АСТРОФИЗИКА

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VIBRATIONAL STABILITY OF DIFFERENTIALLY ROTATING POLYTROPIC STARS

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A method for computing the periods of radial and non-radial modes of oscillations to determine the vibrational stability of differentially rotating polytropic gaseous spheres is represented and incorporated with averaging techniques of Kippenhahn and Thomas. The concepts of Roche-equipotential have also been used for obtaining the distorted structure of different stellar models. Numerical results, based on this study are presented to explain the effect of differential rotation on the oscillations and stability of polytropic stars.

Key words: *Differential rotation: adiabatic and non-adiabatic oscillations:*

1. *Introduction*. The earlier papers of Lal et al. [1], Mohan and Saxena [2], Mohan et al. [3] and Mohan and Singh [4] gave some ideas of the extensive theoretical work which has been done on the theory of oscillations of theoretical models of different types of variable stars. The evidence available thus far shows that except in case of classical Cepheids, it is not possible to explain all the phenomenon of stellar variability purely on the basis of simple radial oscillations. Efforts have, therefore, been often made in literature to explain some of the observed phenomenon of variable stars by invoking the theory of non-radial oscillations. The effects of internal and external forces at work in a star on its modes of radial and non-radial oscillations have also been often investigated for this purpose. Ledoux and Walraven [5] pointed out that dynamical instability leading to explosions in star might be easier to reach for some of the non-radial modes of oscillations rather than for purely radial oscillations. Therefore, non-radial modes of oscillations may be important for the interpretation of some of the more violet types of intrinsic variables.

Most of the authors such as Lal et al.[1], Mohan and Saxena [2], Clement [6], Saio [7], Chandrasekhar and Ferrari [8], Dintrans and Rieutord [9], Lignieres et al [10], Reese et al.[11] and Lovekin and Deupree [12] have studied the oscillations of stars assuming the star to have a solid body rotation, and therefore, rotating uniformly. However, authors such as Woodard [13], Dziembowski and Goode [14], Urpin et al. [15], Mohan et al. [16], Karino and Eriguchi [17], Lovekin et al. [18] and Saini et al. [19] addressed themselves to the problems of differentially rotating stars.

In this study, the problem of eigen-frequencies of radial and non-radial modes of oscillations of differentially rotating polytropic models are determined with the conjunction of an averaging techniques of Kippenhahn and Thomas [20], the law of differential rotation has been assumed of the type[(proposed by Clement [6]]:

$$\omega(s)=\sum_{i=1}^{3} a_i \exp\left(-b_i s^2\right), \tag{1}$$

where $\omega(s)$ is the angular velocity of rotation, s is a non-dimensional cylindrical coordinate while a_i and b_i are constants.

2. *Eigen-value boundary value problem to determine the eigen-frequencies of small oscillations of differentially rotating polytropic models of stars.* In this section, the explicit expressions for determining the pseudo adiabatic radial and non-radial modes of oscillations of differentially rotating polytropic stellar models have been formulated. For this purpose, the relations $P_\psi = P_c\,\theta_\psi^{N+1}$ and $\rho_\psi = \rho_c\theta_\psi^{N}$ have been used in the mathematical modeling of the problem.

2.1. *Radial Oscillations.* To determine pseudo-radial modes of polytropic stellar models, the differential equation for eigen-function is obtained as:

$$\Gamma_1^*\zeta''+\Gamma_2^*\zeta'+\left(\Gamma_3^*\omega^{*2}-\Gamma_4^*\right)\zeta=0, \tag{2}$$

here $\omega^{*2}=R^3 r_{os}^3\,\sigma^2/GM_0$ is the non-dimensional form of the eigen-frequency σ, ζ denotes a suitable average of the relative amplitudes of the pulsation of the fluid elements on the equipotential surface $\psi=\text{const}$, suffixes denote the differentiation with respect to r_0. Mass and radius of undistorted polytropic model are denoted by M_0 and R respectively, r_{os} is the value of r_0 at the surface of the model, while G is the gravitational constant.

