АСТРОФИЗИКА

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EFFECT OF MASS VARIATION ON THE RADIAL OSCILLATIONS OF DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED POLYTROPIC STARS

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A method is proposed to compute the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of differentially rotating and tidally distorted stellar models by taking into effect of mass variation on its equipotential surface inside the stars. The developed approach has been used to compute certain radial modes of oscillations of polytropic models with polytropic indices 1.5, 3.0 and 4.0. The results obtained have been compared with results earlier obtained devoid of taking into account the mass variation. Certain conclusions based on this study have been drawn.

Key words: *Mass variation: equipotential surface: rotation and tidal distortions: polytropic models*

1. *Introduction*. The mathematical problem for determining the eigenfrequencies of oscillations of a rotating star is much intricate. Approximate methods have, therefore, been often used in the literature to study such problems. Most of the authors such as Clement [1], Kochar and Trehan [2], Mohan and Saxena [3], Soofi et al. [4], Dintrans and Rieutord [5], Reese et al. [6], and Lovekin and Deupree [7], have studied the oscillations of stars assuming the star to have solid body rotation and therefore, rotating uniformly. However, some authors such as Ireland [8], Woodard [9], Dziembowski and Goode [10], Mohan et al. [11,12], Lal et al. [13], Karino and Eriguchi [14] and Lovekin et al. [15], Kumar et al. [16] addressed themselves to the problems of differentially rotating stars also. Lal et al. [17] have also discussed the structure and oscillations of polytropic model by taking effects of Coriolis force on the stars. Saini et al. [18] and Kumar et al. [19] have obtained the significant conclusions about the structure of rotationally and tidally distorted stars. Mohan et al. [20] and Kumar et al. [21] studied the barotropic oscillations of differentially rotating and tidally distorted stars and differentially rotating Roche-models, respectively. Their results give some ideas about the structures and oscillations of realistic stars.

In the present paper we have computed the eigenfrequencies of pseudo-radial modes of oscillations of polytropic models of stars including mass variation inside the stars rotating differentially. In the case of gaseous spheres, undergoing periodic oscillations, two types of modes of oscillations to be generated. One of these is called radial modes of oscillation (in which the fluid elements oscillate in the radial direction only) and the other non-radial modes (in which fluid elements oscillate in arbitrary directions). It is expected that in rotating stars, (in which angular velocity of rotation is not too large) these types of modes are still excited but their eigenvalues are get influenced by rotation effects.

The paper is organized as follows: In Section 2, an eigenvalues boundary value problem for determining the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of a differentially rotating polytropic model of a star is mentioned. Numerical computations have then been performed in Section 3 to determine the eigenfrequencies of pseudo-radial modes of oscillations of certain differentially rotating polytropic models of stars at indices 1.5, 3.0 and 4.0. The calculated distorted radii of some polytropic models of stars are given in Section 4. The eigenfrequencies, thus computed, have been compared with the earlier results obtained by Lal et al. [13] without taking the effect of mass variation.

2. *Eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of differentially rotating polytropic models.* Following Mohan et al. [11], an eigenvalue problem determining the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of a differentially rotating polytropic model of a star rotating differentially according to the law (1) may be expressed as:

$$H_1 \frac{d^2\zeta}{dr_0^2} + H_2 \frac{d\zeta}{dr_0} + \left[H_3\,\omega^{*2} - H_4\right]\zeta = 0, \tag{1}$$

where $\omega^{*2} = R^3 r_{0s}^3\,\sigma^2 / GM_0$ and the expression for test of the symbols H_1, H_2, H_3, H_4 are given in the Appendix B, where as ω^{*2} is the non-dimensional form of the actual eigenfrequency of oscillation σ and ζ denotes a suitable average of the relative amplitudes of pulsation of the fluid elements of the equipotential surfaces $\psi = \text{const}$. Also r_{0s} is the value of r_0 at the surface of the model, G universal gravitational constant, M_0 the total mass of the star and R the radius of undistorted polytropic model (necessary details of Eq. (1) are given in the Appendix A for readers' reference).

