АСТРОФИЗИКА

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KALUZA-KLEIN ANISOTROPIC MAGNETIZED DARK ENERGY COSMOLOGICAL MODEL IN BRANS-DICKE THEORY OF GRAVITATION

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We study spatially homogeneous Kaluza-Klein cosmological model with magnetized anisotropic fluid in the scalar tensor theory of Gravitation proposed by Brans-Dicke [1]. Exact solutions of the models are obtained by Volumetric exponential and Power law expansion. The Physical behaviors of the models have been discussed using some physical quantities.

Key words: *Kaluza-Klein model: Electromagnetic field: Dark energy: Brans-Dicke Theory*

1. *Introduction.* Recent cosmological observations show that the universe is accelerating and expanding. It is considered that dark energy is responsible for acceleration of universe [2-13]. The simplest candidate for dark energy (DE) is the cosmological constant (Λ), conventionally associated with the energy of the vacuum. The scalar field's models, such as quintessence [14], phantom [15], quintom [16], K-essence [17], tachyon [18] and dilaton [19] together with interacting dark energy models such as holographic [20] and agegraphic [21] models are the examples of dynamical dark energy models. Various forms of time dependent w have been used for variable models by Mukhopadhyay et al. [22]. Setare [23-25] and Setare et al. [26] have also studied the DE models in different contexts. Dark energy models with variable EoS parameter have been studied by Ray et al. [27]. Yadav et al. [28] investigated the Bianchi Type III dark energy models with constant deceleration parameter. Pradhan et al. [29] obtained anisotropic dark energy Bianchi Type III model with variable EoS parameter in general relativity. Pradhan et al. [30] and Amirhashchi et al. [31-33] investigated dark energy models with variable EoS parameter. Adhav et al. [34] have studied the Bianchi Type I cosmological model with a binary mixture of perfect fluid and dark energy in higher dimensions. Adhav et al. [35] also obtained the Kaluza-Klein cosmological models with anisotropic dark energy in general relativity. Katore et al. [36] has been investigated Kaluza-Klein cosmological models for perfect fluid and dark energy.

In theoretical physics, the Brans-Dicke theory of gravitation is a theoretical framework to explain gravitation. Among the various modification of General Relativity (GR), the Brans-Dicke (BD) theory of gravity [1] is a well known example of a scalar tensor theory in which the gravitational interaction involves a scalar field and the metric tensor. The unification of gravitational forces with other forces in nature is not possible in the usual four dimensional space-times. The study of higher dimensional space-time is important at early stages of evolution of the universe. Witten [37], Appelquist et al. [38], Chodos et al. [39] and Marchiano [40] were attracted to the study of higher dimensional cosmology because it has physical relevance to the early times before the universe has undergone compactification transitions. A five dimensional Kaluza-Klein dark energy model with variable equation of state (EoS) parameter and a constant deceleration parameter is presented in Saez and Ballester scalar-tensor theory of gravitation and scale covariant theory of gravitation by Reddy et al. [41,42]. Rao et al. [43,44] investigated LRS Bianchi Type I dark energy cosmological model in Brans-Dicke and Nordtvedt theory of gravitation. Naidu et al. [45] obtained Bianchi Type III dark energy model in a Saez-Ballester scalar tensor theory of gravitation. Ghate et al. [46] have discussed the cosmological model in Brans-Dicke theory for a Bianchi Type IX space-time filled with dark energy. Anisotropic Dark Energy Bianchi Type III cosmological models in Brans-Dicke theory of Gravity is obtained by Shamir et al. [47]. Katore et al. [48] have studied the solutions of cylindrically symmetric Einstein Rosen universe with variable w in the scalar tensor theory of gravitation proposed by Saez and Ballester in the presence and absence of magnetic field of energy density ρ_B. Also Katore et al. [49,50] have discussed Bianchi Type V and Plane symmetric space- time dark energy model in Brans-Dicke theory of gravitation. Investigation of Bianchi Type III cosmological models with anisotropic dark energy by Akarsu et al. [51] motivates us to investigate Kaluza-Klein universe with anisotropic magnetized dark energy in Brans-Dicke theory of gravitation.