The obtained parameters are as:

$$\Gamma_1^*=\wp_1,\quad \Gamma_2^*=\wp_{12}/r_0+(N+1)\theta'_\psi\wp_{13}/\theta_\psi,$$
$$\Gamma_3^*=(N+1)\xi_{su}^2\overline{\rho}/3\gamma r_{os}^3\rho_c\theta_\psi,\quad \Gamma_4^*=-(3-4/\gamma)(N+1)\theta'_\psi\wp_2/\theta_\psi r_0.$$

2.2. *Non-radial oscillations.* To determine pseudo non-radial modes of oscillations of polytropic stellar models, the eigen-functions ζ, η and ϕ are to be calculated by following differential equations:

$$\begin{gathered}\zeta'+\mathfrak{B}_1^*\zeta+\left(\mathfrak{B}_2^*+\mathfrak{B}_3^*/\omega^{*2}\right)\eta+\omega^{*-2}\mathfrak{B}_3^*\phi=0,\\ \eta'+\left(\mathfrak{C}_1^*\omega^{*2}+\mathfrak{C}_2^*\right)\zeta+\mathfrak{C}_3^*\eta+\mathfrak{C}_4^*\phi+\phi'=0,\\ \phi''+\mathfrak{F}_1^*\phi'+\mathfrak{F}_2^*\zeta+\mathfrak{F}_3^*\eta+\mathfrak{F}_4^*\phi=0.\end{gathered} \tag{3}$$

The dashes denote differentiation with respect to x, where $x=r_0/r_{os}$. ω^{*2} is same as defined in previous section. The parameters $\mathfrak{B}_{1,2,3}^*$, $\mathfrak{C}_{1,2,3,4}^*$ and $\mathfrak{F}_{1,2,3,4}^*$ are as:

$$\mathfrak{B}_1^* = (l+1)/x + (N+1)\theta'_\psi/\gamma\theta_\psi, \qquad \mathfrak{B}_2^* = (N+1)\xi_u^2 r_{as}^3 x \wp_3/2\gamma\theta_\psi,$$
$$\mathfrak{B}_3^* = -3l(l+1)\rho_c r_{as}^4 \wp_4/2x\overline{\rho}, \qquad \mathfrak{C}_1^* = -2\overline{\rho}\wp_5/3x\rho_c r_{as}^4,$$
$$\mathfrak{C}_2^* = 2(\theta'_\psi)^2\{N-(N+1)/\gamma\}\wp_6/\theta_\psi x r_{as}^3 \xi_u^2, \qquad \mathfrak{C}_3^* = l/x + \theta'_\psi\{N-(N+1)/\gamma\}/\theta_\psi,$$
$$\mathfrak{C}_4^* = l/x, \quad \mathfrak{F}_1^* = \{2(l+1)-\wp_7\}/x, \qquad \mathfrak{F}_2^* = 2\theta'_\psi\theta_\psi^{N-1}\{N-(N+1)/\gamma\}\wp_8/x r_{as},$$
$$\mathfrak{F}_3^* = -(N+1)\theta_\psi^{N-1} r_{as}^2 \xi_u^2 \wp_9/\gamma, \qquad \mathfrak{F}_4^* = -l[\wp_{10} + l\wp_{11}]/x^2.$$

The eigen-value problem (3) determines the eigen-frequencies of non-radial modes of oscillations of differentially rotating polytropic models of the star and has to be solved subject to the boundary conditions

$$\eta + \phi = 2\omega^{*2}\overline{\rho}\zeta/3lr_{as}^4\rho_c \quad \text{and} \quad \phi' = 0 \tag{4}$$

at the centre $x = 0$,

$$\eta r_{as}^3 \wp_3^* + 2\theta'_\psi/\xi_u^2 = 0 \quad \text{and} \quad \phi' + \phi\{l + (l+1)\wp_{14}^*\} = 0 \tag{5}$$

at the surface. In equations (5), asterisk on $\wp$ shows the value of these parameters at $x = 1$, while the parameters are defined in appendix.

The system of differential equations (4) has been solved using Chebyshev polynomials expansions technique. The transformation is used as $x = (z+1)/2$ $(-1 \le z \le 1)$, so that the range of integration is renormalized from the original range (0, 1) to (-1, 1).

