АСТРОФИЗИКА

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ANISOTROPIC COSMOLOGICAL MODEL WITH COSMIC STRING IN $f(R)$ THEORY OF GRAVITY

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The paper is dedicated to study the Bianchi type-VI_0 space-time full of cosmic string within the framework of $f(R)$ gravity. The physical and kinematical options of the model are studied and mentioned. The operate $f(R)$ of the Ricci scalar is additionally evaluated for the model

Key words: *cosmic string: Bianchi type -VI_0 metric: $f(R)$ theory*

1. *Introduction*. Modifying gravity is another approach to elucidate the cosmic acceleration providing a attractive force various to dark energy. This provides rise to alternate theories of gravity like $f(T)$ gravity (Ferraro and Fiorini [1]; Bengochea and Ferraro [2]; Linder [3]), Gauss-Bonnet gravity (Carroll et al. [4]; Cognola et al. [5]), etc. Nojiri and Odintsov [6-7] showed that the modified theories of gravity give a natural attractive force various means for dark energy. In these theories, the geometrical a part of Einstein-Hilbert action is changed by adding the higher-order curvature invariants (Schmidt [7]). $f(R)$ gravity [8-10] represents a viable varied to dark energy and naturally provides rise to quick singularity-free solutions in early and late cosmic epochs [11-15].

In $f(R)$ theory, we have a tendency to use a general operate of Ricci scalar because the Lagrangian density. As a result of the looks of the operate of the Ricci scalar the ensuing field equation is fourth order, whereas the field equation in Einstein's relativity theory is second order. $f(R)$ theory was 1st projected by Buchdal [16]. theory gained further quality once a lot of developments by Starobinsky [17] and later following the conclusion of the discrepancy between theory and observation [18-28]. The foremost roomily discovered actual solutions in $f(R)$ gravity area unit the spherically symmetric solutions that were found by Multamaki and Vilja [29]. The precise solutions of $f(R)$ gravity coupled to nonlinear electrodynamics are studied by Hollestein and Lobo [30]. Azadi et al. [31] analyzed the static cylindrically symmetric vacuum solutions in Weyl coordinates in $f(R)$ gravity. The Godel sort models in $f(R)$ gravity are explored by Reboucas and Santos [32] and Santos et al [33]. Shamir [34] analyzed some Bianchi sort cosmological models during this theory. Sebastiani and Zerbini [35] bestowed the non-constant curvature vacuum solutions for the static spherically symmetric metrics in metric $f(R)$ gravity. Sharif and Shamir [36] studied actual vacuum solutions of Bianchi type-I and type-V space times in $f(R)$ theory of gravity. Sharif and Shamir [37] obtained non-vacuum solutions in Bianchi type-I and type-V exploitation perfect fluid in $f(R)$ gravity. Sharif and Shamir [38] investigated the energy distribution of some plane bilaterally symmetrical metrics in $f(R)$ theories of gravity with the belief of constant Ricci scalar. Sharif and Shamir [39] studied the constant curvature vacuum solutions of plane bilaterally symmetrical space time in metric $f(R)$ gravity by considering the power law model. Aktas et al. [40] have studied aeolotropic models in theory of $f(R)$ gravity. Adhav [41] mentioned the Kantowski-Sachs string cosmological model in $f(R)$ gravity. Singh et al. [42] studied purposeful variety of with power-law growth in aeolotropic model. Reddy et al. [43] studied vacuum solutions of Bianchi type-I and V models in $f(R)$ gravity with a special form of deceleration parameter. Recently, Zero Mass field with Bulk consistency for FRW reference system in $f(R)$ theory are mentioned by Katore et al. [44]. Katore and Shaikh [45] studied Bianchi type III Bulk viscous cosmological models in $f(R)$ gravity.

Cosmic strings have attracted many astrophysicists to try and do to realize a possible description of the primary stage of the universe. The study of cosmic strings in high-energy physics arises from the gauge theories with spontaneous broken symmetry. Once the big bang, it's believed that the universe would possibly want tough sort of section transitions by manufacturing vacuum domain structures like domain walls, strings and monopoles. The overall relativistic treatment of strings was initially done by Stachel [46] and Letelier [47-48].Many relativists [49-64] studied numerous aspects of cosmic string cosmological models. Rao and Sireesha [65-67] investigated string cosmological models in scalar tensor theories of gravitation. Katore and Shaikh [68-70] obtained string cosmological models in numerous contexts. Recently, Adhav [71] studied B-III cosmic string cosmological models in $f(R)$ gravity. Recently, Katore [72] investigates Bianchi sort II VIII and IX string cosmological models within the context of $f(R)$ gravity. This motivates us to investigate Bianchi type VI_0 universe with effect of string in $f(R)$ gravity.

