, the central surface density, and the background surface density, respectively.

The limiting radius of this cluster could reasonably be defined as the radius of the observed cluster area, as the stellar background density shows a clear stability level at this border. This radius can be calculated if we consider the border of the background density level to be equal to $\rho_b = f_{bg} + 3\sigma_{bg}$, where σ_{bg} is the uncertainty of f_{bg}.

If we insert this border into Equation 1, the limiting radius r_{lim} of the cluster takes the following form, with its value given in Fig.2.

$$r_{lim} = r_{core}\sqrt{\frac{f_0}{3\sigma_{bg}} - 1}. \tag{2}$$

The stellar concentration of a cluster is represented by the so-called cluster concentration parameter c, which is given by [11] as $c = \log(r_{lim}/r_{core})$. The concentration parameter c, and the limiting and core radius values are listed in Table 1.

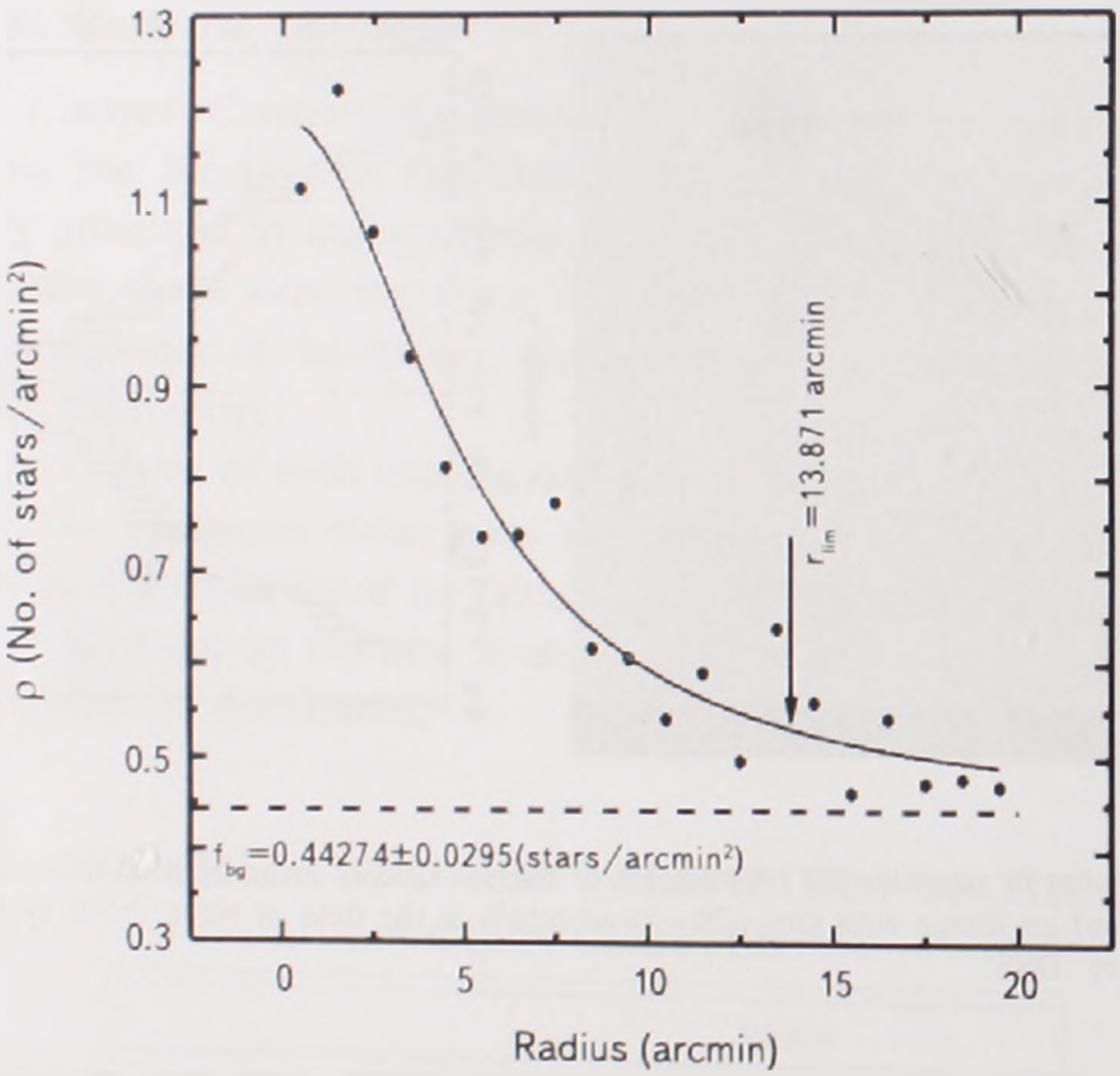

Fig.2. RDP of the NGC 2112 open cluster, with the solid line denoting the fitted density distribution of [10].

2.3. *Probable Cluster Membership*. To estimate the cluster membership probability, we used a star counting technique given by [12] to determine the membership probability for each star in the cluster field. In this technique, the number of neighbors around each star in the cluster can be used to measure the membership probability of a star, p_i.

The parameter p_i for the star i is defined as follows:

$$p_i = \left(N_t - N_f\right)/N_t = 1 - N_f/N_t, \tag{3}$$

where N_t is the total number of stars around the star i and N_f is the average number of field stars. Both numbers are counted within the same aperture size but the first is calculated in the cluster area and the second in the cluster outskirts.