3. *Analysis of numerical results and conclusion.* In this study, the law of differential rotation (1) is taken to observe the effects of parameters s, a_l and b_l (the values are given in Table1).

Table 1

DATA PRESENTED IN THIS TABLE HAS BEEN TAKEN FROM CLEMENT [6]

a_l and b_l	$N = 2.00$	$N = 2.50$	$N = 3.00$	$N = 3.25$
a_1	+0.546668	+0.263144	+0.095155	+0.048836
a_2	+0.544726	+0.720053	+0.555735	+0.400167
a_3	-0.091395	+0.016858	+0.350959	+0.550992
b_1	+0.117936	+0.097485	+0.051248	+0.037318
b_2	+0.387444	+0.290017	+0.203307	+0.153630
b_3	+0.714485	+0.021676	+0.594146	+0.490194

In Table 2, various polytropic stellar models are generated by the combinations of a_l and b_l. The distorted radii and eigen-frequencies of pseudo radial and non-radial modes of oscillations of these polytropic stellar models are computed for polytropic indices 2.0, 2.5, 3.0 and 3.25.

The Fig.1 shows that models 2, 7, 8, 13 and 14 are highly distorted by the effect of differential rotation. The effect increases, if the stars have higher

Table2

COMBINATIONS OF THE PARAMETERS FOR VARIOUS DIFFERENTIALLY ROTATING POLYTROPIC MODELS OF GASEOUS SPHERES

Model	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Parameters	0	0	a_1	a_1	a_1	a_1	0	0	0	a_1	a_1	0	0	a_1	0
	0	a_2	0	0	0	0	a_2	a_2	a_2	0	0	a_2	a_2	a_2	a_2
	0	a_3	a_3	a_3	a_3	a_3	a_3	a_3	0	0	0	0	a_3	a_3	0
	0	b_1	0	b_1	0	b_1	0	b_1	0	0	b_1	b_1	0	b_1	0
	0	0	b_2	0	b_2	b_2	b_2	b_2	0	0	b_2	b_2	0	b_2	b_2
	0	b_3	b_3	b_3	0	0	b_3	0	0	b_3	b_3	b_3	0	b_3	b_3

polytropic index such as 3.0 or 3.25. Except model 14, all assumed stellar models get low distortions if they are of polytropic index 2.0. Model 1 is an undistorted model and does not lose its shape for any polytropic index. However, there are small distortions in models 10 and 11 if they are of polytropic index 3.0 or 3.25. Therefore, more conclusions can also be drawn for the structure of realistic stars which are distorted by differential rotation and their polytropic nature.

The eigen-frequencies of differentially rotating polytropic stellar models for different polytropic indices are represented by Fig.2-5. In case of stars having any polytropic indices 2.0, 2.5 or 3.25, it is observed that model 1 (i.e. undistorted model) has large eigen-frequencies of fundamental, first and second modes of radial oscillations. All distorted polytropic models of polytropic index 2.0 have almost equal eigen-frequencies for fundamental mode of radial

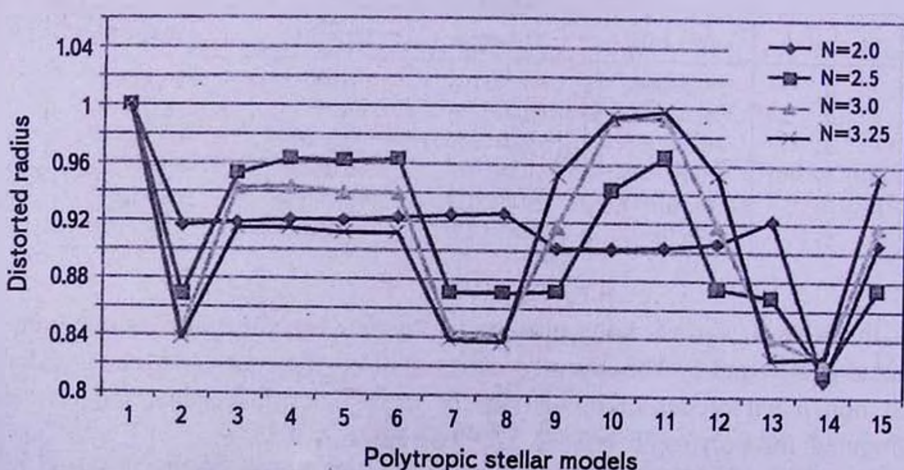

Fig.1. Distorted radii of different differentially rotating polytropic models.