2. $f(R)$ *Gravity Formalism*. The $f(R)$ theory of gravity is the generalization of General Relativity. The action for this theory is given by

$$S=\int\sqrt{-g}\left(\frac{1}{16\pi G}f(R)+L_m\right)d^4x. \tag{1}$$

Here $f(R)$ is a general function of the Ricci Scalar, g is the determinant of the metric $g_{\mu\nu}$ and L_m is the metric Lagrangian that depends on $g_{\mu\nu}$. It is noted that this action is obtained just by replacing R by $f(R)$ in the standard Einstein-Hilbert action.

The corresponding field equations square measure found by varied the action with relevancy the metric $g_{\mu\nu}$

$$F(R)R_{\mu\nu}-\frac{1}{2}f(R)g_{\mu\nu}-\nabla_\mu\nabla_\nu F(R)+g_{\mu\nu}\Theta F(R)=T_{\mu\nu}^{M}. \tag{2}$$

where

$$\Theta=\nabla^\mu\nabla_\mu,\quad F(R)=\frac{df(R)}{dR}. \tag{3}$$

∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m.

3. *Metric and Field Equations*. We take into account the space time metric of the spatially homogeneous and anisotropic Bianchi type VI_0 of the form

$$ds^2=-dt^2+A^2\,dx^2+B^2\,e^{-2qx}\,dy^2+C^2\,e^{2qx}\,dz^2, \tag{4}$$

where A, B and C are the functions of t only and q is non-zero constant.

For Bianchi type VI_0, the corresponding Ricci Scalar curvature model is given by

$$R=2\left[\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}-\frac{q^2}{A^2}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}\right], \tag{5}$$

where dot represents derivative with respect to t.

For cosmic strings, the energy momentum tensor T_{ij} is given by

$$T_{ij}=\rho u_i u_j-\lambda x_i x_j. \tag{6}$$

Here ρ is the rest energy density of the system of strings with massive particles attached to the strings and λ the tension density of the system of strings. As pointed out by Latelier [48] λ may be positive or negative, u^i describes the system of four velocities and x^i represents a direction of anisotropy, i.e. the direction of strings, we have

$$u^i u_i=-x^i x_i=-1 \quad \text{and} \quad u^i x_i=0. \tag{7}$$

We consider $\rho=\rho_p+\lambda$ is the rest energy cloud of strings with particles attached to them, λ is the tension density of the string and ρ_p is the rest density of the particles u^i the cloud four-velocity and x^i to be along x-axis.

From Eqs. (6) and (7), in the co-moving co-ordinate system, we have

$$T_1^1=-\lambda,\quad T_2^2=T_3^3=0,\quad T_4^4=-\rho,\quad T_i^j=0,\quad i\neq j. \tag{8}$$

Using Eqs. (6)-(8), the corresponding field Eq. (2) for cosmic string in respect of the Bianchi type VI_0 space-time reduce to the following set of equations

$$\left(\frac{\ddot{A}}{A}-2\frac{q^2}{A^2}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}\right)F+\frac{1}{2}f(R)+\ddot{F}+\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{F}=\lambda, \tag{9}$$

$$\left(\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}\right)F+\frac{1}{2}f(R)+\ddot{F}+\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\dot{F}=0, \tag{10}$$

$$\left(\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}\right)F+\frac{1}{2}f(R)+\ddot{F}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{F}=0. \tag{11}$$

$$\left(\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}\right)F+\frac{1}{2}f(R)+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{F}=\rho, \tag{12}$$

$$q\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)F=0. \tag{13}$$

From Eq. (13), we get

$$B=c_1\,C,$$

where c_1 is the constant of integration.

Without loss of generality, we can consider $c_1=1$ for the sake of simplicity. Hence we get

$$B=C \tag{14}$$

4. *Solutions of the Field Equations*. Using Eq. (14) in the field Eqs. (9)-(12), we get

$$\left(\frac{\ddot{A}}{A}+2\frac{\dot{A}\dot{B}}{AB}-2\frac{q^2}{A^2}\right)F+\frac{1}{2}f(R)+\ddot{F}+\left(2\frac{\dot{B}}{B}\right)\dot{F}=\lambda, \tag{15}$$

$$\left(\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right)F+\frac{1}{2}f(R)+\ddot{F}+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{F}=0, \tag{16}$$

$$\left(\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}\right)F+\frac{1}{2}f(R)+\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)\dot{F}=\rho. \tag{17}$$

The physical quantities of observational interest in cosmology are spatial volume V, mean Hubble parameter H, the expansion scalar θ, the mean anisotropy parameter A_m. the shear scalar σ^2 and the deceleration parameter q. They are defined as:

Spatial volume $$V=ABC. \tag{18}$$

Mean Hubble parameter $$H=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right). \tag{19}$$

Scalar expansion $$\theta=3H. \tag{20}$$

Anisotropic parameter $$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{21}$$

where H_i $(i=1,\ 2,\ 3)$ along directions of x, y and z axes which are the directional Hubble parameters.