2.4. *Analysis of the Color Magnitude Diagram (CMD).* The main photometric parameters (e.g. age, reddening, and distance) can be determined from the color-magnitude diagram (CMD) using fitting with the theoretical Padova isochrones[1], as in an example given by [13].

We cleaned the observed CMDs (J, J-H and K_s, J-K_s) of NGC 2112 of field star contamination and then fitted them with several isochrones of different ages. The best isochrone was then visually matched to the observed CMD. This was then used to give the same absolute distance modulus and color excess following [14], using the relations between the two CMDs. Fig.3 shows these CMDs

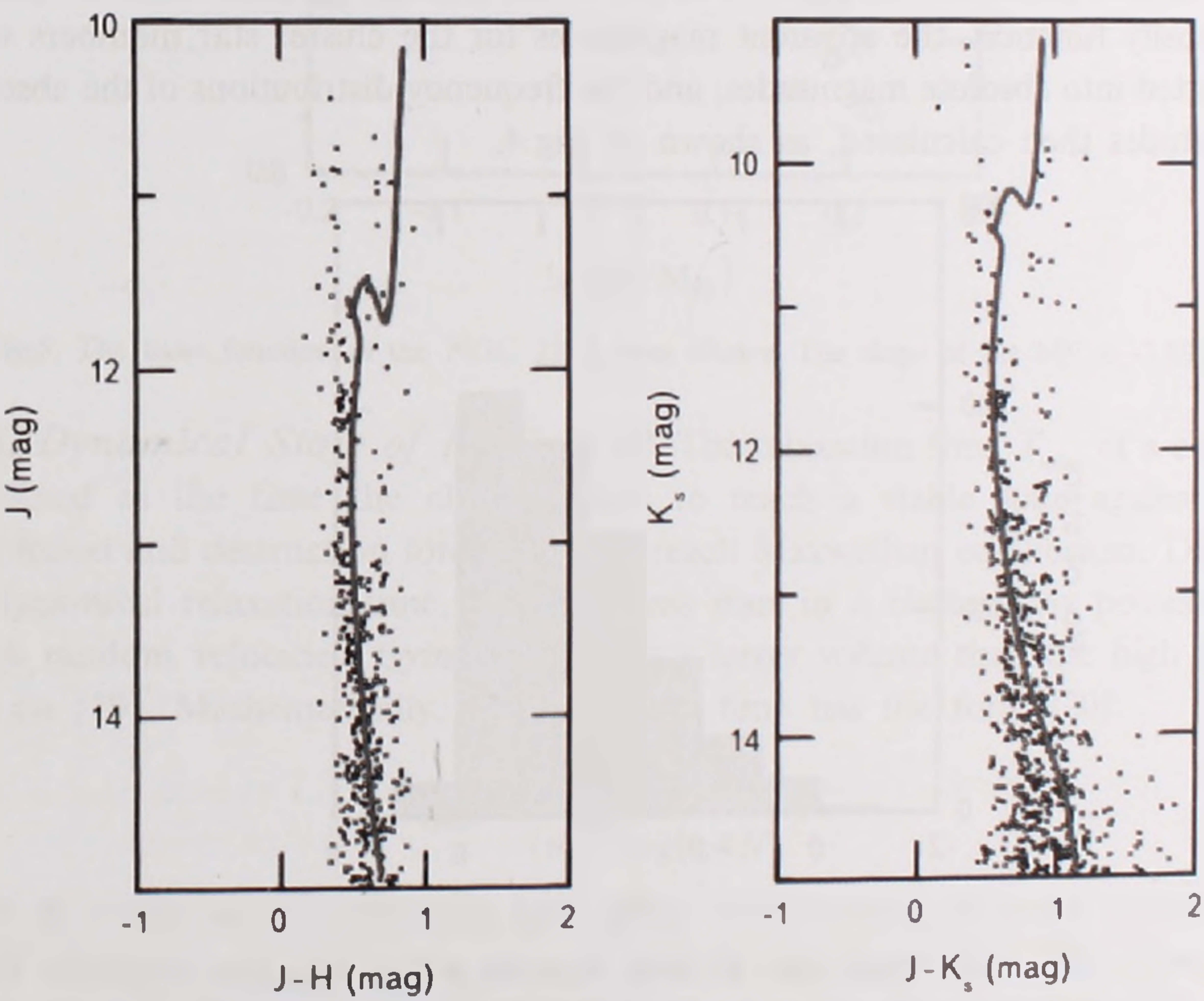

Fig.3. Padova solar-like star ($Z = 0.019$) isochrones with an age (log) of 9.25 for K_s, J-K_s and J, J-H CMDs of the NGC 2112 open cluster.

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[1] *http://stev.oapd.inaf.it/cgi-bin/cmd*

superposed with the selected isochrone, which has an age of log(age) = 9.25 assuming solar metallicity.

To calculate the interstellar reddening, E(B - V), we adopted the interstellar extinction coefficient ratios $A_J / A_V = 0.276$ and $A_H / A_V = 0.176$ for color excess transformations derived from absorption ratios in [15], while the ratio $A_{K_s} / A_V = 0.118$ was derived from [16]. We obtained the following results: $E_{J-H} / E_{B-V} = 0.309 \pm 0.130$, $E_{J-K_s} / E_{B-V} = 0.485 \pm 0.150$, assuming $R_V = A_V / E_{B-V} = 3.1$.