Eq. (1) determines the eigenfrequencies of small adiabatic pseudo-radial modes of oscillation of differentially rotating polytropic models of stars rotating differentially according to the law $\omega = b_1 + b_2 s^2$ by taking the effect of mass variation inside the star.

3. *Numerical evaluation of the eigenfrequencies.* Eigenvalue problems developed in Sections 2 have been solved numerically to compute

eigenvalues of pseudo-radial modes of oscillations of certain differentially rotating polytropic models. The eigenvalue problem of Section 2 is of the Sturm-Liouville type. In order to compute the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of differentially rotating polytropic models, Eq. (1) has been integrated numerically subject to the boundary conditions which require ζ being finite at points corresponding to the center and the free surface of the model. Computations are started with some trial values of ω^{*2}. For this chosen value of ω^{*2} series solution was first developed at a point close to the centre ($x=0.005$). This solution is then used to carry the integration of the pulsation Eq. (1) outward using the fourth order Runge-Kutta method. Using the same numerical value of ω^{*2}, the series solution is also developed at points near the surface which is then used to carry the integration of Eq. (1) inward. The values of $\varsigma(d\varsigma/dx)^{-1}$ obtained from the outward and inward integrations of (1) is then matched at some preselected point in the interior of the model. The process is continued iteratively with different choices of the value of ω^{*2}, till a value of ω^{*2} is found for which the two solutions agree to a specified accuracy.

In order to start integrations from points near the center and the surface, series solutions were developed at $x=0.01$ and $x=0.99$. Outward and inward integrations were performed using a step length of $x=0.01$. Trials with different values of ω^{*2} were continued till the absolute difference in the value of $\varsigma(d\varsigma/dx)^{-1}$ at the preselected point in the interior of the model from the outward and inward integrations was found to be less than 0.0005. Computations have been performed to compute the fundamental and the first mode of pseudo-radial oscillation of differentially rotating and tidally distorted polytropic models of indices 1.5, 3.0, 4.0.

4. *Observations.* The values of distorted radii, due to the effect of differential rotation and tidal distortion, are calculated for different polytropic stellar models at different indices as mentioned in Table 1.

Table 1

DISTORTED RADII r_{0s} OF CERTAIN DIFFERENTIALLY ROTATING AND TIDALLY DISTORTED STARS

Model	Rotational and tidal distortion parameters			r_{0s} at different polytropic indices		
	b_1	b_2	q	1.5	3.0	4.0
1	0.0000	0.0000	0.1000	0.499815	0.499935	0.499955
2	0.3162	0.0000	0.1000	0.496235	0.498620	0.499475
3	0.0000	0.3162	0.1000	0.499805	0.499936	0.499956
4	0.3162	0.3162	0.1000	0.495895	0.498510	0.499430
5	0.2000	0.0200	0.1000	0.498665	0.499500	0.499820
6	0.1000	-0.0600	0.1500	0.499245	0.499730	0.499850

5. *Conclusion*. The obtained results have been compared (Fig.1-7) with the results obtained by Lal et al. [13]. Model 1 is non-rotating but tidally distorted, which gives largest radius in comparison of other models. Model 2 is a solid body rotating under the effects of tidal distortion as well as mass variation. This gives largest radius in comparison of other differentially rotating and tidally distorted models, for polytropic index 2.0 but not for 3.0 and 4.0. Therefore, it may be concluded that the radius will be maximum for non-rotating and tidally distorted stars. However, if it is rotating as a solid body then radius will be a large only for polytropic index 1.5 but less than from non-rotating. Model 4, has much distortion due to differential rotation in comparison of others, gives large radius for indices 3.0 and 4.0. Hence, a differentially rotating and tidally distorted stars increase their radii for higher polytropic indices with the effect of mass variation. It is also observed that radii vary much for index 1.5 but varies slightly for large index (such as 4.0).