We would like to investigate the effect of magnetic field on the Kaluza-Klein model in the presence of anisotropic dark energy. We find the solutions using the assumption of Exponential law and Power law. The paper has the following format: In section 2, metric and momentum-tensor of Kaluza-Klein are described. The field equations of Kaluza-Klein are presented in section 3. The models are presented in section 4 (Exponential Law Model) and Section 5 (Power Law Model). Section 6 concludes the findings.

2. *Metric and Energy Momentum Tensor*. We consider five dimensional Kaluza-Klein metric in the form

$$ds^2 = -dt^2 + A^2\left(dx^2 + dy^2 + dz^2\right) + B^2\, d\phi^2, \tag{1}$$

where A, B are functions of cosmic time t. The fifth co-ordinate is taken to be space-like, unlike Wesson [54]. Here the spatial curvature has been taken as zero by Gron [55]. We assume that the universe is filled with anisotropic fluid and that there is no electric field while the magnetic field is oriented along X-axis. King and Coles [52] used the magnetized perfect fluid energy-momentum tensor to discuss the effects of magnetic field on the evolution of the universe.

Brans-Dicke [1] introduced a scalar-tensor theory of gravitation involving a scalar function in addition to the familiar general relativistic metric tensor. In this theory the scalar field has the dimension of inverse of the gravitational constant and its role is confined to its effects on gravitational field equations. Brans-Dicke field equations for the combined scalar and tensor field are given by

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}-\frac{\omega}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu}-\frac{1}{2}g_{\mu\nu}\phi_{,k}\phi^{,k}\right)-\frac{1}{\phi}\left(\phi_{;\mu\nu}-g_{\mu\nu}\phi^{;k}_{;k}\right)=\frac{8\pi T_{\mu\nu}}{\phi}, \tag{2}$$

$$\phi^{;k}_{;k}=\frac{8\pi T}{(3+2\omega)\phi}, \tag{3}$$

where ω is a dimensionless coupling constant. It is satisfies the equation (3) given by ϕ is known as BD scalar field while T is the trace of the matter energy-momentum tensor. It is mentioned here that the general relativity is recovered in the limiting case $\omega\to\infty$. Thus we can compare our results with experimental tests for significantly large value of ω.

Preserving the diagonal form of the energy-momentum tensor in a consistent way with the metric (1), the simplest generalization of EoS parameter of perfect fluid is to determine the EoS parameter separately on each spatial axis by Reddy et al. [41]. Hence the combined energy-momentum tensor for anisotropic fluid and magnetic field is taken in the following form (King et al. [52], Sharif et al. [56]).

$$T^i_j=\text{diag}\left[T^1_1, T^2_2, T^3_3, T^4_4, T^5_5\right]. \tag{4}$$

Then we may parameterize it as follows

$$T^i_j=\text{diag}\left[P_x-\rho_B, P_y-\rho_B, P_z+\rho_B, P_\phi+\rho_B, -\rho+\rho_B\right], \tag{5}$$

$$T^i_j=\text{diag}\left[w_x\rho-\rho_B, w_y\rho-\rho_B, w_z\rho+\rho_B, w_\phi\rho+\rho_B, -\rho+\rho_B\right], \tag{6}$$

where ρ is the energy density; P_x, P_y, P_z, P_ϕ are pressure on x, y, z and ϕ axes respectively; ρ_B stands for energy density of magnetic field where as w_x, w_y, w_z, w_ϕ are the directional EoS parameter along x, y, z, ϕ axes respectively. By setting $w_x=w+\delta$, $w_y=w+\gamma$, $w_\phi=w$, $w_z=w+\eta$.

We have

$$T^i_j=\text{diag}\left[(w+\delta)\rho-\rho_B, (w+\gamma)\rho-\rho_B, (w+\eta)\rho+\rho_B, w\rho+\rho_B, -\rho+\rho_B\right], \tag{7}$$

where w is deviation free EoS parameter and δ, γ, η are the skewness parameters. If the deviation parameters are equal then (6) represents the energy-momentum tensor for the isotropic fluid and magnetic field (King et al. [52]).

For the magnetic field to be zero, (6) is reduced to the energy-momentum tensor for anisotropic fluid (Akarsu et al. [51]).