oscillations, but models 9, 10, 11 and 12 have somelarge eigen-frequencies of first mode of radial oscillations in comparison of other distorted stellar models of same polytropic index. For second mode of radial oscillations of distorted stars of polytropic index 2.0 Comma model 3 gives lowest eigen-frequencies, which are slightly different with eigen-frequencies of model 5 of same index. The stellar models 2, 7, 8, 9, 12, 13 and 14 with polytropic index 2.5 do not give any mode of radial oscillation except fundamental mode. In case of polytropic index 3.0, first and second modes of radial oscillations could not find for models 2, 7, 8, 9, 12, 13, 14 and 15. For polytropic index 3.25,

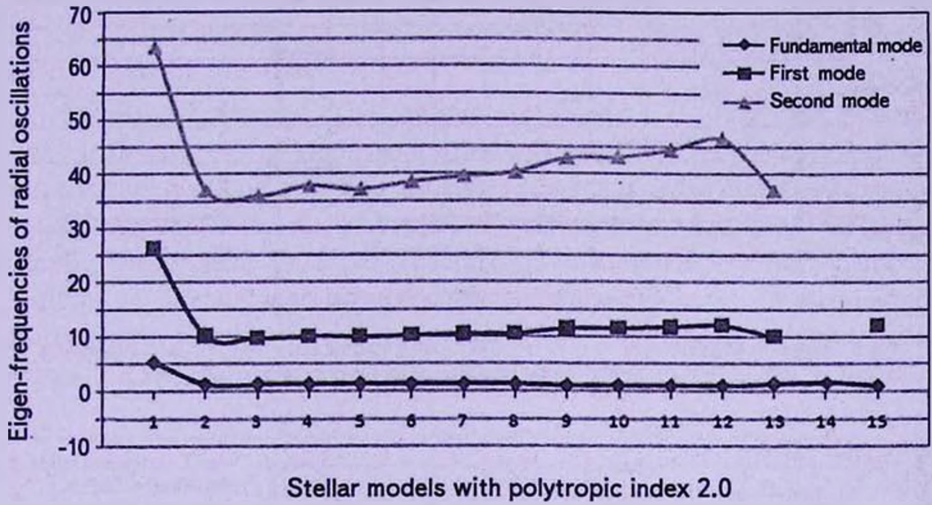

Fig.2. Eigen-frequencies ($\omega^2 = r_\omega^3 R^3 \sigma^2 / GM_0$) of fundamental, first and second modes of oscillations of differentially rotating stellar models of polytropic index 2.0.

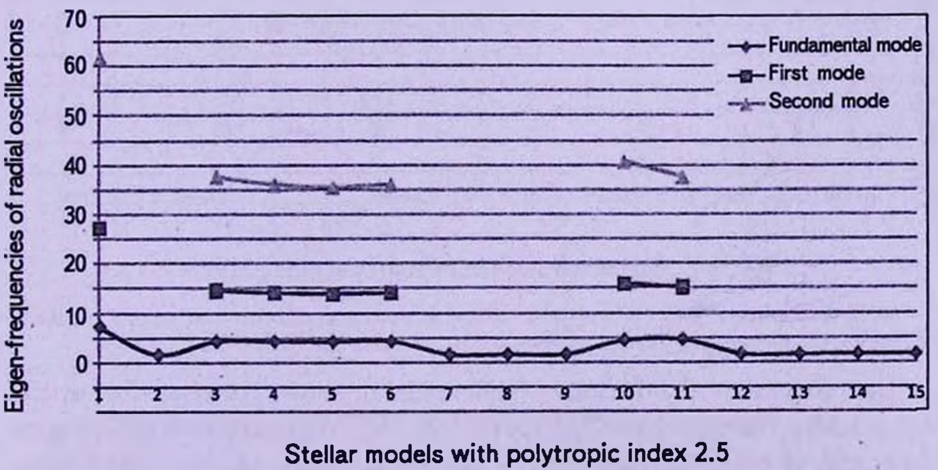

Fig.3. Eigen-frequencies ($\omega^2 = r_\omega^3 R^3 \sigma^2 / GM_0$) of fundamental, first and second modes of oscillations of differentially rotating stellar models of polytropic index 2.5.