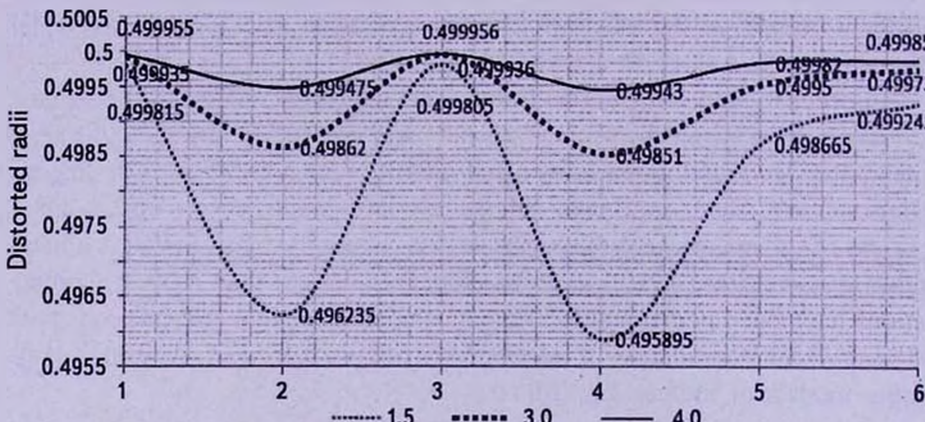

Fig.1. Radius distrotion in polytropic models for index.

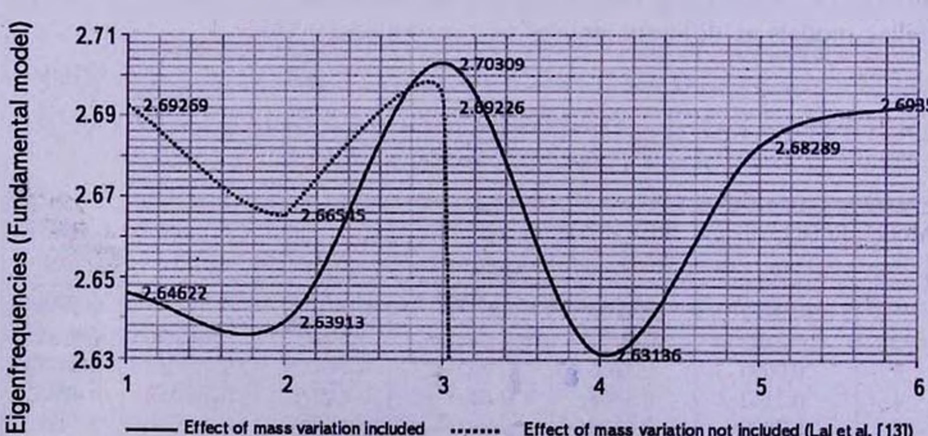

Fig.2. Differentially rotating stellar models of polytropic index 1.5.

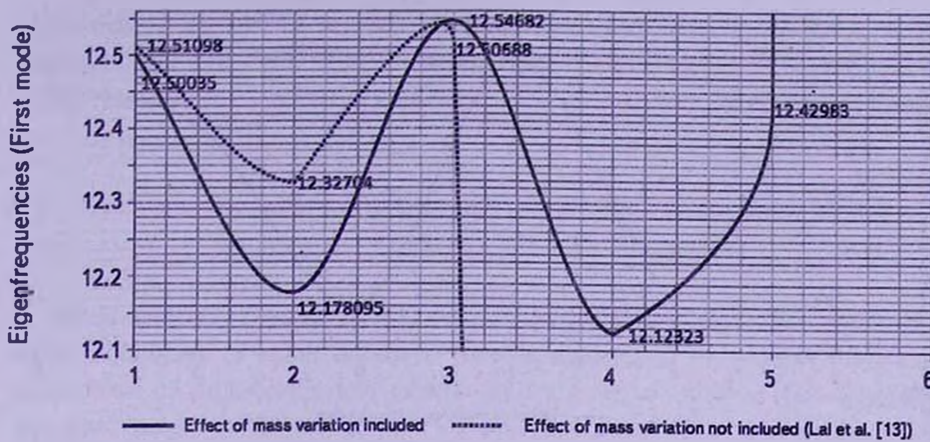

Fig.3. Differentially rotating stellar models of polytropic index 1.5.