3. *Field Equation.* In the co-moving co-ordinate system, the Brans-Dicke field equations (2)-(3) for the metric (1) with help of equations (4) - (7) becomes

$$2\frac{A_{55}}{A}+\frac{B_{55}}{B}+\frac{A_5^2}{A^2}+2\frac{A_5}{A}\frac{B_5}{B}+\frac{\omega}{2}\left(\frac{\phi_5}{\phi}\right)^2+\frac{\phi_5}{\phi}\left(2\frac{A_5}{A}+\frac{B_5}{B}\right)+\frac{\phi_{55}}{\phi}=-\frac{8\pi}{\phi}\left[(w+\delta)\rho-\rho_B\right], \quad (8)$$

$$2\frac{A_{55}}{A}+\frac{B_{55}}{B}+\frac{A_5^2}{A^2}+2\frac{A_5}{A}\frac{B_5}{B}+\frac{\omega}{2}\left(\frac{\phi_5}{\phi}\right)^2+\frac{\phi_5}{\phi}\left(2\frac{A_5}{A}+\frac{B_5}{B}\right)+\frac{\phi_{55}}{\phi}=-\frac{8\pi}{\phi}\left[(w+\gamma)\rho-\rho_B\right], \quad (9)$$

$$2\frac{A_{55}}{A}+\frac{B_{55}}{B}+\frac{A_5^2}{A^2}+2\frac{A_5}{A}\frac{B_5}{B}+\frac{\omega}{2}\left(\frac{\phi_5}{\phi}\right)^2+\frac{\phi_5}{\phi}\left(2\frac{A_5}{A}+\frac{B_5}{B}\right)+\frac{\phi_{55}}{\phi}=-\frac{8\pi}{\phi}\left[(w+\eta)\rho-\rho_B\right], \quad (10)$$

$$3\frac{A_{55}}{A}+3\frac{A_5^2}{A^2}+\frac{\omega}{2}\left(\frac{\phi_5}{\phi}\right)^2+3\frac{\phi_5}{\phi}\frac{A_5}{A}+\frac{\phi_{55}}{\phi}=-\frac{8\pi}{\phi}\left[w\rho+\rho_B\right], \quad (11)$$

$$3\frac{A_5^2}{A^2}+3\frac{A_5}{A}\frac{B_5}{B}-\frac{\omega}{2}\left(\frac{\phi_5}{\phi}\right)^2+\frac{\phi_5}{\phi}\left(3\frac{A_5}{A}+\frac{B_5}{B}\right)=-\frac{8\pi}{\phi}\left[-\rho+\rho_B\right]. \quad (12)$$

Here the subindex 5' in A, B, ϕ and elsewhere denotes derivatives with respect to time t.

Using equation (3), we get

$$\phi_{55}+\phi_5\left(3\frac{A_4}{A}+\frac{B_4}{B}\right)=\frac{8\pi\left[(1-4w-\delta-\gamma-\eta)\rho-\rho_B\right]}{(3+2\omega)\phi}. \quad (13)$$

From equation (8) and (9), we obtain

$$\gamma=\delta. \quad (14)$$

From equations (9), (10) and (14), we found that

$$\eta=\delta-\frac{2\rho_B}{\rho}. \quad (15)$$

Thus the system of equations (8)-(12) reduce to

$$2\frac{A_{55}}{A}+\frac{B_{55}}{B}+\frac{A_5^2}{A^2}+2\frac{A_5}{A}\frac{B_5}{B}+\frac{\omega}{2}\left(\frac{\phi_5}{\phi}\right)^2+\frac{\phi_5}{\phi}\left(2\frac{A_5}{A}+\frac{B_5}{B}\right)+\frac{\phi_{55}}{\phi}= =-\frac{8\pi}{\phi}\left[(w+\eta)\rho+\rho_B\right], \quad (16)$$

$$3\frac{A_{55}}{A}+3\frac{A_5^2}{A^2}+\frac{\omega}{2}\left(\frac{\phi_5}{\phi}\right)^2+3\frac{\phi_5}{\phi}\frac{A_5}{A}+\frac{\phi_{55}}{\phi}=-\frac{8\pi}{\phi}\left[w\rho+\rho_B\right], \quad (17)$$

$$3\frac{A_5^2}{A^2}+3\frac{A_5}{A}\frac{B_5}{B}-\frac{\omega}{2}\left(\frac{\phi_5}{\phi}\right)^2+\frac{\phi_5}{\phi}\left(3\frac{A_5}{A}+\frac{B_5}{B}\right)=-\frac{8\pi}{\phi}\left[-\rho+\rho_B\right], \quad (18)$$

$$\phi_{55}+\phi_5\left(3\frac{A_5}{A}+\frac{B_5}{B}\right)=\frac{8\pi\left[(1-4w-\delta-\gamma-\eta)\rho-\rho_B\right]}{(3+2\omega)\phi}, \quad (19)$$