Shear scalar $$\sigma^2 = \frac{3}{2} A_m H^2 . \quad (22)$$

Deceleration parameter $$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \quad (23)$$

Here the deceleration parameter q measures the rate of expansion of the universe. The sign of q indicates the state of expanding universe. If $q<0$ then it represents inflation otherwise deflation if $q>0$. While $q=0$ shows expansion with constant velocity.

For Bianchi type VI_0 model, the average scale factor is defined as

$$a = \left(V^{1/3}\right). \quad (24)$$

Here we've 3 freelance field equations containing six unknowns A, B, λ , ρ , F, f, thus we tend to shall assume further conditions to get distinctive solutions of the field equations. For the whole determination of the precise solutions, extra constraints relating these parameters area unit needed. Solutions to the field equations may be generated by applying a law of variation for Hubble's parameter, that was initial planned by Berman [71] in FRW models which yields a constant value of deceleration parameter (DP). Cunha and Lima [72] favors recent acceleration and past slowing with high degree of applied mathematics confidence level by analyzing 3 SNe kind Ia samples. Singh and Debnath [73], Adhav et al. [74], Katore et al. [75] has outlined a special variety of deceleration parameter, so as to match this observation, as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{\alpha}{1+a^\alpha}, \quad (25)$$

where $a>0$, is a constant and a is mean scale factor of the universe.

After solving (25) one can obtain the mean Hubble parameter H as

$$H = \frac{\dot{a}}{a} = k\left(1 + a^{-\alpha}\right), \quad (26)$$

where k is a constant of integration.

On integrating equation (26), we obtain the mean scale factor as

$$a = \left(e^{k\alpha t} - 1\right)^{1/\alpha}. \quad (27)$$

Using Eqs. (18), (24) and (27), we have

$$\left(e^{\alpha kt} - 1\right)^{3/\alpha} = AB^2 . \quad (28)$$

In order to solve the above equations, we use a physical condition that the expansion scalar is proportional to shear scalar. We assume that the expansion θ in the model is proportional to the shears σ. This condition leads to

$$B = A^n , \quad (29)$$

where n is arbitrary constant.

According to Thorne [76] observations of rate redshift relation for collection sources recommend that Hubble expansion of the universe is identical concerning half-hour vary roughly (Kantowski and Sachs [77], Kristian and Sachs [78]) and redshift studies place the limit $\sigma/H \leq 0.30$. Collins et al. [79] mentioned the physical significance of this condition for perfect fluid and barotropic equation of state in an exceedingly additional general case. Recently, several authors have assumed this condition so as to get solutions of the field equations for various type's cosmological models [80-87].

Using Eqs. (28) and (29), we obtain

$$A=\left(e^{\alpha kt}-1\right)^{3/\alpha(2n+1)}, \tag{30}$$

$$B=\left(e^{\alpha kt}-1\right)^{3n/\alpha(2n+1)}. \tag{31}$$

Hence a Bianchi type VI_0 space-time describe by line element (1) corresponding to equations (30) and (31) can be written in the form

$$\begin{aligned} ds^2 = -dt^2 + \left(e^{\alpha kt}-1\right)^{6/\alpha(2n+1)} dx^2 + \left(e^{\alpha kt}-1\right)^{6n/\alpha(2n+1)} e^{-2qx} dy^2 + \\ + \left(e^{\alpha kt}-1\right)^{6n/\alpha(2n+1)} e^{2qx} dz^2 . \end{aligned} \tag{32}$$

We also assume that the result established by Sharif and Shamir [36] in $f(R)$ gravity which shows that

$$F \alpha a^m, \tag{33}$$

where m is an arbitrary constant, $a=(V)^{1/3}$ is a scale factor.