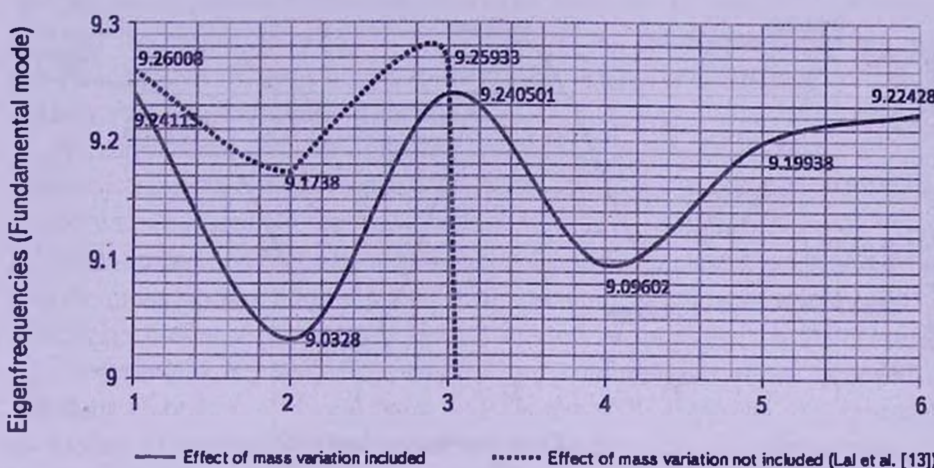

Fig.4. Differentially rotating stellar models of polytropic index 3.0.

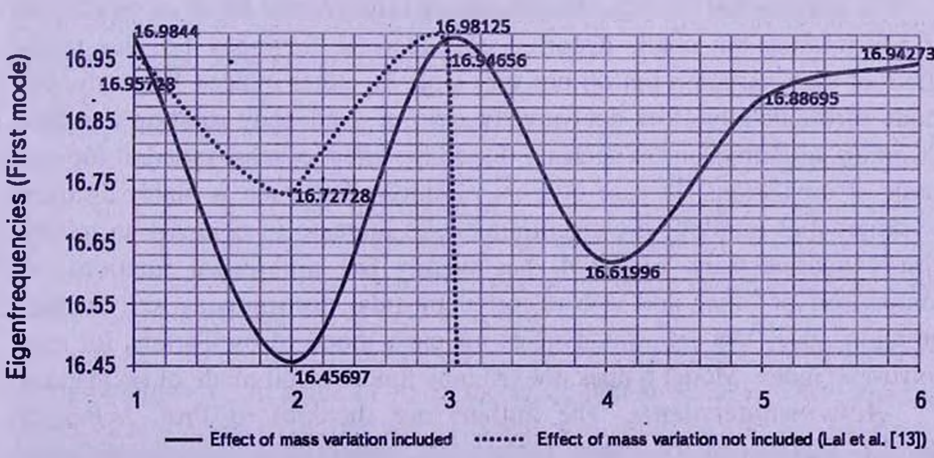

Fig.5. Differentially rotating stellar models of polytropic index 3.0.

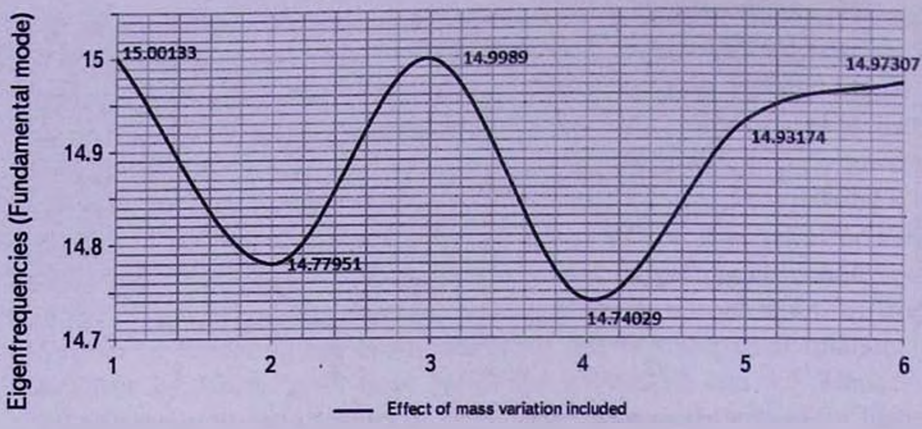

Fig.6. Differentially rotating stellar models of polytropic index 4.0.