Subtracting (16) from (17), we get

$$\frac{A_{55}}{A}-\frac{B_{55}}{B}+2\frac{A_5^2}{A^2}-2\frac{A_5}{A}\frac{B_5}{B}+\frac{\phi_5}{\phi}\left(\frac{A_5}{A}-\frac{B_5}{B}\right)=\eta\rho. \tag{20}$$

From equation (20), we obtain

$$H_z-H_\phi=\frac{A_5}{A}-\frac{B_5}{B}=\frac{\lambda}{V\phi}+\frac{1}{V\phi}\iint[(\gamma\rho-2\rho_B)V\phi]dt, \tag{21}$$

where λ is the constant of integration. The integral term vanish for

$$\gamma=\frac{2\rho_B}{\rho} \quad \text{or} \quad \eta=0. \tag{22}$$

For this term the equation (21) becomes

$$\frac{A}{c_2 B}=\exp\int\frac{\lambda}{A^3B\phi}dt. \tag{23}$$

The power law relation between scale factor a and scalar field ϕ has already been used by Johri et al. [53] in the context of Robertson Walker Brans-Dicke models. Thus the power law relation between ϕ and a i.e. $\phi \propto a^n$ where n is any integer implies that

$$\phi=ba^n, \tag{24}$$

where b is the constant of proportionality.

We assumed that the magnetized dark energy is minimally interacting, hence the Bianchi identity has been split into two separately additive conserved components: namely, the conservation of the energy-momentum tensor for the anisotropic fluid and for the magnetic field (King et al. [52], Katore et al. [48]).

The equations of motion

$$T^{ij}_{;j}=0 \tag{25}$$

are consequence of the field equations (1) and (2).

Equation (25) gives as

$$\rho_5+(1+w)\rho\left(3\frac{A_5}{A}+\frac{B_4}{B}\right)+\rho(\delta+\gamma+\eta)\frac{A_5}{A}=0 \tag{26}$$

$$(\rho_B)_5+4\frac{A_5}{A}\rho_B=0. \tag{27}$$

Equation (27) leads to

$$\rho_B=\frac{c}{A^4} \tag{28}$$

where c is constant of integration.

Now we have four linearly independent equations (16)-(19) and seven unknown variables A, B, ρ, ρ_B, ϕ, η, γ. Thus we can introduce more conditions either by an assumption corresponding to some physical situation or an arbitrary mathematical supposition; however these procedures have some draw-backs. Physical situation may lead to differential equations which will be difficult to integrate and mathematical supposition may lead to a non-physical situation. The system is initially undermined and we need additional constraints to close the system. To do that we have used two different volumetric expansion laws (Akarsu et al. [51])

$$V = c_1 e^{4kt}, \tag{29}$$

$$V = c_1 t^{4m}, \tag{30}$$

where c_1, k and m are arbitrary positive constant. The model with exponential expansion exhibit accelerating volumetric expansion, where as the model of power law gives constant deceleration parameter q for $0 < m < 1$ and accelerated expansion for $m > 1$. For $m = 1$, the average scale factor has a linear growth with constant velocity and $q = 0$ that is universe is in inflationary phase.

The anisotropy of the expansion can be parameterized after defining the directional Hubble parameters and the mean Hubble parameter of the expansion. The directional Hubble parameters in the direction of x, y, z and ϕ axes for the Kaluza-Klein metric may be defined in (1) as follows

$$H_x = H_y = H_z = \frac{A_5}{A}, \quad H_\phi = \frac{B_5}{B}. \tag{31}$$

The mean Hubble Parameter is given by

$$H = \frac{1}{4}\frac{V_5}{V} = \frac{1}{4}\left(3\frac{A_5}{A} + \frac{B_5}{B}\right), \tag{32}$$

where $V = A^3 B$ is the volume of the universe. The anisotropy parameter of the expansion is defined as

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2, \tag{33}$$

where H_i $(i = 1, 2, 3, 4)$ represent the directional Hubble parameters in the direction of x, y, z and ϕ respectively. The equation (33) further reduces to

$$\Delta = \frac{3}{16H^2}\left(H_x - H_\phi\right)^2, \tag{34}$$

$\Delta = 0$ corresponds to isotropic expansion. The space approaches isotropy, in case of diagonal energy-momentum tensor if $\Delta = 0$, $V \to +\infty$ as $t \to +\infty$ (Collins et al. [57]).