The cluster-Sun distance was then used to determine the distance from the cluster to the galactic center R_{gc}, the projected distances on the galactic plane ($X_{\odot}$ & $Y_{\odot}$) and the distance from the galactic plane $Z_{\odot}$, based on the sketch-chart of [17].

2.5. *Luminosity and Mass Functions*. The stellar luminosity function (LF) is a description of the relative number of stars of different absolute magnitudes. It is used to study the properties of large groups or classes of objects such as stars in clusters or galaxies in the Local Group. To determine the cluster luminosity function, the apparent magnitudes for the cluster star members were converted into absolute magnitudes, and the frequency distributions of the absolute magnitudes then calculated, as shown in Fig.4.

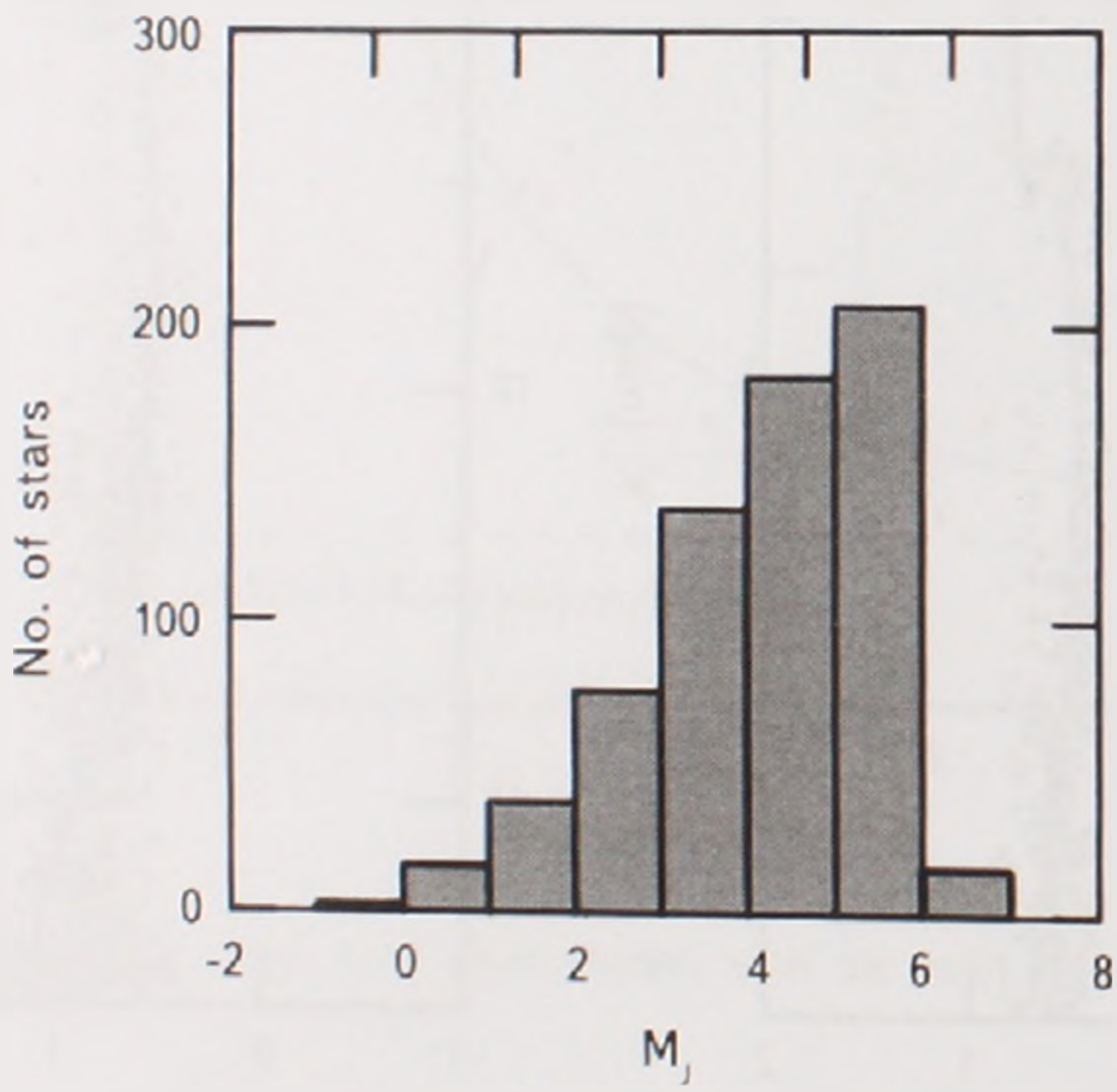

Fig.4. The LF of the NGC 2112 open cluster.

The LF can be converted into a mass function (MF) per the mass-luminosity relation (MLR), which is derived based on the data of the adopted isochrone.

The MLR of this cluster was constructed based on the adopted isochrone [13]. It is a polynomial relation of second order between the mass in $M_{\odot}$ and the absolute magnitude M_K, given as follows:

$$M/M_{\odot} = 1.617385326 - 0.2204258896 M_K + 0.004586772674 M_K^2 . \quad (4)$$

Fig.5 shows the MF of NGC 2112. The slope of this mass function on a logarithmic scale agrees with that given by [18] for the field stars.