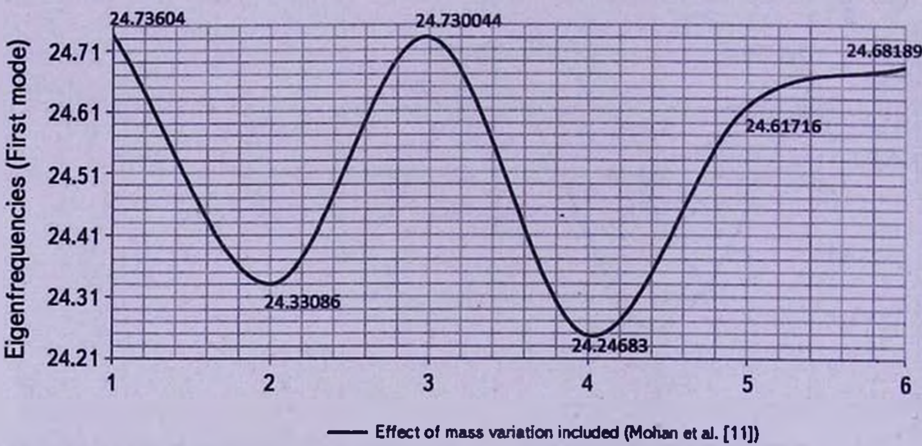

Fig.7. Differentially rotating stellar models of polytropic index 4.0.

It is noticed that the eigenfrequencies of fundamental mode of oscillations of non-rotating but tidally distorted star, decrease for index 1.5, due to the effect of mass variation but do not vary large for other indices. Similarly, first mode of oscillations does not vary. In case of solid body rotation and tidal distortion, eigenfrequencies decrease due to the effect of mass variation for each mode of oscillations. In case of mass variation, if the star is highly distorted by differential rotation, the eigenfrequencies increase in comparison to uniformly rotating stars. Model 4, has slightly fast differential rotations in comparison of other and also distorted by tidal distortion as well as mass variation, gives low eigenfrequencies for each mode of oscillations for each polytropic index. Model 6 does not give any fundamental mode of oscillations.

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Appendix

Eigenvalue boundary problems for computing pseudo-radial modes of oscillations of differentially rotating models

Mohan et al. [20] formulated eigenvalue problems which determine the eigenfrequencies of small adiabatic pseudo-radial and non-radial modes of oscillations of rotationally and tidally distorted stellar models. The approach was later used by Mohan et al. [11] and Lal et al. [13] to determine the eigenfrequencies of small adiabatic pseudo-radial and non-radial modes of oscillations of certain differentially rotating stars. In this section, we present in brief the approach adopted by Mohan et al. [11,20] to determine the eigenfrequencies of small adiabatic barotropic modes of oscillations of differentially rotating and tidally distorted stars.

A1: *Eigenvalue boundary problem determining the eigenfrequencies of small adiabatic barotropic pseudo-radial modes of oscillations of differentially rotating stars.* Assuming that during the oscillations, the fluid elements on an equipotential surface oscillate in unison, the eigenfrequencies of small adiabatic pseudo-radial modes of oscillations of the actual rotating star rotating differentially according to the implicit law of differential rotations; can be obtained from its topologically equivalent spherical model developed on the basis of the averaging technique of Kippenhahn and Thomas [22]. Following the approach of Mohan et al. [11,12], the equation determining the eigenfrequencies of pseudo-radial modes of oscillations of differentially rotating and tidally distorted stellar models which correspond to the eigenvalue problem determining the eigenfrequencies of radial modes of oscillations of the topologically equivalent spherical model may be expressed as:

$$\frac{d^2\eta}{dr_{0\psi}^2}+\frac{4-\mu}{r_{0\psi}}\frac{d\eta}{dr_{0\psi}^2}+\left[\frac{\rho_{0\psi}}{\gamma P_{0\psi}}\sigma^2-\left(3-\frac{4}{\gamma}\right)\frac{\mu}{r_{0\psi}^2}\right]\eta=0, \quad \text{where} \quad \mu=-\frac{r_{0\psi}}{P_{0\psi}}\frac{dP_{0\psi}}{dr_{0\psi}}. \quad (A_1)$$

