АСТРОФИЗИКА

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KINEMATICS AND VELOCITY ELLIPSOID PARAMETERS OF STELLAR GROUPS AND OPEN STAR CLUSTERS: II COOL STARS

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Based on the galactic space velocity components (*U, V, W*) and with aid of the vector and matrix analyses, we computed the velocity ellipsoid parameters for 790 late-type stars from CoRoT (Convection Rotation and Transits) observations and 290 *L* dwarf stars. We ran the calculations for spectral types *F, G* and *K* for late-type stars and *L0, L1, L2* and *L3* for *L* dwarf stars. We found that the ratio of the middle to the major axis in the galaxy ranged from 0.35 to 0.73. The vertex deviation from the galactic center was very small for the samples under investigation, which agrees well with earlier calculations.

Key words: *Velocity ellipsoid: CoRoT stars: L dwarfs: Vertex deviation: Oort's constants*

1. *Introduction*. The motions of nearby stars can reveal important information about the spiral structures and star formations in our galaxy [1]. The characteristic feature of these motions is that the peculiar velocities have an axis of greatest mobility. This characteristic is most conveniently represented using the ellipsoidal law of velocity distribution [2].

The velocity ellipsoid parameters are important because they are connected to the most important mathematical function of stellar astronomy, that is, the phase density function.

There are three methods that determine the velocity ellipsoid parameters: radial velocities, proper motion, and space velocities [3].

This paper is the second in a series that investigate the velocity ellipsoid of stellar groups and open clusters. In the first paper [4], we computed the velocity ellipsoid for the solar neighborhood white dwarfs. Here, we compute the velocity ellipsoid parameters for two samples of cool (CoRoT FGK type stars) and ultra-cool stars (*L* dwarf). The structure of this paper is as follows. Section 2 describes the data, and the computational method is outlined in Section 3. Section 4 contains our results and conclusions.

2. *Data*. We used two data samples in our investigation: the CoRoT (Convection Rotation and Transits) observations for late type stars and the SDSS (Sloan Digital Sky Survey) observations of *L*-type dwarfs.

We used the kinematical properties of CoRoT stars; the CoRoT space telescope [5] is an astronomical experiment dedicated to stellar seismology and the search for extrasolar planets. The mission is led by CNES (Centre national d'etudesspatiales) in association with French laboratories and with significant international participation. The ESA (Science Programme), RSSD/ESTEC, Austria, Belgium, and Germany contribute to the payload, whereas Spain and Brazil contribute to the ground segment [6]. The CoRoT was mainly designed to study the internal structures of many families of stars [7]. It can provide a complete kinematical description of 754 stars in three CoRoT fields. The kinematical criteria were used to identify the galactic populations in their sample and study their characteristics, particularly their chemistry. Here, we selected approximately

i) $346\,\mathrm{CoRoT\ stars} \le 1000\,\mathrm{pc}$, and

ii) $1000\,\mathrm{pc} \le 250\,\mathrm{CoRoT\ stars} \le 2000\,\mathrm{pc}$.

L dwarfs (LDs) are ultra-cool objects; they are cooler than *M* dwarfs. Most LDs are expected to be brown dwarfs, i.e., they do not reach the hydrogen burning phase. A sample of very cool dwarfs (LDs) were presented in [8], which contains 484 LD from the Sloan Digital Sky Survey SDSS. Here, we used approximately 290 LDs, classified as follows.

a) Approximately 176 *L0* ($\approx 2550\,\mathrm{K}$), ranging from 10.8 to 113.7 pc.

b) Approximately 59 *L1* ($\approx 2100\,\mathrm{K}$), ranging from 8.6 to 109.9 pc.

c) Approximately 20 *L2* ($\approx 1960\,\mathrm{K}$), ranging from 28.8 to 107.7 pc.

d) Approximately 35 *L3* ($\approx 1830\,\mathrm{K}$), ranging from 20 to 109.4 pc.

3. *Computational Method*. We used the computational algorithm in [9] to compute the velocity ellipsoid and its parameters for the above sample of data.

For the galactic space velocities (*U*, *V*, *W*), we can define the mean velocities

$$\overline{U} = \frac{1}{N}\sum_{i=1}^{N} U_i; \quad \overline{V} = \frac{1}{N}\sum_{i=1}^{N} V_i; \quad \overline{W} = \frac{1}{N}\sum_{i=1}^{N} W_i, \tag{1}$$

where N is the total number of stars.