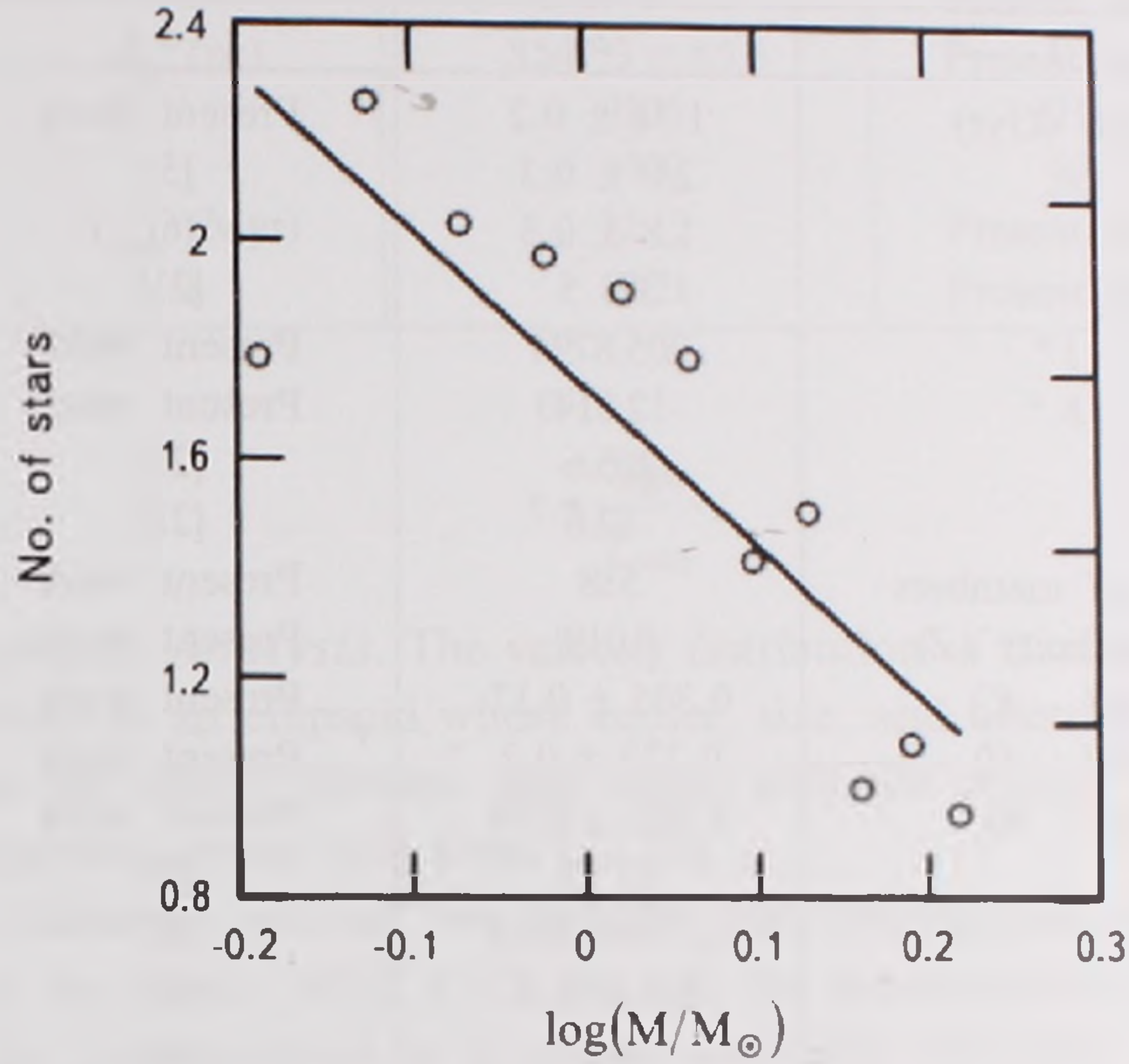

Fig.5. The mass function of the NGC 2112 open cluster. The slope of the MF is -2.89 ± 0.3.

3. *Dynamical State of NGC 2112*. The relaxation time T_{relax} of a cluster is defined as the time the cluster needs to reach a stable state against the contraction and destruction forces, i.e., to reach Maxwellian equilibrium. During the dynamical relaxation time, the low mass stars in a cluster may possess the largest random velocities, trying to occupy a larger volume than the high mass stars do [19]. Mathematically, the relaxation time has the form [20]:

$$T_{relax} = \frac{8.9 \times 10^9 \sqrt{N}\, R_h^{3/2}}{\langle m \rangle^{1/2} \log(0.4 N)} , \quad (5)$$

where R_h is the radius containing half of the cluster mass, N is the number of cluster members and $\langle m \rangle$ is the average mass of the cluster stars. We estimated the dynamical relaxation time for NGC 2112 using the above formula. We also calculated the dynamical evolution parameter τ for the cluster using Eq. (6), and determined that the cluster may be considered dynamically relaxed ($\tau >> 1.0$). The results we obtained in this section and those presented by others are summarized in Table 2.