Eq. (33) leads to $F = la^m$, where the proportionality constant is l. Hence

$$F = l\left(e^{\alpha kt}-1\right)^{m/\alpha}. \tag{34}$$

5. *Physical and Kinematical Parameters of the model.* It is well known that one can study the behavior of the physical and kinematical parameters either by observing the analytical expressions or by graphical representation. Using Eqs. (30) and (31), the spatial volume V of the universe is given by

$$V = \left(e^{k\alpha t}-1\right)^{3/\alpha}. \tag{35}$$

The spatial volume V vanishes at $t=0$. It expands exponentially as t increases and becomes infinitely large as $t \to \infty$ as shown in Fig.1.

Using Eqs. (30) and (31), the directional Hubble parameters are found as

$$H_x = \left(\frac{3k}{2n+1}\right)\frac{e^{\alpha kt}}{e^{\alpha kt}-1}, \tag{36}$$

$$H_y = H_z = \left(\frac{3nk}{2n+1}\right)\frac{e^{\alpha kt}}{e^{\alpha kt}-1}. \tag{37}$$

Using Eqs. (16), (30) and (31), we obtain

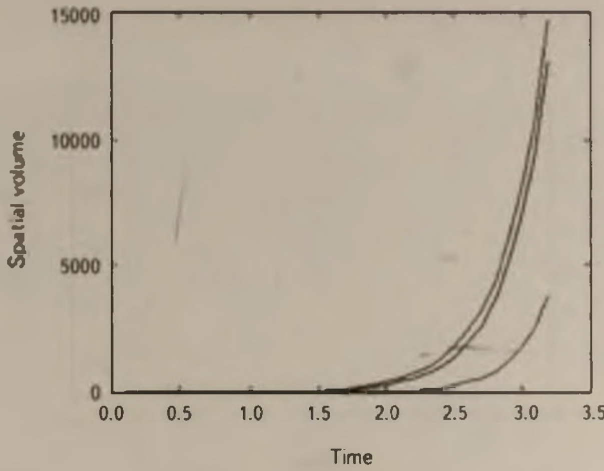

Fig 1 Spatial volume vs time (for $\alpha = 0.5, 1, 2$).

$$\frac{1}{2}f(R) = -\left\{\frac{3\alpha n k^2}{(2n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left[\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left(\frac{3n}{\alpha(2n+1)}-1\right)+1\right] + (1+n)\frac{9nk^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2}\right\}\left[l_1\left(e^{\alpha kt}-1\right)^{m/\alpha}\right] - \left[\frac{3k(n+1)}{(2n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\right]\left[l_1 mk\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)^{1-(m/\alpha)}}\right] - \left[l_1 m\alpha k^2\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)^{1-(m/\alpha)}}\right]\left[1+\left(\frac{m}{\alpha}-1\right)\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)^{1-(m/\alpha)}}\right]. \tag{38}$$

Using Eqs. (17), (30) and (31), the Energy density as

$$\rho = \left\{\frac{3\alpha k^2}{(2n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left[\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left(\frac{3}{\alpha(2n+1)}-1\right)+1\right] - \frac{3\alpha nk^2}{(2n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)} \times \left[\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\left(\frac{3n}{\alpha(2n+1)}-1\right)+1-(1+n)\frac{9nk^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{(e^{\alpha kt}-1)^2}\right]\right\}\left[l_1\left(e^{\alpha kt}-1\right)^{m/\alpha}\right] + \left[\frac{3nk}{(2n+1)}\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)}\right]\left[l_1 mk\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)^{1-(m/\alpha)}}\right] - \left[l_1 m\alpha k^2\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)^{1-(m/\alpha)}}\right]\left[1+\left(\frac{m}{\alpha}-1\right)\frac{e^{\alpha kt}}{(e^{\alpha kt}-1)^{1-(m/\alpha)}}\right] \tag{39}$$

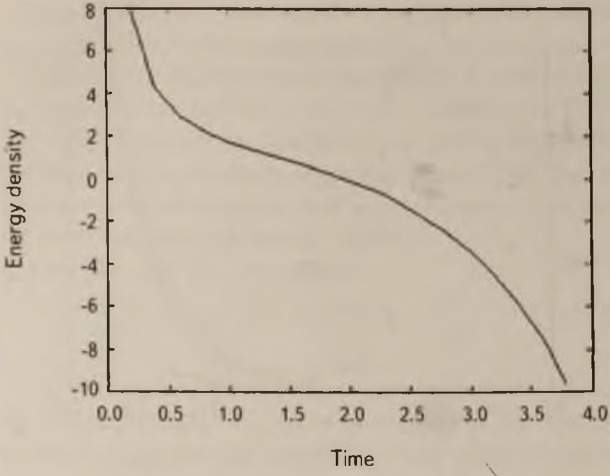

Fig.2. Energy density vs time.