Let ξ be an arbitrary axis, with a zero point coincident to the center of the distribution. Let l, m, and n be the directional cosines of the axis with respect to the shifted axis. Then, the coordinates Q_i of the point i, with respect to the ξ axis are

$$Q_i = l\left(U_i - \overline{U}\right) + m\left(V_i - \overline{V}\right) + n\left(W_i - \overline{W}\right). \tag{2}$$

Let us adopt, as the measured of the scatter components Q_i, a generalization of the mean square deviation. It is defined as

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} Q_i^2. \tag{3}$$

From Equations (1), (2) and (3), we can deduce after some calculations that

$$\sigma^2 = \underline{x}^T B \underline{x}, \tag{4}$$

where $\underline{x}$ is the (3×1) direction cosines vector and B is a (3×3) symmetric matrix μ_{ij}, with elements

$$\left.\begin{aligned} \mu_{11} &= \frac{1}{N}\sum_{i=1}^{N} U_i^2 - \left(\bar{U}\right)^2; \quad \mu_{12} = \frac{1}{N}\sum_{i=1}^{N} U_i V_i - \bar{U}\bar{V}; \\ \mu_{13} &= \frac{1}{N}\sum_{i=1}^{N} U_i W_i - \bar{U}\bar{W}; \quad \mu_{22} = \frac{1}{N}\sum_{i=1}^{N} V_i^2 - \left(\bar{V}\right)^2; \\ \mu_{23} &= \frac{1}{N}\sum_{i=1}^{N} V_i W_i - \bar{V}\bar{W}; \quad \mu_{33} = \frac{1}{N}\sum_{i=1}^{N} W_i^2 - \left(\bar{W}\right)^2. \end{aligned}\right\} \tag{5}$$

The necessary conditions for an extremum are now

$$(B - \lambda I)\underline{x} = 0. \tag{6}$$

These are three homogenous equations in three unknowns, which have a nontrivial solution if, and only if,

$$D(\lambda) = |B - \lambda I| = 0, \tag{7}$$

where λ is the eigenvalue, and $\underline{x}$ and B are

$$\underline{x} = \begin{bmatrix} l \\ m \\ n \end{bmatrix} \quad \text{and} \quad B = \begin{vmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{23} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{vmatrix}.$$

Equation (7) is a characteristic equation for the matrix B. The required roots (i.e., eigenvalues) are

$$\left.\begin{aligned} \lambda_1 &= 2\rho^{1/3}\cos\frac{\phi}{3} - \frac{k_1}{3}; \\ \lambda_2 &= -\rho^{1/3}\left\{\cos\frac{\phi}{3} - \sqrt{3}\sin\frac{\phi}{3}\right\} - \frac{k_1}{3}; \\ \lambda_3 &= -\rho^{1/3}\left\{\cos\frac{\phi}{3} + \sqrt{3}\sin\frac{\phi}{3}\right\} - \frac{k_1}{3}. \end{aligned}\right\} \tag{8}$$

Here,

$$\left.\begin{aligned} k_1 &= -(\mu_{11} + \mu_{22} + \mu_{33}), \\ k_2 &= \mu_{11}\mu_{22} + \mu_{11}\mu_{33} + \mu_{22}\mu_{33} - \left(\mu_{12}^2 + \mu_{13}^2 + \mu_{23}^2\right), \\ k_3 &= \mu_{12}^2\mu_{33} + \mu_{13}^2\mu_{22} + \mu_{23}^2\mu_{11} - \mu_{11}\mu_{22}\mu_{33} - 2\mu_{12}\mu_{13}\mu_{23}. \end{aligned}\right\} \tag{9}$$

$$q = \frac{1}{3}k_2 - \frac{1}{9}k_1^2; \quad r = \frac{1}{6}(k_1 k_2 - 3k_3) - \frac{1}{27}k_1^3 \tag{10}$$

$$\rho = \sqrt{-q^3} \tag{11}$$

$$x = \rho^2 - r^2 \tag{12}$$

and

$$\phi = \tan^{-1}\left(\frac{\sqrt{x}}{r}\right). \tag{13}$$

Using the matrix that controls the eigenvalue problem (Equation (6)) for the velocity ellipsoid, we can establish the analytical expressions of some parameters for the correlation studies in terms of the matrix elements (μ_{ij}) (i.e., the velocity ellipsoid parameters, VEPs).