models 10 and 11 have largest eigen-frequencies (25.80463 for first mode and 55.29806 for second mode). No model gives first and second modes of radial oscillations except for models 1, 9, 10, 11, 12 and 15. For polytropic index 2.0 model 14 does not give first and second modes while model 15 does not give second mode of radial oscillations.

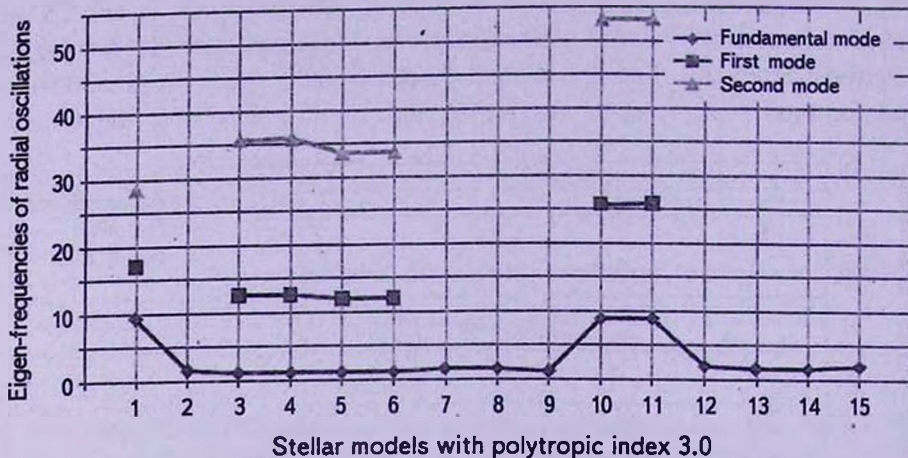

Fig.4. Eigen-frequencies ($\omega^2 = r_{\omega}^3 R^3 \sigma^2 / GM_0$) of fundamental, first and second modes of oscillations of differentially rotating stellar models of polytropic index 3.0.

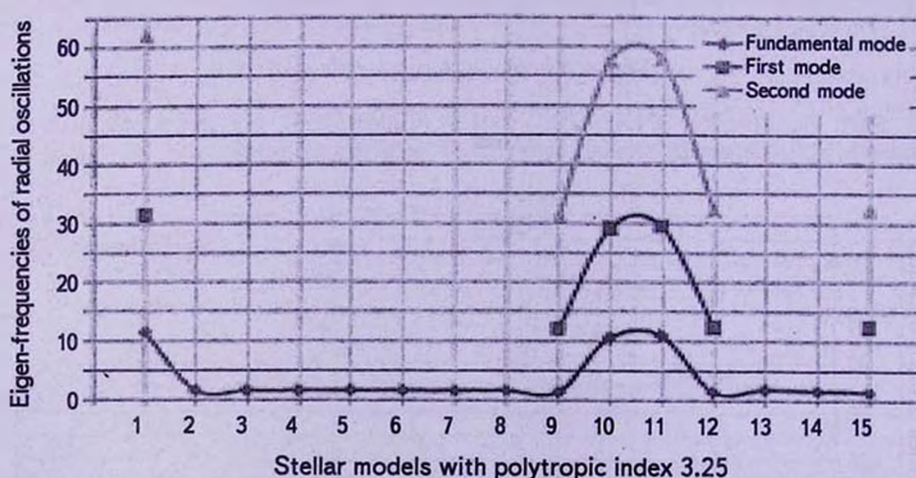

Fig.5. Eigen-frequencies ($\omega^2 = r_{\omega}^3 R^3 \sigma^2 / GM_0$) of fundamental, first and second modes of oscillations of differentially rotating stellar models of polytropic index 3.25.