4. *Exponential Expansion model.* Using equations (23) and (29), we get

$$A = (c_1 c_2)^{1/4} \exp\left[kt - \frac{\lambda e^{-4k((n+3)/3)t}}{16bk((n+3)/3)c_1^{(n+3)/3}}\right], \tag{35}$$

$$B=\left(\frac{c_1}{c_2^3}\right)^{1/4}\exp\left[kt+\frac{3\lambda e^{-4k((n+3)/3)t}}{16bk((n+3)/3)c_1^{(n+3)/3}}\right]. \tag{36}$$

The magnetized energy density using equations (28) and (35) is obtained as

$$\rho_B=\left(\frac{c}{c_1c_2}\right)\exp\left[-4kt+\frac{\lambda e^{-4k((n+3)/3)t}}{4bk((n+3)/3)c_1^{(n+3)/3}}\right]. \tag{37}$$

The energy density of the model using equations (18), (24), (28), (35) and (36) is found to be

$$\frac{8\pi\rho}{\phi}=\left\{\frac{8\pi}{bc_1^{n/3}e^{4nkt/3}}\left[\left(\frac{c}{c_1c_2}\right)\exp\left[-4kt+\frac{\lambda e^{-4k((n+3)/3)t}}{4bk((n+3)/3)c_1^{(n+3)/3}}\right]\right]+\right.$$
$$\left.+\left[\frac{27-4\omega n^2+24n}{9}\right]2k^2-\left[\frac{3\lambda^2 e^{-8k((n+3)/3)t}}{8b^2c_1^{2((n+3)/3)}}\right]\right\}. \tag{38}$$

The deviation free Eos parameter from equations (17), (24), (28), (35), (36) and (38) is obtained as

$$\frac{8\pi\rho}{\phi}=-\left\{\frac{8\pi}{bc_1^{n/3}e^{4nm/3}}\left[\left(\frac{c}{c_1c_2}\right)t^{-4m}\exp\left[\frac{-\lambda t^{[1-4m((n+3)/3)]}}{b[1-4m((n+3)/3)]c_1^{(n+3)/3}}\right]\right]+\right.$$
$$\left.+\left[\frac{27+4\omega n^2+24n}{9}\right]+\left[\frac{3\lambda^2 t^{-8m(n+3/3)}}{8b^2c_1^{2(n+3)/3}}\right]\right\} \tag{39}$$

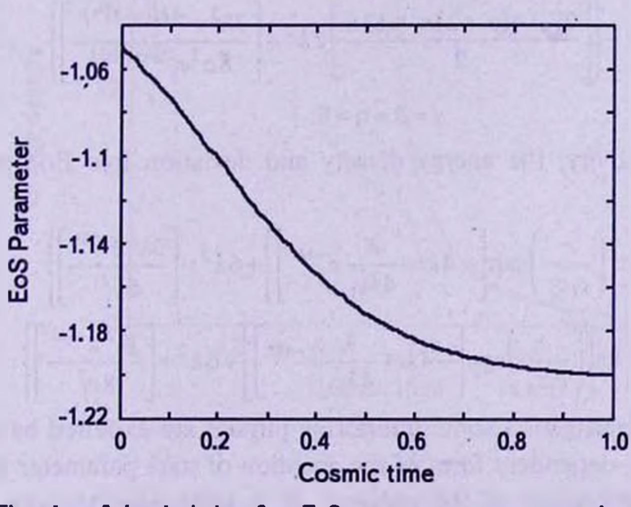

Fig.1. The plot of the deviation free EoS parameter w versus cosmic time in exponential law. Here $b=n=k=c=c_1=c_2=\omega=\lambda=1$.

In the graph one can observe that the value of the deviation free EoS parameter $w<-1$ which represents the phantom fluid dominated universe.