- σ_i; $i = 1, 2, 3$ parameters

The σ_i; $i = 1, 2, 3$ parameters are defined as

$$\sigma_i = \sqrt{\lambda_i}\,. \tag{14}$$

- l_i, m_i and n_i parameters

(l, m, n) are the directional cosines for the eigenvalue problem. They can be calculated using

$$l_i = \left[\mu_{22}\mu_{33} - \sigma_i^2\left(\mu_{22} + \mu_{33} - \sigma_i^2\right) - \mu_{23}^2\right]/D_i; \quad i = 1, 2, 3, \tag{15}$$

$$m_i = \left[\mu_{23}\mu_{13} - \mu_{12}\mu_{33} + \sigma_i^2\mu_{12}\right]/D_i; \quad i = 1, 2, 3, \tag{16}$$

and

$$n_i = \left[\mu_{12}\mu_{23} - \mu_{13}\mu_{22} + \sigma_i^2\mu_{13}\right]/D_i; \quad i = 1, 2, 3, \tag{17}$$

where

$$\begin{aligned} D_i^2 &= \left(\mu_{22}\mu_{33} - \mu_{23}^2\right)^2 + \left(\mu_{23}\mu_{13} - \mu_{12}\mu_{33}\right)^2 + \left(\mu_{12}\mu_{23} - \mu_{13}\mu_{22}\right)^2 + \\ &+ 2\left[\left(\mu_{22} + \mu_{33}\right)\left(\mu_{23}^2 - \mu_{22}\mu_{33}\right) + \mu_{12}\left(\mu_{23}\mu_{13} - \mu_{12}\mu_{33}\right) + \mu_{13}\left(\mu_{12}\mu_{23} - \mu_{13}\mu_{22}\right)\right]\sigma_i^2 + \\ &+ \left(\mu_{33}^2 + 4\mu_{22}\mu_{33} + \mu_{22}^2 - 2\mu_{23}^2 + \mu_{12}^2 + \mu_{13}^2\right)\sigma_i^4 - 2\left(\mu_{22} + \mu_{33}\right)\sigma_i^6 + \sigma_i^8 . \end{aligned}$$

- Solar motion

Let $S_\odot$ be the absolute value of the Sun's velocity relative to the stars under consideration,

i.e.,

$$S_\odot = \sqrt{\overline{U}^2 + \overline{V}^2 + \overline{W}^2} \quad \text{km/s}.$$

The galactic longitude (l_A) and latitude (b_A) of the solar apex are

$$l_A = \tan^{-1}\left(-\overline{V}/\overline{U}\right),$$

$$b_A = \sin^{-1}\left(-\overline{W}/S_\odot\right).$$

Table 1

VEPs FOR 790 CoRoT *FGK* STARS

VEPs	140 *F*-type stars ([illegible] K)			321 *G*-type stars (5200-6000 K)			329 *K*-type stars (3700-5200 K)		
$(\overline{U},\overline{V},\overline{W})$ km/s	-11.76	-16.13	-7.654	-10.122	-20.94	-3.5168	-7.835	-19.85	-1.147
$(\sigma_1,\sigma_2,\sigma_3)$ km/s	41.37	36.03	30.29	42.35	40.42	29.54	154.20	55.54	88.68
$(\lambda_1,\lambda_2,\lambda_3)$ km/s	1712.2	1298.47	917.737	1794.09	1634.29	875.879	23779.1	7864.69	3084.81
(l_1, m_1, n_1)	0.935	-0.116	0.334	0.920	-0.3898	-0.0399	0.591	-0.757	-0.278
(l_2, m_2, n_2)	0.3460	0.1020	-0.9326	0.0356	0.1848	-0.9821	0.7748	0.628	-0.064
(l_3, m_3, n_3)	-0.0741	-0.987	-0.135	-0.3902	-0.9021	-0.1839	-0.22	0.177	-0.958