At initial epoch, the energy density was positive and as time increases, it tends to be negative. The negative energy density does not violate any law of physics but this violates the weak energy condition. The negative energy density indicates that there is vacuum instability in the inertial frames at early stage of the evolution.The energy density decayed to be negative which resembles with Frampton [88], Katore and Shaikh [89].

Using Eqs. (15), (30) and (31), we have

$$\lambda=\left\{\frac{3\alpha k^2}{(2n+1)}\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\left[\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\left(\frac{3}{\alpha(2n+1)}-1\right)+1\right]-\frac{3\alpha nk^2}{(2n+1)}\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\times\right.$$
$$\left.\times\left[\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\left(\frac{3n}{\alpha(2n+1)}-1\right)+1\right]+(1-n)\frac{9nk^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}-\frac{2q^2}{\left(e^{\alpha kt}-1\right)^{6/\alpha(2n+1)}}\right\}\times$$
$$\times\left[l_1\left(e^{\alpha kt}-1\right)^{m/\alpha}\right]+\left[\frac{3k(n-1)}{(2n+1)}\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\right]\left[l_1 mk\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)^{1-(m/\alpha)}}\right]. \tag{40}$$

The Mean Hubble parameter H is given by

$$H=\frac{ke^{\alpha kt}}{e^{\alpha kt}-1}. \tag{41}$$

The expansion scalar is given by

$$\theta=\frac{3ke^{\alpha kt}}{e^{\alpha kt}-1}. \tag{42}$$

From Fig.3, we observe that the expansion scalar start with infinite value at $t=0$ and then rapidly becomes constant after some finite time.

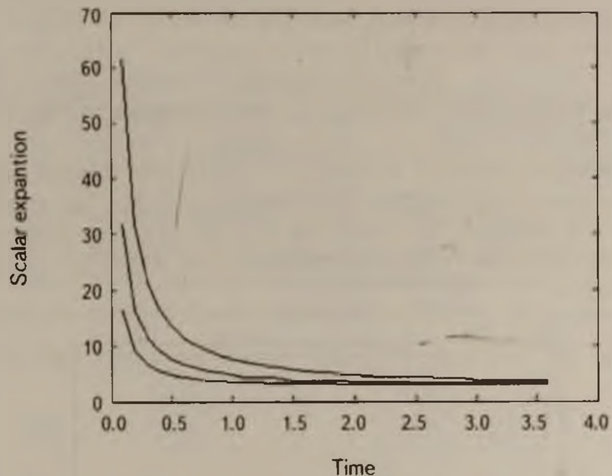

Fig.3. Scalar expansion vs time (for $\alpha = 0.5, 1, 2$).

The mean anisotropy parameter is given by

$$A_m = \frac{2(n-1)^2}{(2n+1)^2}. \tag{43}$$

The average anisotropic parameter remains constant throughout the evolution of the universe and it becomes zero when $n = 1$.

The shear scalar is given by

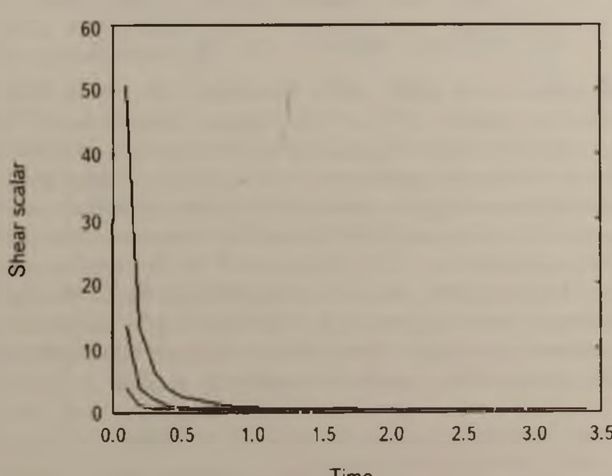

Fig.4. Shear scalar vs time (for $\alpha = 0.5, 1, 2$).

$$\sigma^2 = \frac{3k^2(n-1)^2}{(2n+1)^2}\frac{e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2} \tag{44}$$

The shear scalar $\sigma \to \infty$ as time $t \to 0$ and decreases to null as time increases.

The deceleration parameter is within $-1 \le q \le 0.5$ that matches with the observations created by (Riess et al. [90]; Permutter et al. [91]) reveal that this universe is accelerating. The model has a transition of the universe from the first slowing part to the recent acceleration part (see, Fig.5) that agrees with recent observations.