Here $r_{0\psi}$, $\rho_{0\psi}$ and $P_{0\psi}$ are the values of r_ψ, ρ_ψ and P_ψ on the equipotential ($\psi=\text{const}$) in its equilibrium position, σ the eigenfrequency of oscillation and η some average of the relative amplitudes of pulsation of the fluid elements on the equipotential surface $\psi=\text{const}$. Using r_ψ, ρ_ψ and P_ψ in place of $r_{0\psi}$, $\rho_{0\psi}$ and $P_{0\psi}$ to denote the equilibrium values on the equipotential surfaces, taking $r_0=z/(\psi-q)$ in place of r_ψ as the independent variable, and assuming $\omega^2=b_1^2+2b_1b_2s^2+b_2^2s^4$ as the law of differential rotation, the Eq. (A_1) governing the small adiabatic pseudo-radial modes of oscillations of a differentially rotating and tidally distorted gas sphere may be expressed as:

$$A(z,b_1,b_2,q)\frac{d^2\eta}{dr_0^2}+\left[\frac{4-\mu}{r_0}B(z,b_1,b_2,q)-C(z,b_1,b_2,q)\right]\frac{d\eta}{dr_0}+ \\ +\left[\frac{R^2\sigma^2\rho_\psi}{rP_\psi}-\left(3-\frac{4}{r}\right)\frac{\mu}{r_0^2}E(z,b_1,b_2,q)\right]\eta=0 \tag{A$_2$}$$

where

$$A(z,b_1,b_2,q)=\left[1-\frac{8b_1^2r_0^3}{3z}-\frac{16b_1b_2r_0^5}{15z}-\frac{28q^2r_0^6}{5z^2}-\frac{128b_2^2r_0^7}{105z}-\frac{90q^2r_0^8}{7^2}-\frac{44q^2r_0^{10}}{3z^2}+\dots\right],$$

$$B(z,b_1,b_2,q)=\left[1-\frac{5b_1^2r_0^3}{3z}-\frac{28b_1b_2r_0^5}{15z}-\frac{32q^2r_0^6}{5z^2}-\frac{24b_2^2r_0^7}{35z}-\frac{50q^2r_0^8}{7z^2}-\frac{2q^2r_0^{10}}{3z^2}+\dots\right],$$

$$C(z,b_1,b_2,q)=\frac{1}{r_0}\left[\frac{4b_1^2r_0^3}{z}+\frac{8b_1b_2r_0^5}{5z}+\frac{168q^2r_0^6}{5z^2}+\frac{64b_2^2r_0^7}{z}+\frac{360q^2r_0^8}{7z^2}+\frac{220q^2r_0^{10}}{3z^2}+\dots\right],$$

$$E(z,b_1,b_2,q)=\left[1-\frac{2b_1^2r_0^3}{3z}-\frac{8b_1b_2r_0^5}{15z}-\frac{8q^2r_0^6}{5z^2}-\frac{16b_2^2r_0^7}{35z}-\frac{10q^2r_0^8}{7z^2}-\frac{4q^2r_0^{10}}{3z^2}+\dots\right].$$

Also

$$\mu=-\frac{r_\psi}{P_\psi}\frac{dP_\psi}{dr_0}\frac{dr_0}{dr_\psi}=-F(z,b_1,b_2,q)\frac{r_0}{P_\psi}\frac{dP_\psi}{dr_0},$$

where

$$F(z,b_1,b_2,q)=\left[1-\frac{b_1^2r_0^3}{z}-\frac{4b_1b_2r_0^5}{15z}-\frac{24q^2r_0^6}{5z^2}-\frac{56b_2^2r_0^7}{105z}-\frac{40q^2r_0^8}{7z^2}-\frac{20q^2r_0^{10}}{3z^2}+\dots\right].$$