Table 2

VEPs FOR 290 *L* DWARFS

VEPs	176 of *L0* - type, $10.8 \le$ distance(pc) ≤ 113.7			59 of *L1* - type, $8.6 \le$ distance(pc) ≤ 109.9			20 of *L2* - type, $28.8 \le$ distance(pc) ≤ 107.7			35 of *L3* - type, $20 \le$ distance(pc) ≤ 109.4		
$(\overline{U},\overline{V},\overline{W})$ km/s	1.71	-10.43	-0.82	1.26	-10.50	3.62	-9.63	-22.47	-0.33	13.89	-11.74	4.21
$(\sigma_1,\sigma_2,\sigma_3)$ km/s	35.09	26.20	23.03	31.53	27.27	21.67	45.22	37.14	15.92	32.93	31.40	17.98
$(\lambda_1,\lambda_2,\lambda_3)$ km/s	1231.52	686.41	530.37	994.32	743.46	469.50	2044.6	1379.0	253.47	1084.4	985.68	323.2
(l_1, m_1, n_1)	0.94	0.34	0.06	0.85	0.53	-0.02	0.73	0.69	0.02	0.80	0.60	-0.01
(l_2, m_2, n_2)	0.23	-0.46	-0.86	0.21	-0.31	0.93	0.11	-0.15	0.98	0.01	-0.03	-1.00
(l_3, m_3, n_3)	-0.26	0.82	-0.51	-0.48	0.79	0.38	-0.68	0.71	0.18	-0.60	0.80	-0.03

4. *Results and Conclusion.* Based on the previously described model, we developed a Mathematica routine that computes the kinematics and velocity ellipsoid parameters. Fig.1 to 4 show the distribution of the space velocities of the two samples (*FGK* CoRoT and *L* dwarf stars). We ran the routine for the following samples.

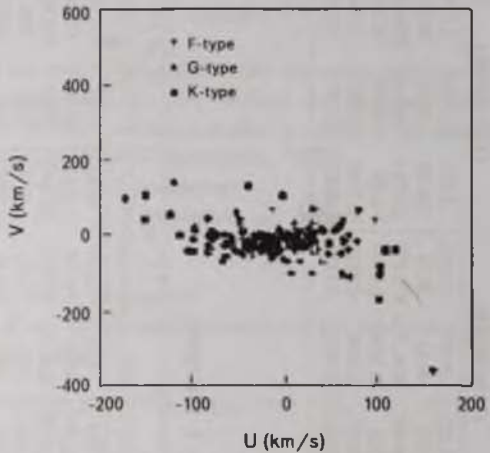

Fig.1. The U - V plane for *FGK* stars.

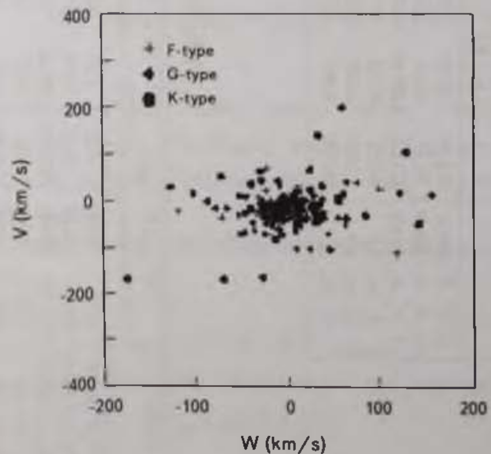

Fig.2. The W - V plane for *FGK* stars.

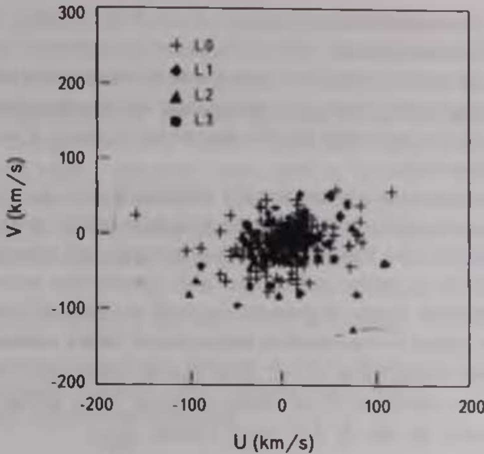

Fig.3. *U* - *V* plane for the *L* dwarf stars.

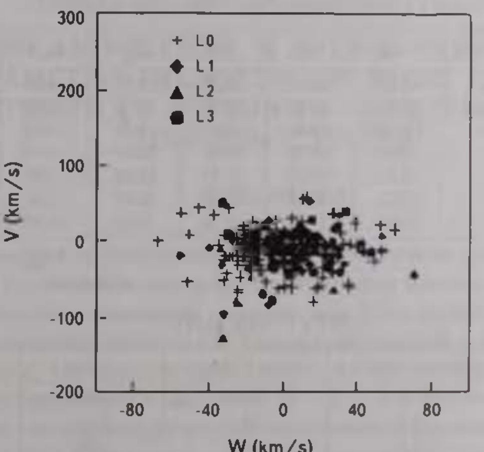

Fig.4. *W* - *V* plane for the *L* dwarf stars.