The skewness parameter using equation (22) becomes

$$\gamma=\delta=\frac{1}{\rho}\left(\frac{2c}{c_1c_2}\right)\exp\left[-4kt+\frac{\lambda e^{-4k((n+3)/3)t}}{4bk((n+3)/3)c_1^{(n+3)/3}}\right], \tag{40}$$

$$\eta=0. \tag{41}$$

Using equation (31), (35) and (36) the directional Hubble parameter of the model is found to be

$$H_x=H_y=H_z=k+\frac{\lambda e^{-4k((n+3)/3)t}}{4bc_1^{(n+3)/3}}, \quad H_\phi=k-\frac{3\lambda e^{-4k((n+3)/3)t}}{4bc_1^{(n+3)/3}}. \tag{42}$$

The mean Hubble parameter of this model is given by

$$H=k. \tag{43}$$

The anisotropy parameter leads to

$$\Delta=\frac{3\lambda^2 e^{-8k((n+3)/3)t}}{16k^2b^2c_1^{2((n+3)/3)}}. \tag{44}$$

In the absence of magnetic field i.e. $c\to 0$, the energy density for magnetic field of the model, the energy density of the model, deviation free EoS parameter and the skewness parameter are

$$\rho_B=0 \tag{45}$$

$$\frac{8\pi\rho}{\phi}=\left\{\left[\frac{27-4\omega n^2+24n}{9}\right]2k^2-\left[\frac{3\lambda^2 e^{-8k((n+3)/3)t}}{8b^2c_1^{2((n+3)/3)}}\right]\right\}, \tag{46}$$

$$\frac{8\pi w\rho}{\phi}=-\left\{\left[\frac{27+4(\omega+2)n^2+18n}{9}\right]2k^2+\left[\frac{3\lambda^2 e^{-8k((n+3)/3)t}}{8b^2c_1^{2((n+3)/3)}}\right]\right\}, \tag{47}$$

$$\gamma=\delta=\eta=0. \tag{48}$$

In General Relativity, the energy density and deviation free EoS parameter becomes

$$8\pi\rho=\left\{8\pi\left[\left(\frac{c}{c_1c_2}\right)\exp\left[-4kt+\frac{\lambda}{4kc_1}e^{-4kt}\right]\right]+6k^2-\left[\frac{3\lambda^2 e^{-8kt}}{8c_1^2}\right]\right\}, \tag{49}$$

$$8\pi w=-\frac{1}{\rho}\left\{8\pi\left[\left(\frac{c}{c_1c_2}\right)\exp\left[-4kt+\frac{\lambda}{4kc_1}e^{-4kt}\right]\right]+6k^2+\left[\frac{3\lambda^2 e^{-8kt}}{8c_1^2}\right]\right\}. \tag{50}$$

In the present investigation some interesting physics are explored by assuming a particular time -dependent form of the equation of state parameter w to track down the time evolution of the universe. It is interesting to note that the present work has also been successful in justifying the idea of Adhav et al. [35] that time-dependent equation of state parameter is essential for studying the complete time-evolution of the cosmos and in the absence of magnetic field and scalar field tends to 1, this results are resembles to that of investigated results of the Akarsu et al. [51] in general relativity.

5. *Power Law Model.*Using equation (23) and (30), we obtain

$$A=(c_1c_2)^{1/4}t^m\exp\left[\frac{\lambda t^{[1-4m((n+3)/3)]}}{4b[1-4m((n+3)/3)]c_1^{(n+3)/3}}\right], \tag{51}$$

$$B=\left(\frac{c_1}{c_2^3}\right)^{1/4}t^m\exp\left[\frac{-3\lambda t^{[1-4m((n+3)/3)]}}{4b[1-4m((n+3)/3)]c_1^{(n+3)/3}}\right]. \tag{52}$$

The magnetized energy density using equations (28) and (51) is obtained as

$$\rho_B=\left(\frac{c}{c_1c_2}\right)^{1/4}t^{-4m}\exp\left[\frac{-\lambda t^{[1-4m((n+3)/3)]}}{b[1-4m((n+3)/3)]c_1^{(n+3)/3}}\right]. \tag{53}$$