Table 2

THE PRESENT PHOTOMETRIC PARAMETERS OF THE NGC 2112 OPEN CLUSTER

Parameter	Value	Author
1	2	3
Age (Gyr)	1.78 ± 0.2	Present work
	2.0 ± 0.3	[5]
	1.8 ± 0.3	[6]
	3 - 5	[21]
l °	205.8794	Present work
b °	-12.6143	Present work
	205.6	[21]
	-12.6	[21]
No. of members	538	Present work
Metallicity (Z)	0.019	Present work
$E(J - K_s)$	0.305 ± 0.17	Present work
$E(J - H)$	0.273 ± 0.2	Present work
$(m - M)_0$	9.765 ± 0.05	Present work
	11.35	[5]
	11.75 ± 0.15	[6]
	9.3 ± 0.4	[2]
	9.2	[1]
$E(B - V)$	0.756 ± 0.03	Present work
	0.63 ± 0.14	[5]
	0.6 ± 0.1	[6]
	0.6	[21]
	0.6 ± 0.1	[2]
	0. 5	[1]
Distance (pc)	898 ± 41	Present work
	850 ± 100	[5]
	940 ± 40	[6]
	700 - 800	[2]
	700	[1]
r_{lim} (pc)	3.623 ± 0.3	Present work
R_h (pc)	1.8115 ± 0.2	Present work
r_{core} (pc)	1.329 ± 0.21	Present work
C	0.44	Present work
M_c ($M_\odot$)	641.145	Present work
$\langle m \rangle$ ($M_\odot$)	0.947	Present work
r_t (pc)	12.589 ± 1.2	Present work
$X_\odot$ (pc)	-788.442 ± 12	Present work
$Y_\odot$ (pc)	-382.497 ± 35.4	Present work
	-400	[6]

Table 2 (The end)

1	2	3
$Z_\odot$ (pc)	-196.111 ± 20.02	Present work
	-200	[6]
α	2.89 ± 0.3	Present work
R_{gc} (pc)	8547.3 ± 85.5	Present work
	9200	[21]
	9300	[6]
T_{relax} (Myr)	22.17	Present work
τ	80.21	Present work

$$\tau = \frac{\text{age}}{T_{relax}}. \qquad (6)$$

4. *Kinematic Analysis*. The velocity distribution of stars in a stellar cluster is characterized as an ellipsoid whose center, size, and orientation vary systematically with the cluster lifetime (and hence with the colors), while the stellar velocity dispersion grows with time [22,23].

In the following section, we describe our investigation of the kinematic properties of the cluster NGC 2112, through the determination of its convergent point and the characteristics of its stellar ellipsoidal velocities.

4.1. *The Cluster Convergent Point (A, D)*. To determine the coordinates of the cluster apex, we used the apex diagram (AD diagram) method as demonstrated by [24-27]. The apexes of the stars in a cluster are plotted on the so-called AD diagram. Each apex gives a position on the celestial sphere where the spatial velocity vector of a star intersects it. The apex equatorial coordinates are A (right ascension) and D (declination), as shown in Fig.6. A and D are calculated using coordinates, proper motions, radial velocities, and the determined distance of the cluster for all stars.

The AD diagram can be used to study the kinematic structure of the cluster and to determine its inner kinematical substructures.

We assumed a stellar distance of the cluster of $r_i = 898$ pc, as mentioned previously in the photometric section. Assuming that all the cluster members have the same spatial velocity V, the components of V (V_x, V_y, V_z) along the x, y, and z axes of a coordinate system centered at the Sun can be expressed using the following equations, according to [28].

$$V_x = -4.74\, r_i\, \mu_\alpha \cos\delta \sin\alpha - 4.74\, r_i\, \mu_\delta \sin\delta \cos\alpha + V_r \cos\delta \sin\alpha\,, \qquad (7)$$

$$V_y = +4.74\, r_i\, \mu_\alpha \cos\delta \cos\alpha - 4.74\, r_i\, \mu_\delta \sin\delta \sin\alpha + V_r \cos\delta \sin\alpha\,, \qquad (8)$$