In absence of any distortion: $z=1$, $b_1=b_2=0$, $\rho_\psi=\rho$, $P_\psi=P$, $r_0=x$, the above Eq. (A$_2$) reduces to $\frac{d^2\eta}{dx^2}+\frac{4-\mu}{x}\frac{d\eta}{dx}+\left[\frac{R^2\sigma^2\rho}{r\rho}-\left(3-\frac{4}{r}\right)\frac{\mu}{x^2}\right]\eta=0$ with $\mu=-\frac{x}{p}\frac{dP}{dx}$, which is the usual equation determining the eigenfrequencies of small adiabatic radial modes of oscillations of a gaseous sphere.

Eq. (A$_2$) forms an eigenvalue problem in the eigenfrequency of oscillation σ. As usual, this eigenvalue problem is of Sturm-Liouville type having singularities both at the centre and the surface of the model. It has to be solved subject to the boundary conditions which require η being finite at the centre as well as at the free surface.

In reality Eq. (A$_2$) determines the periods of small adiabatic radial modes of oscillations of the topologically equivalent spherical model. However, equipotential surfaces of the actual differentially rotating distorted model are also the surfaces of equipressure and equidensity, the values of pressure and density on the equipotential surfaces of the differentially rotating star are same as on the corresponding equipotential surfaces of the equivalent spherical model. Hence the eigenfrequencies of the radial modes of oscillations determined by solving the eigenvalue problem for the topologically equivalent spherical model are indeed the eigenfrequencies of the radial modes of oscillation of the undistorted model which have got influenced by the rotational effects of the star. However, the values of the eigenfunction η obtained on solving (A_2) for the equivalent spherical model are not the actual values of amplitudes of pulsation η for the distorted model but rather some averages of the true values of eigenfunction η on the differentially rotating model.

We may thus use Eq. (A_2) to determine the effects of differential rotation and the tidal distortions on the periods of small adiabatic radial modes of oscillations of a stellar model. The effects of differential rotation and tidal distortions have been incorporated through introduction of terms $A(z,b_1,b_2,q)$, $B(z,b_1,b_2,q)$, $C(z,b_1,b_2,q)$, $E(z,b_1,b_2,q)$ and $F(z,b_1,b_2,q)$, and dependence of ρ_ψ and P_ψ on ψ. The present method in fact incorporates the effects of distortional forces both while computing the equilibrium structure (in computing the values of P_ψ, ρ_ψ etc.) as well as in the coefficients A, B and C of the Eq. (A_2) which determines the periods of adiabatic small radial modes of oscillations.

The eigenvalue problem (A_2) together with the boundary conditions which require η being finite both at the centre as well as the free surface of the star may be solved numerically in the usual manner as is done in the case of undistorted models. For convenience in numerical work it is sometimes found convenient to set

$$\eta = \frac{\zeta}{r_0} \quad \text{and} \quad r_0 = x r_{OS} \, . \tag{A3}$$

(r_{os} being the value of r_0 on the outermost surface) in Eq. (A_2) and treat x as the independent variable and ζ as the dependent variable. With these substitutions x is now zero at the centre and one at the free surface. The boundary condition $\eta =$ finite at the centre now gets replaced by $\zeta = 0$ at the centre. The boundary condition $\eta =$ finite at the free surface now becomes ζ finite at $x = 1$. Using (A_3), Eq. (A_2) gets transformed in terms of the variables ζ and x as

$$A^*(z,b_1,b_2,q,x)\frac{d^2\zeta}{dx^2} + B^*(z,b_1,b_2,q,x)\frac{d\zeta}{dx} + C^*(z,b_1,b_2,q,x)\zeta = 0 \tag{A_4}$$

where

$$A^{*}(z,b_1,b_2,q,x)=A(z,b_1,b_2,xr_{OS}).$$

$$B^{*}(z,b_1,b_2,q,x)=\frac{4-\mu}{x}B(z,b_1,b_2,xr_{OS})-r_{OS}C(z,b_1,b_2,xr_{OS})-\frac{2}{x}A(z,b_1,b_2,xr_{OS})$$

and

$$C^{*}(z,b_1,b_2,q,x)=\frac{r_{OS}^2R^2\rho_\psi}{\gamma P_\psi}\sigma^2-\left(3-\frac{4}{\gamma}\right)\frac{\mu}{x^2}E(z,b_1,b_2,xr_{OS})-\frac{1}{x}B^{*}(z,b_1,b_2,xr_{OS}).$$