The energy density of the model using equations (18), (24), (28), (51) and (52) is found to be

$$\frac{8\pi\rho}{\phi}=\left\{\frac{8\pi}{bc_1^{n/3}t^{4nm/3}}\left[\left(\frac{c}{c_1c_2}\right)t^{-4m}\exp\left[\frac{-\lambda t^{[1-4m((n+3)/3)]}}{b[1-4m((n+3)/3)]c_1^{(n+3)/3}}\right]\right]+\right.$$
$$\left.+\left[\frac{27-4\omega n^2+24n}{9}\right]-\left[\frac{3\lambda^2t^{-8m((n+3)/3)}}{8b^2c_1^{2(n+3)/3}}\right]\right\}. \tag{54}$$

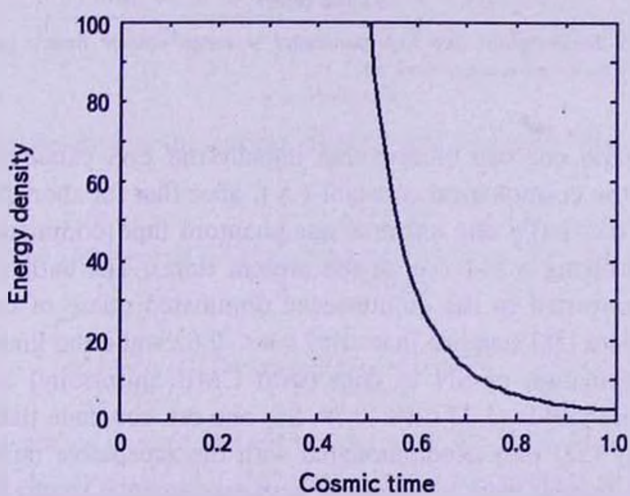

Fig.2. The plot of energy density ρ versus cosmic time in power law expansion. Here $b=n=k=c=c_1=c_2=\omega=\lambda=1$.

From Fig.2, it can be deduced that at an early stage of the universe, the energy density of the universe is large and at late time it decreases.

The deviation free EoS parameter from equations (17), (24), (28), (51), (52) and (54) is obtained as

$$\frac{8\pi w\rho}{\phi}=-\left\{\begin{array}{l}\frac{8\pi}{bc_1^{n/3}t^{4mn/3}}\left[\left(\frac{c}{c_1c_2}\right)t^{-4m}\exp\left[\frac{-\lambda t^{[1-4m((n+3)/3)]}}{b[1-4m((n+3)/3)]c_1^{(n+3)/3}}\right]+\right]\\ +\left[\frac{27+4(\omega+2)n^2+18n}{9}\right]\frac{2m^2}{t^2}-\left(\frac{4n+9}{3}\right)\frac{m}{t^2}+\left[\frac{3\lambda^2 t^{-8m((n+3)/3)}}{8b^2c_1^{2(n+3/3)}}\right]\end{array}\right\} \tag{55}$$

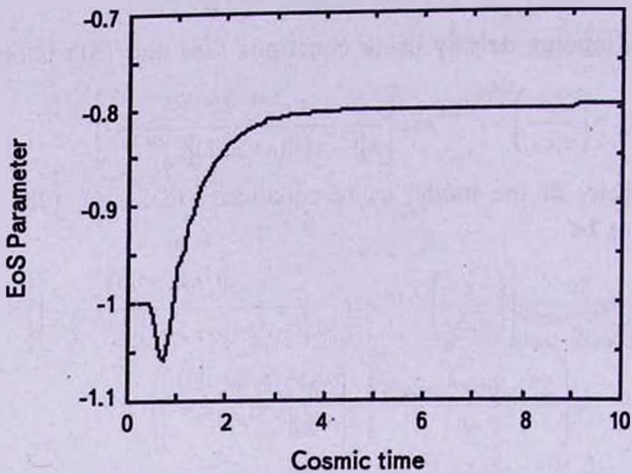

Fig.3. The plot the deviation free EoS parameter w versus cosmic time t in power law expansion. Here $b=n=k=c=c_1=c_2=\omega=\lambda=1$.

From the graph one can observe that initially the EoS parameter $w=-1$, is equivalent to the cosmological constant (Λ), after that for short period, the EoS parameter $w<-1$ (i.e. the universe was phantom fluid dominated) and at late time it is evolving $w>-1$ (i.e. at the present time). The earlier phantom fluid later on converted to the quintessence dominated phase of universe.