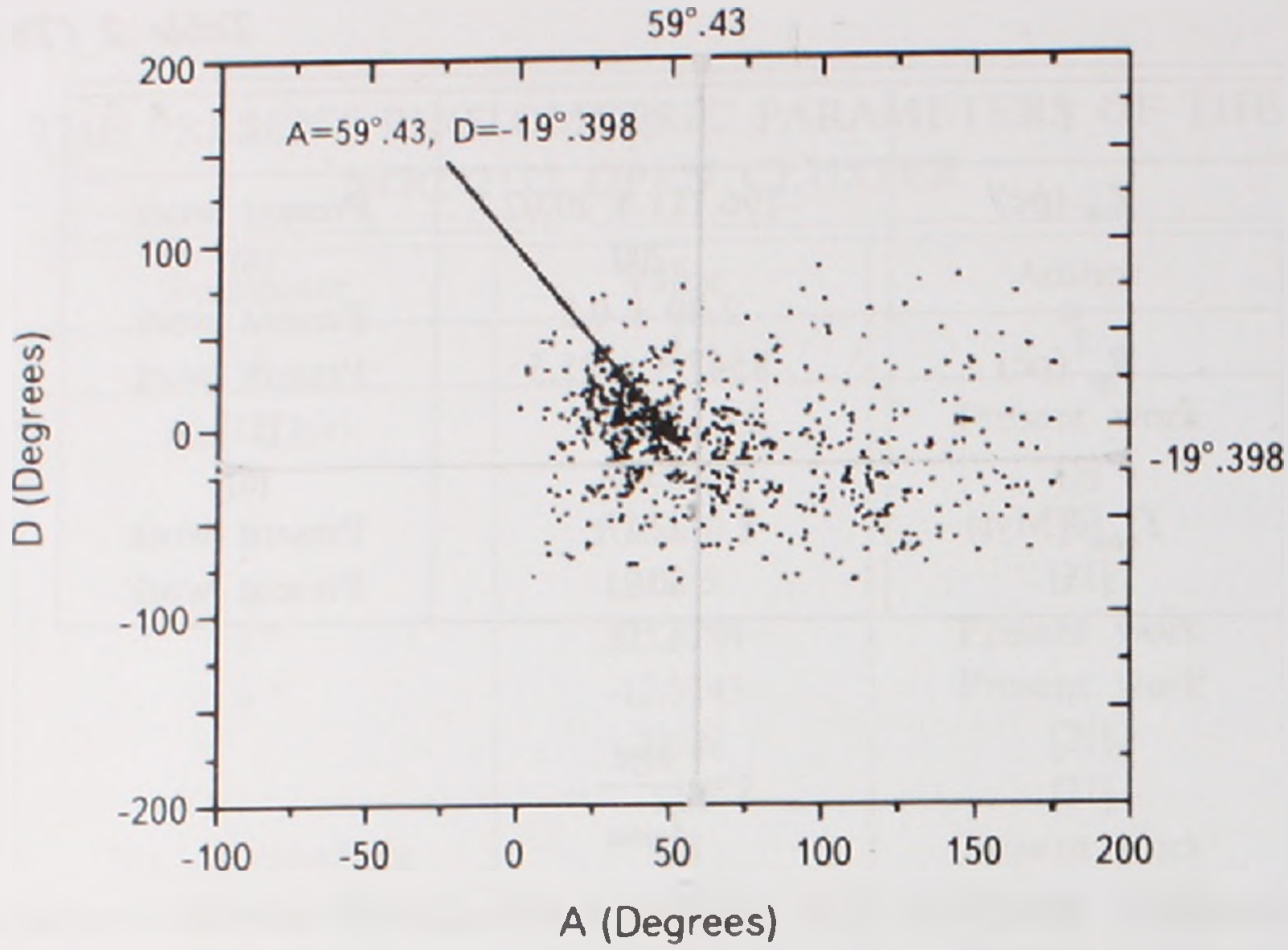

Fig.6. The AD diagram for 538 stars in the NGC 2112 open cluster.

$$V_z = +4.74 r_i \mu_\delta \cos\delta + V_r \sin\delta . \qquad (9)$$

After averaging (V_x, V_y, V_z), we calculated the equatorial coordinates of the convergent point following [22], i.e.

$$A_{conv} = \tan^{-1}\left(\overline{V}_y / \overline{V}_x\right), \qquad (10)$$

$$D_{conv} = \tan^{-1}\left(\overline{V}_z \Big/ \sqrt{\overline{V}_x^2 + \overline{V}_y^2}\right). \qquad (11)$$

The cluster apex coordinates were determined to be: A = 59°.43 ± 0°.002 and D = -19°.398 ± 0°.015. Fig.1 shows a higher stellar concentration close to the location of the cluster apex. These stars have a higher probability of being members than stars far away from this position. This indicates that they have parallel motions. The stars with smaller membership probabilities are further away from the location of the cluster apex.

4.2. *The Velocity Ellipsoid Parameters (VEPs)*. To compute the parameters of the ellipsoidal velocities of the probable members (i.e., 538 stars) of NGC 2112, we used a computational algorithm presented in [29]. The spatial velocities (U, V, W) and their averages ($\overline{U}, \overline{V}, \overline{W}$) were computed.

The coordinates Q_i of point i with respect to an arbitrary axis ξ centered on the center of the stellar distribution were also determined. Following the above-mentioned algorithm, the generalized form of the mean square deviation, σ^2, and the direction of the cosines vector B were calculated, from which a symmetric matrix with elements μ_{ij} and eigenvalues $\lambda_{1,2,3}$ were obtained.

Depending on the matrix controlling the eigenvalue problem for the velocity ellipsoid, we established analytical expressions for some parameters of the correlation studies in terms of the matrix elements μ_{ij} of the eigenvalue problem for the velocity ellipsoid (i.e., VEPs).

- The σ_i parameters ($i = 1, 2, 3$)

The σ_i; $i = 1,2,3$ parameters are defined as

$$\sigma_i = \sqrt{\lambda_i} \tag{12}$$

- The l_i, m_i, and n_i parameters

The direction cosines l, m, and n are mathematically represented as follows:

$$l_i = \left[\mu_{22}\mu_{33} - \sigma_i^2\left(\mu_{22} + \mu_{33} - \sigma_i^2\right) - \mu_{23}^2\right] / D_i; \quad i = 1, 2, 3 \tag{13}$$

$$m_i = \left[\mu_{23}\mu_{13} - \mu_{12}\mu_{33} + \sigma_i^2\mu_{12}\right] / D_i; \quad i = 1, 2, 3 \tag{14}$$

$$n_i = \left[\mu_{12}\mu_{23} - \mu_{13}\mu_{22} + \sigma_i^2\mu_{13}\right] / D_i; \quad i = 1, 2, 3 \tag{15}$$