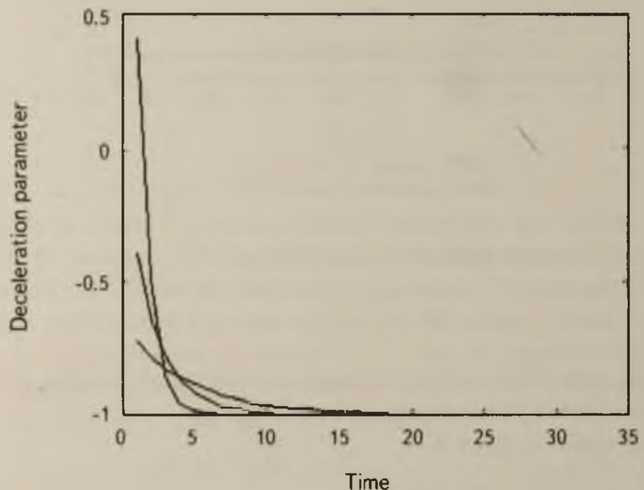

Fig.5. Deceleration Parameter vs time. (for $\alpha = 0.5, 1, 2$)

6. *Conclusion*. In this paper, we've got studied the cosmic string cosmological model for Bianchi - VI_0 in $f(R)$ gravity with a special form of deceleration parameter. During this model we tend to determined that the spatial volume is finite at $t=0$ and it expands as t will increase and becomes infinitely giant as $t \to \infty$ as shown in Fig.1. It's determined that the Expansion scalar starts with a finite worth at $t=0$, and as time will increase it decreases to a continuing worth and remains constant as $t \to \infty$. The value of the deceleration parameter q of the universe is in the range $-1 \le q \le 0.5$, as shown in Fig.5, which matches with the observations made by Knop et al. [92] that the present day universe is undergoing accelerated expansion. It's seen that, the results obtained here area unit kind of like the results obtained earlier by Reddy et al. [93].

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АНИЗОТРОПНАЯ КОСМОЛОГИЧЕСКАЯ МОДЕЛЬ С КОСМИЧЕСКИМИ СТРУНАМИ В $f(R)$ ТЕОРИИ ГРАВИТАЦИИ

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Работа посвящена изучению пространства-времени типа Бианки VI_0, наполненной космическими струнами, в рамках $f(R)$ теории гравитации. Изучены и указаны физические и кинематические свойства модели. Для этой модели дополнительно проведено действие $f(R)$ скаляра Риччи.

Ключевые слова: *космическая струна: тип Бианки VI_0: теория* $f(R)$

REFERENCES

1. *R.Ferraro, F.Fiorini*, Phys. Rev. D, **75**, 084031, 2007.
2. *G.R.Bengochea, R.Ferraro*, Phys. Rev. D, **79**, 124019, 2009.
3. *E.V.Linder*, Phys. Rev. D, **81**, 127301, 2010.
4. *S.M.Carroll, A. De Felice, V.Duvvuri et al.*, Phys. Rev. D, **71**, 063513, 2005.
5. *G.Cognola, E.Elizalde, S.Nojiri et al.*, Phys. Rev. D, **73**, 084007, 2006.
6. *S.Nojiri, S.D.Odintsov*, Phys. Rev. D, **74**, 086005, 2006.
7. *S.Nojiri, S.D.Odintsov*, J. Phys. A, **40**, 6725, 2007a.
8. *H.J.Schmidt*, Int. J. Geom. Methods Mod. Phys., **4**, 209, 2007.
9. *S.Nojiri, S.D.Odintsov*, Int. J. Geom. Meth. Mod. Phys., 4, 115, 2007, [arXiv:hep-th/0601213].
10. *R.Kerner*, Gen. Rel. Grav., **14**, 453, 1982.
11. *J.Barrow, A.Ottewill*, J. Phys., **A16**, 2757, 1983.
12. *S.Capozziello*, Int. J. Mod. Phys. D, **11**, 483, 2002.
13. *S.Capozziello, S.Carloni, A.Troisi*, in Recent Research Developments in Astron. Astrophys., RSP/AA/21-2003, [arXiv:astro-ph/0303041], 2003.
14. *S.M.Carroll, V.Duvvuri, M.Trodden, M.S.Turner*, Phys. Rev. D, **70**, 043528, 2004.
15. *S.Nojiri, S.D.Odintsov*, Phys. Rev. D, **68**, 123512, 2003.
16. *H.A.Buchdahl*, Mon. Not. Roy. Astron. Soc., **150**, 1, 1970.
17. *A.A.Starobinsky*, Phys. Lett. B, **91**, 99, 1980.
18. *T.Biswas, E.Gerwick, T.Koivisto, A.Mazumdar*, Phys. Rev. Lett., **108**, 031101, 2012, (Preprint 1110.5249).
19. *S.Carroll, V.Duvvuri, M.Turner, M.Trodden*, Phys. Rev. D, **70**, 2004.
20. *V.Faraoni*, arXiv preprint arXiv:0810.2602, 2008.
21. *T.P.Sotiriou, S.Liberati*, Ann. Phys., **322**, 935, 2007.
22. *S.Nojiri, S.D.Odintsov*, Phys. Reports, **505**, 59-144, 2011.