The results show that because of differential rotations, the eigen-frequencies of f, p and g-modes of oscillations decrease in comparisons with the eigen-frequencies of the same mode of non-rotating polytropic models. These results appear to be contrary to the commonly accepted fact that rotation stabilizes (increases) the eigen-frequencies of g-modes. However, before attributing this contrary to expected behavior of the effect of rotation on g-modes to neglect the Coriolis force or barotropic nature, it should be noted that rotation stabilizes the eigen-frequencies of g-modes is based on the assumption that the mass of star does not change due to differential rotation.

The present study has shown that, in general, with the dependence of angular velocity of differential rotation on the parameters, the eigen-frequencies of pseudo-radial and non-radial modes (f and p-modes) of oscillations for differentially rotating polytropic models of indices 2.0, 2.5, 3.0 and 3.25. If the parameters of the law of differential rotation (1) are chosen in such a way that the angular velocity could find as a realistic star such as Sun, 1 Mon, 12 Lac or 16 Lac stars etc. then more useful conclusions can also be drawn on the basis of this study.

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Appendix

Detail of the parameters which has been used in the expressions of radial and non-radial modes of oscillations

$$\wp_1 = 1-\frac{8}{3}\mathcal{A}r_0^3+\frac{8}{5}\mathcal{B}r_0^5-\frac{26}{45}\mathcal{C}r_0^6-\frac{64}{105}\mathcal{D}r_0^7+\frac{16}{7}\mathcal{F}r_0^8+\left(\frac{32}{189}\mathcal{D}+\frac{3796}{405}\mathcal{G}\right)r_0^9-$$
$$-\left(\frac{928}{315}\mathcal{H}+\frac{328}{315}\mathcal{J}\right)r_0^{10}+\ldots,$$

$$\wp_2 = 1-\frac{5}{3}\mathcal{A}r_0^3+\frac{14}{15}\mathcal{B}r_0^5-\frac{47}{45}\mathcal{C}r_0^6-\frac{12}{35}\mathcal{D}r_0^7+\frac{136}{63}\mathcal{F}r_0^8+\left(\frac{88}{945}\mathcal{D}+\frac{1651}{405}\mathcal{G}\right)r_0^9-$$
$$-\left(\frac{8}{7}\mathcal{H}+\frac{268}{315}\mathcal{J}\right)r_0^{10}+\ldots,$$

$$\wp_3 = 1+2\mathcal{A}x^3r_{os}^3-\frac{16}{15}\mathcal{B}x^5r_{os}^5+\frac{24}{5}\mathcal{C}x^6r_{os}^6+\frac{8}{21}\mathcal{D}x^7r_{os}^7-\frac{44}{7}\mathcal{F}x^8r_{os}^8+$$
$$+\left(\frac{32}{5}\mathcal{D}-\frac{32}{315}\mathcal{G}\right)x^9r_{os}^9+\left(\frac{832}{315}\mathcal{H}+\frac{208}{105}\mathcal{J}\right)x^{10}r_{os}^{10}+\ldots,$$

$$\wp_4 = 1+\frac{4}{3}\mathcal{A}x^3r_{os}^3-\frac{4}{5}\mathcal{B}x^5r_{os}^5+\frac{133}{45}\mathcal{C}x^6r_{os}^6+\frac{32}{105}\mathcal{D}x^7r_{os}^7-\frac{152}{35}\mathcal{F}x^8r_{os}^8+$$
$$+\left(\frac{194}{81}\mathcal{D}-\frac{16}{189}\mathcal{G}\right)x^9r_{os}^9+\left(\frac{88}{45}\mathcal{H}+\frac{2332}{1575}\mathcal{J}\right)x^{10}r_{os}^{10}+\ldots,$$