- CoRoT stars of spectral type *F* (140 candidates).
- CoRoT stars of spectral type *G* (321 candidates).
- CoRoT stars of spectral type *K* (329 candidates).
- *L0* dwarf (176 candidates).
- *L1* dwarf (59 candidates).

- *L2* dwarf (20 candidates).
- *L3* dwarf (35 candidates).

The results are shown in Tables 1 and 2, which contains the mean space velocities, the velocity dispersion, the eigenvalues, and the parameters (l_i, m_i, n_i, $i=1$, 2 and 3). As an initial test for our code, we have $l_i^2+m_i^2+n_i^2=1$ for the two samples used.

Table 3 shows the values of the velocity dispersions σ_1, σ_2, and σ_3, the ratio σ_2/σ_1, the solar velocity $S_{\odot}$, and the longitude of the vertex l_2 for the present work and from the literature. The Oort constants are among the most important quantities in stellar kinematics. The relationship between these constants and the ratio σ_2/σ_1 is given by $(\sigma_2/\sigma_1)^2=-B/(A-B)$. This should be satisfied if the galaxy is considered to be stationary. Oort's constants (*A* and *B*) and the corresponding ratios σ_2/σ_1 from various calculations are listed in Table 4 [10]. Most values of σ_2/σ_1 listed in Table 3 are within the range of 0.55-0.75, except for the *K*, *L2*, and *L3* dwarf stars.

Table 3

VELOCITY DISPERSIONS FOR THE SAMPLES

Target/Spectral type	σ_1	σ_2	σ_3	$S_{\odot}$	σ_2/σ_1	l_2
F CoRot stars	41.37	36.03	30.29	21.37	0.87	0.346
G CoRot stars	42.35	40.42	29.54	23.52	0.95	0.035
K CoRot stars	154.20	88.68	55.54	21.38	0.58	0.778
LD0	35.09	26.20	23.03	10.60	0.75	0.230
LD1	31.53	27.27	21.67	11.18	0.86	0.210
LD2	45.22	37.14	15.92	24.44	0.82	0.110
LD3	32.93	31.40	17.98	18.67	0.95	0.010

Table 4

OoRT's CONSTANTS

A (km s^{-1} kpc^{-1})	*B* (km s^{-1} kpc^{-1})	σ_2/σ_1
14.5	-12	0.65
12.6	-13.2	0.71
14.8	-12.4	0.67
11.3	-13.9	0.74

Considering the symmetry, one would expect that one axis of the velocity ellipsoid of stars in the galactic plane to point to the galactic center, i.e., l_2 should be zero. However, Table 3 shows that the longitude of the vertex l_2 was not zero for the two samples. Our samples represent cool and very cool stars, so the vertex deviation is very small compared with the youngest stars. For *L* dwarfs, the vertex deviation clearly diminishes as we go from *L0* to

L3, which implies that there is a temperature dependence.

We also calculated the parameters of the velocity ellipsoid for late type stars (*FGK* CoRoT stars with 790 candidates, and *L* dwarf stars with 290 candidates).

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КИНЕМАТИЧЕСКИЕ И ЭЛЛИПСОИДАЛЬНЫЕ ПАРАМЕТРЫ СКОРОСТИ ЗВЕЗДНЫХ ГРУПП И ОТКРЫТОГО ЗВЕЗДНОГО СКОПЛЕНИЯ: II ХОЛОДНЫЕ ЗВЕЗДЫ

В.ЭЛСАНОРИ

Основываясь на компоненты (*U*, *V*, *W*) галактически пространственной скорости и с помощью векторного и матричного анализа, мы вычислили эллипсоидальные параметры скорости для 790 поздних типов звезд из CoRoT (Convection Rotation and Transits) наблюдений и 290 карликовых звезд типа *L*. Мы сделали вычисления для спектральных типов *F*, *G* и *K* поздних типов звезд и для типов *L0*, *L1*, *L2* и *L3* карликовых *L* звезд. Мы нашли, что отношение средней оси к большой изменяется в галактике от 0.35 до 0.73. Наибольшее смещение от галактического центра было очень маленьким для исследуемых образцов, что хорошо согласуется с прежними вычислениями.

Ключевые слова: *эллипсоид скорости: CoRoT звезды: L-карлики: постоянные Оорта*

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