23. *S.Nojiri, S.D.Odintsov*, Phys. Rev. D, **74**, 086005, 2006.
24. *S.Nojiri, S.D.Odintsov*, Phys. Lett. B, **657**, 238, 2007.
25. *S.Srivastava*, International J. Theor. Phys., **47**, 1966, 2008.
26. *S.Capozziello, V.Cardone, A.Troisi*, J. Cosmology and Astroparticle Phys., 2006001, 2006.
27. *A. De Felice, S.Tsujikawa*, Living Rev. Rel., **13**, 1002-4928, 2010.
28. *G.Magnano, M.Ferraris, M.Francaviglia*, General relativity and gravitation **19**, 465, 1987.
29. *T.Multamäki, I.Vilja*, Phys. Rev. D, **74**, 064022, 2006.
30. *L.Hollestein, F.S.N.Lobo*, Phys. Rev. D, **78**, 124007, 2008.
31. *A.Azadi, D.Momeni, M.Nouri-Zonoz*, Phys. Lett. B, **670**, 210, 2008.
32. *M.J.Reboucas, J.Santos*, Phys. Rev. D, **80**, 063009, 2009.
33. *J.Santos, M.J.Reboucas, T.B.R.F.Oliveira*, Phys. Rev. D, **81**, 123017, 2010.
34. *M.F.Shamir*, Astrophys. Space Sci., **330**, 183, 2010.
35. *L.Sebastiani, S.Zerbini*, Eur. Phys. J. C, **71**, 1591, 2011.
36. *M.Sharif, M.F.Shamir*, Class. Quantum Grav., **26**, 235020, 2009.
37. *M.Sharif, M.F.Shamir*, arXiv:1005.2798v1 [gr-qc], 17 May 2010.
38. *M.Sharif, M.F.Shamir*, Gen. Relativ. Gravit., **42**, 1557, 2010a.
39. *M.Sharif, M.F.Shamir*, Mod. Phys. Lett. A, **25**, 1281, 2010b.
40. *C.Aktas et al.*, Phys. Lett. B, **707**, 237, 2012.
41. *K.S.Adhav*, Can. J. Phys., **90**, 119, 2012.
42. *V.Singh, C.P.Singh*, Astrophys. Space Sci., **346**, 285, 2013.
43. *D.R.K.Reddy et al.*, Int. Jou. Sci. Adv. Tech., **4**, 3, 23, 2014.
44. *S.D.Katore, A.Y.Shaikh, N.K.Sarkate*, National Conference on Engineering Applications of Mathematics, 219-235, 2014.
45. *S.D.Katore, A.Y.Shaikh*, The African Rev. Phys., **9**, 0054, 2014.
46. *J.Stachel*, Phys. Rev. D, **21**, 2171, 1980.
47. *P.S.Letelier*, Phys. Rev. D, **20**, 1249, 1979.
48. *P.S.Letelier*, Phys. Rev. D, **28**, 2414, 1983.
49. *A.Banerjee, A.K.Sanyal, S.Chakraborty*, Pramana - J. Phys., **34**, 1, 1990.
50. *K.D.Krori, T.Chaudhury, C.R.Mahanta, A.Mazumder*, Gen. Relativ. Gravit., **22**, 123, 1990.
51. *X.X.Wang*, Chin. Phys. Lett., **20**, 615, 2003.
52. *X.X.Wang*, Chin. Phys. Lett., **20**, 1205, 2003.
53. *R.Bali, R.D.Upadhaya*, Astrophys. Space Sci., **283**, 97, 2002.
54. *A.Pradhan, A.K.Yadav, L.Yadav*, Czech. J. Phys., **55**, 503, 2005.
55. *M.K.Yadav, A.Rai, A.Pradhan*, Int. J. Theor. Phys., **46**, 2677, 2007.
56. *M.K.Yadav, A.Pradhan, S.K.Singh*, Astrophys. Space Sci., **311**, 423, 2007
57. *R.Bali, Anjali*, Astrophys. Space Sci., **302**, 201, 2006.
58. *R.Bali*, Electron. J. Theor. Phys., 5, 105, 2008.
59. *I.Yilmaz*, Gen. Relativ. Gravit., **38**, 1397, 2006,
60. *H.Baysal, I.Yavuz, I.Tarhan et al.*, Turk. J. Phys., **25**, 283, 2001.
61. *A.Pradhan*, Fizika B (Zagreb), **16**, 205, 2007.
62. *A.Pradhan, A.K.Yadav, R.P.Singh, V.K.Singh*, Astrophys. Space Sci., **312**, 145, 2007.