$$\wp_5 = 1 + \frac{2}{3}\mathcal{A}x^3 r_{as}^3 - \frac{8}{15}\mathcal{B}x^5 r_{as}^5 + \frac{14}{9}\mathcal{C}x^6 r_{as}^6 + \frac{8}{35}\mathcal{D}x^7 r_{as}^7 - \frac{124}{45}\mathcal{F}x^8 r_{as}^8 + \\ + \left(\frac{536}{405}\mathcal{D} - \frac{64}{945}\mathcal{G}\right)x^9 r_{as}^9 + \left(\frac{88}{35}\mathcal{H} + \frac{184}{175}\mathcal{J}\right)x^{10} r_{as}^{10} + \dots,$$

$$\wp_6 = 1 - 2\mathcal{A}x^3 r_{as}^3 + \frac{16}{15}\mathcal{B}x^5 r_{as}^5 - \frac{4}{5}\mathcal{C}x^6 r_{as}^6 - \frac{8}{21}\mathcal{D}x^7 r_{as}^7 + \frac{212}{105}\mathcal{F}x^8 r_{as}^8 + \\ + \left(\frac{64}{5}\mathcal{D} - \frac{32}{15}\mathcal{G}\right)x^9 r_{as}^9 + \left(\frac{352}{315}\mathcal{H} + \frac{1328}{1575}\mathcal{J}\right)x^{10} r_{as}^{10} + \dots,$$

$$\wp_7 = 2\mathcal{A}x^3 r_{as}^3 - \frac{8}{3}\mathcal{B}x^5 r_{as}^5 + 8\mathcal{C}x^6 r_{as}^6 + \frac{8}{5}\mathcal{D}x^7 r_{as}^7 + \frac{352}{7}\mathcal{F}x^8 r_{as}^8 - \\ - \left(\frac{44}{5}\mathcal{D} - \frac{32}{15}\mathcal{G}\right)x^9 r_{as}^9 + \left(\frac{256}{21}\mathcal{H} - \frac{2864}{315}\mathcal{J}\right)x^{10} r_{as}^{10} + \dots,$$

$$\wp_8 = 1 + \frac{2}{3}\mathcal{A}x^3 r_{as}^3 - \frac{8}{15}\mathcal{B}x^5 r_{as}^5 + \frac{14}{9}\mathcal{C}x^6 r_{as}^6 + \frac{8}{35}\mathcal{D}x^7 r_{as}^7 - \frac{124}{45}\mathcal{F}x^8 r_{as}^8 + \\ + \left(\frac{536}{405}\mathcal{D} - \frac{64}{945}\mathcal{G}\right)x^9 r_{as}^9 - \left(\frac{48}{35}\mathcal{H} + \frac{184}{175}\mathcal{J}\right)x^{10} r_{as}^{10} + \dots,$$

$$\wp_9 = 1 + \frac{8}{3}\mathcal{A}x^3 r_{as}^3 - \frac{8}{5}\mathcal{B}x^5 r_{as}^5 + \frac{346}{45}\mathcal{C}x^6 r_{as}^6 + \frac{64}{105}\mathcal{D}x^7 r_{as}^7 - \frac{1136}{105}\mathcal{F}x^8 r_{as}^8 + \\ + \left(\frac{6092}{105}\mathcal{D} - \frac{32}{189}\mathcal{G}\right)x^9 r_{as}^9 + \left(\frac{496}{105}\mathcal{H} + \frac{5672}{1575}\mathcal{J}\right)x^{10} r_{as}^{10} + \dots,$$

$$\wp_{10} = 2 + 4\mathcal{A}x^3 r_{as}^3 - 4\mathcal{B}x^5 r_{as}^5 + \frac{67}{5}\mathcal{C}x^6 r_{as}^6 + \frac{32}{15}\mathcal{D}x^7 r_{as}^7 - \frac{964}{35}\mathcal{F}x^8 r_{as}^8 - \\ - \left(\frac{19}{5}\mathcal{D} - \frac{16}{21}\mathcal{G}\right)x^9 r_{as}^9 + \left(\frac{5048}{315}\mathcal{H} + \frac{3796}{105}\mathcal{J}\right)x^{10} r_{as}^{10} + \dots,$$