The boundary conditions now are:

$$\zeta=0 \quad \text{at the centre} \quad x=0 \quad \zeta=\text{finite} \quad \text{at the surface} \quad x=1. \qquad (A_5)$$

For computing an eigenvalue σ (A_4) has to be solved numerically subject to the specified boundary conditions (A_5). Centre and the free surface of the star being singularities of this differential equitation it may be advisable to write the series solutions of (A_4) near the singularities to start numerical integrations. If we assume ζ to be normalized to have value 1 at the free surface, Eq. (A_4) can be integrated near these two singularities by the series solutions of the type

$$\zeta=\sum_{J=0}^{\infty}a_J x^{J+\lambda} \quad \text{near the centre} \quad x=0, \qquad (A_6)$$

and

$$\zeta=1+\sum_{J=0}^{\infty}b_J(1-x)^{J+\lambda} \quad \text{near the surface} \quad x=1. \qquad (A_7)$$

For obtaining an eigenfrequency of pseudo-radial mode of oscillation, the Eq. (A_4) has to be integrated numerically for trial values of σ till a value of σ is obtained for which both the boundary conditions are satisfied. One way to achieve this objective could be to integrate Eq. (A_4) numerically from the surface towards the centre using say fourth-order Runge-Kutta method. Starting values near the surface may be obtained from series solution (A_7). Similarly we can integrate Eq. (A_4) numerically outwards from the centre starting from a point near the centre. The starting values near the centre may be obtained from the series solution (A_6). Trials with different values of σ may be continued till a value of σ is found for which the value $\varsigma(d\varsigma/dx)^{-1}$ from the inward and outward integrations match to desired accuracy at some suitably selected point inside the model.

The quantities ρ_ψ, P_ψ and the eigenfrequencies σ are still in dimensional form. For determining the eigenfrequencies it is recommended that these be first converted into suitable non-dimensional forms keeping in view the physical nature of the model under investigation.

It may be noted that the eigenvalued boundary value problem set up in this section determines the eigenfrequencies of the pseudo-radial modes of oscillations of a differentially rotating and tidally distorted gas spheres rotating differentially according to the law $\omega^2 = b_1^2 + 2b_1 b_2 s^2 + b_2^2 s^4$. For pseudo-radial oscillations of a rotating model having solid body rotation we may set $b_1^2 = 2$, $b_2^2 = 0$, $z = 1$ ($2n$ being the square of the angular velocity of rotation in Eq. (A_4)).

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ВЛИЯНИЕ ВАРИАЦИИ МАССЫ НА РАДИАЛЬНЫЕ КОЛЕБАНИЯ ДИФФЕРЕНЦИАЛЬНО ВРАЩАЮЩИХСЯ И ПРИЛИВНО ДЕФОРМИРОВАННЫХ ПОЛИТРОПИЧЕСКИХ ЗВЕЗД

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Предложен метод расчета собственных частот малых адиабатических псевдорадиальных мод колебаний дифференциально вращающихся и приливно деформированных звездных моделей, с учетом влияния вариаци массы на эквипотенциальные поверхности внутри звезды. Разработанный подход использован для расчета некоторых радиальных мод колебаний политропных моделей с политропными индексами 1.5, 3.0 и 4.0. Полученные результаты сравнены с результатами, полученными ранее без учета вариации массы. Приведены некоторые выводы, основанные на этом исследовании.

Ключевые слова: *варации массы: эквипотенциальные поверхности: вращение и приливные деформации: политропические модели и т.д.*

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