63. *A.Pradhan*, *M.K.Mishra*, *A.K.Yadav*, Rom. J. Phys., **54**, 747, 2009, arXiv:0705.1765 [gr-qc].
64. *A.K.Yadav*, *V.K.Yadav*, *L.Yadav*, Int. J. Theor. Phys., **48**, 568, 2009.
65. *V.U.M.Rao*, *K.V.S.Sireesha*, The European Phys. J. Plus, **127**, article 33, 2012.
66. *V.U.M.Rao*, *K.V.S.Sireesha*, Int. J. Theor. Phys., **51**, 10, 3013, 2012.
67. *V.U.M.Rao*, *K.V.S.Sireesha*, The European Phys. J. Plus, **127**, article 49, 2012.
68. *S.D.Katore*, *A.Y.Shaikh*, Int. J. Theor. Phys., **51(6)**, 1881, 2012.
69. *S.D.Katore*, *A.Y.Shaikh*, Int. J. Modern Phys. A, **26**(09), 1651, 2011.
70. *S.D.Katore*, *A.Y.Shaikh*, Rom. J. Phys., **59**, 7-8, 715, 2014.
71. *M.S.Bermann*, Nuovo Cimento B, **74**, 182, 1983.
72. *J.V.Cunha*, *J.A.S.Lima*, Mon. Not. Roy. Astron. Soc., **390**, 210, 2008.
73. *A.K.Singh*, *U.Debnath*, Int. J. Theor. Phys., **48**(2), 351, 2009.
74. *Adhav et al.*, Astrophys. Space Sci., **345**(2), 405, 2013.
75. *S.D.Katore*, *A.Y.Shaikh*, *S.A.Bhaskar*, Bulg. J. Phys., **41**, 34, 2014.
76. *K.S.Thorne*, Astrophys. J., **148**, 51, 1967.
77. *R.Kantowaski*, *R.K.Sachs*, J. Math. Phys., **7**, 433, 1966.
78. *J.Kristian*, *R.K.Sachs*, Astrophys. J., **143**, 379, 1966.
79. *C.B.Collins*, *E.N.Glass*, *D.A.Wilkinson*, Gen. Relat. Gravit., **12**, 805, 1980.
80. *X.X.Wang*, Chin. Phys. Lett., **22**, 29, 2005.
81. *A.Pradhan*, Fizika B, **16**, 205, 2007.
82. *A.Pradhan*, *A.K.Yadav*, *R.R.Singh*, *V.K.Singh*, Astrophys. Space Sci., **312**, 3, 2007.
83. *A.Pradhan*, *A.K.Jotani*, *A.Singh*, Braz. J. Phys., **38**, 167, 2008.
84. *M.F.Shamir*, Gen. Relat. Grav., **42**, 1557, 2010.
85. *R.Venkateswarlu*, *J.Satish*, *K.P.Kumar*, Elect. J. Theor. Phys., **8**, 353, 2011
86. *M.Sharif*, *S.Waheed*, Eur. Phys. J. C, **72**, 1876, 2012.
87. *S.D.Katore*, *A.Y.Shaikh*, Bulg. J. Phys., **39**, 241, 2012.
88. *P.H.Frampton*, Mod. Phys. Lett. A, **19**, 801, 2004.
89. *S.D.Katore*, *A.Y.Shaikh*, Astrophys. Space Sci., 357:27, 2015.
90. *A.G.Riess et al.*, Astron. J., **116**, 1009, 1998.
91. *S.Perlmutter et al.*, Astrophys. J., **517**, 565, 1999.
92. *R.K.Knop et al.*, Astrophys. J., **598**, 102, 2003.
93. *D.R.K.Reddy et al.*, Int. J. Sci. Adv. Tech., **4**, 3, 23, 2014.