$$\wp_{11} = 2\mathcal{A}x^3 r_{as}^3 - \frac{4}{3}\mathcal{B}x^5 r_{as}^5 + \frac{27}{5}\mathcal{C}x^6 r_{as}^6 + \frac{56}{105}\mathcal{D}x^7 r_{as}^7 - \frac{292}{35}\mathcal{F}x^8 r_{as}^8 + \\ + \left(\frac{37}{5}\mathcal{D} - \frac{16}{105}\mathcal{G}\right)x^9 r_{as}^9 + \left(\frac{1208}{315}\mathcal{H} + \frac{932}{315}\mathcal{J}\right)x^{10} r_{as}^{10} + \dots,$$

$$\wp_{12} = 4 - \frac{32}{3}\mathcal{A}r_0^3 + \frac{116}{15}\mathcal{B}r_0^5 - \frac{266}{45}\mathcal{C}r_0^6 - \frac{368}{105}\mathcal{D}r_0^7 + \frac{1120}{63}\mathcal{F}r_0^8 + \\ + \left(\frac{1072}{945}\mathcal{D} + \frac{14914}{315}\mathcal{G}\right)r_0^9 - \left(\frac{752}{63}\mathcal{H} + \frac{904}{105}\mathcal{J}\right)r_0^{10} + \dots,$$

$$\wp_{13} = 1 - \frac{8}{3}\mathcal{A}r_0^3 + \frac{8}{5}\mathcal{B}r_0^5 - \frac{26}{45}\mathcal{C}r_0^6 - \frac{64}{105}\mathcal{D}r_0^7 + \frac{16}{7}\mathcal{F}r_0^8 + \\ + \left(\frac{32}{189}\mathcal{D} + \frac{1622}{315}\mathcal{G}\right)r_0^9 - \left(\frac{752}{63}\mathcal{H} + \frac{904}{105}\mathcal{J}\right)r_0^{10} + \dots,$$

$$\wp_{14} = 1 + \mathcal{A}x^3 r_{as}^3 - \frac{2}{3}\mathcal{B}x^5 r_{as}^5 + \frac{11}{5}\mathcal{C}x^6 r_{as}^6 + \frac{8}{15}\mathcal{D}x^7 r_{as}^7 - \frac{368}{105}\mathcal{F}x^8 r_{as}^8 + \\ + \left(\frac{32}{5}\mathcal{D} - \frac{32}{315}\mathcal{G}\right)x^9 r_{as}^9 + \left(\frac{104}{63}\mathcal{H} + \frac{44}{63}\mathcal{J}\right)x^{10} r_{as}^{10} + \dots,$$

where

$$\mathcal{A}=\sum_{i=1}^{3}a_i\,,\quad \mathcal{B}=\sum_{i=1}^{3}a_ib_i\,,\quad \mathcal{C}=\sum_{i=1}^{3}\sum_{j=1}^{3}a_ia_j\,,\quad \mathcal{D}=\sum_{i=1}^{3}a_ib_i^2\,,\quad \mathcal{F}=\sum_{i=1}^{3}\sum_{j=1}^{3}a_ib_ia_j\,,$$

$$\mathcal{G}=\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}a_ia_ja_k\,,\quad \mathcal{H}=\sum_{i=1}^{3}\sum_{j=1}^{3}a_ib_i^2a_j\,,\quad \mathcal{J}=\sum_{i=1}^{3}\sum_{j=1}^{3}a_ib_ia_jb_j\,.$$

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ВИБРАЦИОННАЯ УСТОЙЧИВОСТЬ ДИФФЕРЕНЦИАЛЬНО ВРАЩАЮЩИХСЯ ПОЛИТРОПНЫХ ЗВЕЗД

С.КУМАР[1], А.К.ЛАЛ[2], С.САИНИ[3]

Предложен метод для вычисления периодов радиальных и нерадиальных мод осцилляций с целью определения вибрационной устойчивости дифференциально вращающихся политропных газовых сфер. Он сопоставляется с усредненной процедурой Киппененанд Томаса. Для получения возмущенной структуры различных звездных моделей, использовано также понятие эквипотенциалов Роше. Численные результаты, основанные на данном исследовании, приводятся для объяснения влияния дифференциального вращения на осцилляции и стабильность политропических звезд.

Ключевые слова: *дифференциальное вращение: адиабатик и не-адиабатик осцилляции*

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