where

$$\begin{aligned} D_i^2 = &\left(\mu_{22}\mu_{33} - \mu_{23}^2\right)^2 + \left(\mu_{23}\mu_{13} - \mu_{12}\mu_{33}\right)^2 + \left(\mu_{12}\mu_{23} - \mu_{13}\mu_{22}\right)^2 + \\ &+ 2\left[\left(\mu_{22} + \mu_{33}\right)\left(\mu_{23}^2 - \mu_{22}\mu_{33}\right) + \mu_{12}\left(\mu_{23}\mu_{13} - \mu_{12}\mu_{33}\right) + \mu_{13}\left(\mu_{12}\mu_{23} - \mu_{13}\mu_{22}\right)\right]\sigma_i^2 + \\ &+ \left(\mu_{33}^2 + 4\mu_{22}\mu_{33} + \mu_{22}^2 - 2\mu_{23}^2 + \mu_{12}^2 + \mu_{13}^2\right)\sigma_i^4 - 2\left(\mu_{22} + \mu_{33}\right)\sigma_i^6 + \sigma_i^8 . \end{aligned}$$

- Elements of solar motion

The solar motion $S_{\odot}$ can be defined as the absolute value of the Sun's velocity relative to the group of stars under consideration,

i.e.,

$$S_{\odot} = \left(\overline{U}^2 + \overline{V}^2 + \overline{W}^2\right)^{1/2} \text{ km s}^{-1} . \tag{16}$$

The direction in which the Sun moves with respect to the local standard of rest, i.e., the galactic longitude (l_A) and galactic latitude (b_A) of the solar apex, are

$$l_A = \tan^{-1}\left(-\overline{V}/\overline{U}\right), \tag{17}$$

$$b_A = \sin^{-1}\left(-\overline{W}/S_{\odot}\right). \tag{18}$$

These three parameters may be considered the elements of solar motion with respect to the stellar group under consideration.

5. *Results*. A computational routine using Mathematica software was developed to compute the VEPs based on the model described in the last section. Fig.7 shows the Vector Point Diagram (VPD) of 538 stars. The obtained results are given in Table 3 in the following format:

Table 3

KINEMATIC PARAMETERS AND VEPs OF NGC 2112

Parameter	Value	Author
$(\overline{V}_x, \overline{V}_y, \overline{V}_z)_{\mathrm{km\,s^{-1}}}$	(18.0126, 30.5009, -12.4726)	Present work
(A, D)	(59.43 ± 0.002, -19.398 ± 0.015)	Present work
$(\overline{U}, \overline{V}, \overline{W})_{\mathrm{km\,s^{-1}}}$	(-21.5943, -13.9843, -27.3577)	Present work
$(\lambda_1, \lambda_2, \lambda_3)_{\mathrm{km\,s^{-1}}}$	(11356.4, 4686.26, 871.086)	Present work
$(\sigma_1, \sigma_2, \sigma_3)_{\mathrm{km\,s^{-1}}}$	(106.566, 68.4563, 29.5142)	Present work
$(l_1, m_1, n_1)_{\mathrm{deg}}$	(0.4281, -0.691, -0.583)	Present work
$(l_2, m_2, n_2)_{\mathrm{deg}}$	(-0.2096, 0.552, -0.8074)	Present work
$(l_3, m_3, n_3)_{\mathrm{deg}}$	(0.8791, 0.4678, 0.0913)	Present work
$S_\odot$ (km s^{-1})	37.5542 ± 5.4	Present work
l_A°	147.084 ± 0.00	Present work
b_A°	46.763 ± 0.001	Present work

Row 1: The velocities along the *x*, *y*, and *z* axes of a coordinate system whose center is at the Sun.

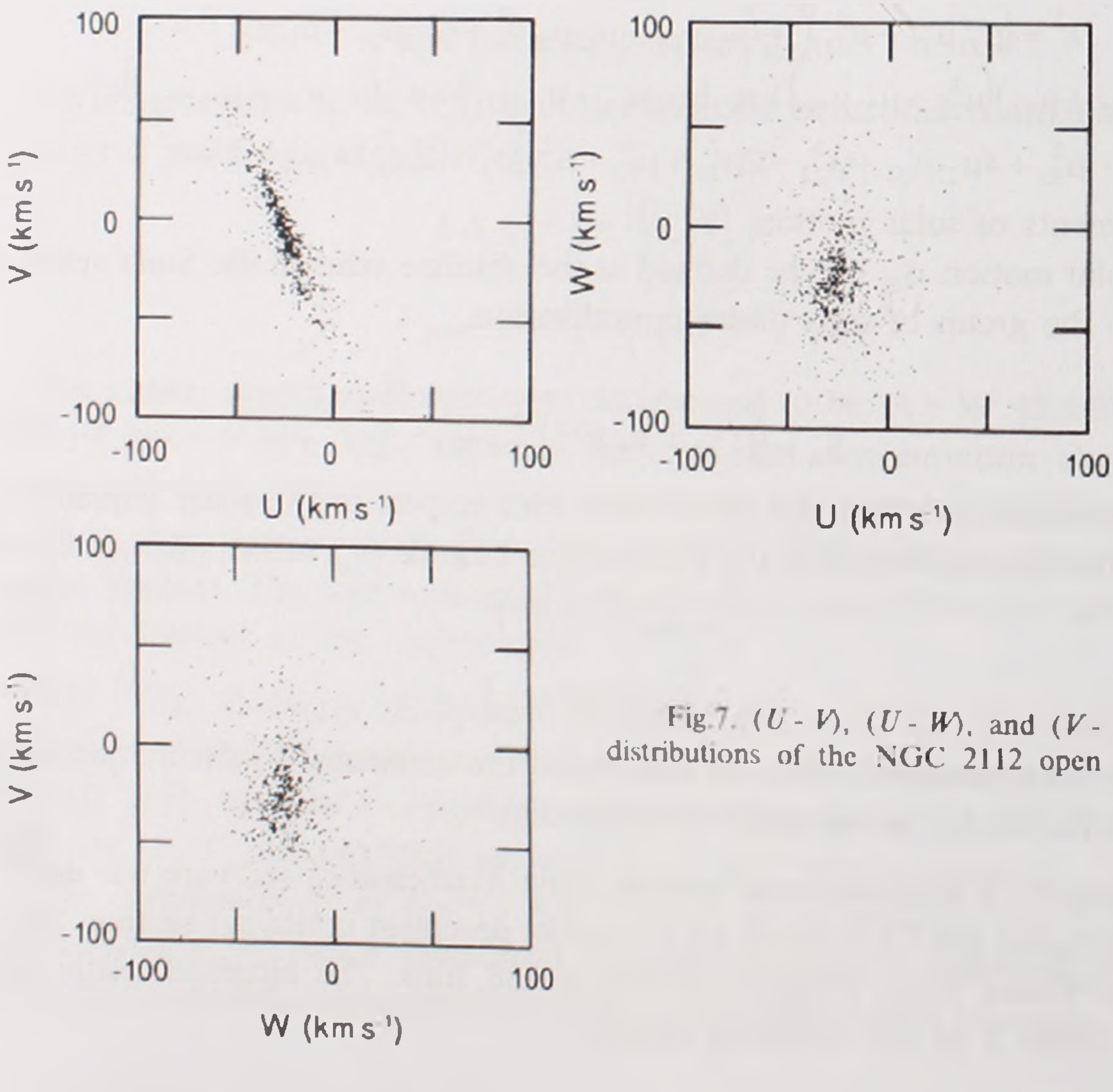

Fig.7. $(U - V)$, $(U - W)$, and $(V - W)$ velocity distributions of the NGC 2112 open cluster.

Row 2: The convergent point (i.e., vertex), with row 3 showing the mean spatial velocities.

Row 4: The eigenvalues, with row 5 showing the velocity dispersion values.

Rows 6, 7, and 8: The direction cosine parameters.

Rows 9, 10, and 11: The elements of the solar motion.

The ratio σ_2/σ_1 is used to determine the Oort constants (A and B), with $(\sigma_2/\sigma_1)^2 = -B/(A-B)$. The values of the Oort constants (A and B) and the ratio (σ_2/σ_1) are presented by [30]. Their ratio (σ_2/σ_1) value is in the range of 0.65-0.74, in good agreement with our value (0.64).

6. *Conclusions*. The main conclusions of our photometric and kinematic study of the open cluster NGC 2112 can be summarized as follows:

- We have obtained the cluster structure parameters based on the derived radial density profile.
- We have obtained values for fundamental physical parameters (e.g., distance, age, and distance modulus) from the cleaned CMD.
- The cluster LF shows a gradual increase towards low-luminosity stars, and its dimensionless exponent (i.e., $\alpha = 2.89 \pm 0.3$) agrees with that provided by Salpeter.
- The cluster may be considered dynamically relaxed, and the mass segregation effect caused by its dynamical evolution must be considered an important factor.
- We computed the convergent point ($A = 59^\circ.43 \pm 0^\circ.002$ and $D = -19^\circ.398 \pm 0^\circ.015$) using the AD diagram technique.
- We also computed the VEPs, the ratio of σ_2/σ_1 and the Oort constants, which are in good agreement with the previously published values.
- Finally, the velocity dispersion and solar velocity were also computed.

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ФОТОМЕТРИЧЕСКИЕ И КИНЕМАТИЧЕСКИЕ СВОЙСТВА БЛИЖАЙШЕГО ОТКРЫТОГО ЗВЕЗДНОГО СКОПЛЕНИЯ NGC 2112

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На основе данных, собранных из каталога PPMXL, оцениваются фотометрические параметры (ограниченность ядра, приливной радиус, расстояние, светимость и функции масс, полная масса, время релаксации) открытого звездного скопления среднего возраста NGC 2112. Ближайшее скопление находится на расстоянии 898 ± 41 пк от Солнца с логарифмом возраста, равным 9.25 ± 0.05 в предположении, что солнечная металличность $Z = 0.019$. Оценочное время релаксации скопления короче оценочного возраста, что указывает на возможность пребывания его в состоянии динамической релаксации. Кроме того рассчитана точка сходимости: найденные с использованием диаграммы AD ее экваториальные координаты равны $59^{\circ}.430$, $-19^{\circ}.398$. Определены также параметры эллипсоидальной скорости скопления.

Ключевые слова: *NGC 2112: открытые скопления: фотометрический анализ: кинематическое исследование: постоянная Оорта: апекс: точка сходимости*

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