The SN Ia data [58] suggests that $-1.67<w<-0.62$ while the limit imposed on w by a combination of SN Ia data (with CMB anisotropy) and galaxy clustering statistics [59] is $-1.33<w<-0.79$. So, one can conclude that the limit of w provided by (52) may accommodated with the acceptable range of EoS parameter, if the present work is compared with experimental results mentioned above.

The skewness parameter by using equation (22) becomes

$$\gamma=\delta=\frac{1}{\rho}\left(\frac{2c}{c_1c_2}\right)t^{-4m}\exp\left[\frac{-\lambda t^{[1-4m((n+3)/3)]}}{b[1-4m((n+3)/3)]c_1^{(n+3)/3}}\right], \tag{56}$$

$$\eta=0. \tag{57}$$

Using equations (31), (51) and (52) the directional Hubble parameter of the model is found to be

$$H_x = H_y = H_z = \frac{m}{t} + \frac{\lambda t^{-4m((n+3)/3)}}{4bc_1^{(n+3)/3}}, \quad H_\phi = \frac{m}{t} - \frac{3\lambda t^{-4m((n+3)/3)}}{4bc_1^{(n+3)/3}}. \tag{58}$$

The mean Hubble parameter of this model is given by

$$H = \frac{m}{t}. \tag{59}$$

The anisotropy parameter leads to

$$\Delta = \frac{3t^2\lambda^2 t^{-8m((n+3)/3)}}{16m^2b^2c_1^{2((n+3)/3)}}. \tag{60}$$

In the absence of magnetic field i.e. $c \to 0$, the energy density for magnetic field of the model, the energy density of the model, deviation free EoS parameter and the skewness parameter are

$$\rho_B = 0, \tag{61}$$

$$\frac{8\pi\rho}{\phi} = \left\{\left[\frac{27 - 4\omega n^2 + 24n}{9}\right]2\frac{m^2}{t^2} - \left[\frac{3\lambda^2 t^{-8m((n+3)/3)}}{8b^2c_1^{2((n+3)/3)}}\right]\right\}, \tag{62}$$

$$\frac{8\pi w\rho}{\phi} = -\left\{\left[\frac{27 + 4(\omega+2)n^2 + 18n}{9}\right]2\frac{m^2}{t^2} - \left(\frac{4n+9}{3}\right)\frac{m}{t^2} + \left[\frac{3\lambda^2 t^{-8m((n+3)/3)}}{8b^2c_1^{2((n+3)/3)}}\right]\right\}, \tag{63}$$

$$\gamma = \delta = \eta = 0 \tag{64}$$

In General Relativity, the energy density and deviation free EoS parameter becomes

$$8\pi\rho = \left\{8\pi\left[\left(\frac{c}{c_1c_2}\right)t^{-4m}\exp\left[\frac{-\lambda t^{(1-4m)}}{(1-4m)c_1}\right]\right] + 6\frac{m^2}{t^2} - \left[\frac{3\lambda^2 t^{-8m}}{8c_1^2}\right]\right\}, \tag{65}$$

$$8\pi w = -\frac{1}{\rho}\left\{8\pi\left[\left(\frac{c}{c_1c_2}\right)t^{-4m}\exp\left[\frac{-\lambda t^{(1-4m)}}{(1-4m)c_1}\right]\right] + 6\frac{m^2}{t^2} - 3\frac{m}{t^2} + \left[\frac{3\lambda^2 t^{-8m}}{8c_1^2}\right]\right\}. \tag{66}$$

It is interesting to note that the present work has been successful in justifying the idea of Adhav et al. [35] that time-dependent equation of state parameter is essential for studying the complete time-evolution of the cosmos and in the absence of magnetic field and scalar field tends to 1, this results are resembles to that of investigated results of the Akarsu et al. [51] in general relativity.

6. *Conclusion.* We have investigated the Kaluza-Klein cosmological model in presence of anisotropic magnetized dark energy. In which, we consider the energy momentum-tensor consist of anisotropic fluid with anisotropic equation of state $p = w\rho$ and a uniform magnetic field of energy density ρ_B. Then we have made assumption on the anisotropy of the fluid in a way

to reduce the anisotropy parameter of the expansion to a simple form and obtained a hypothetical fluid with a special anisotropic EoS parameter. The exact solutions of the Brans-Dicke field equations have been obtained by assuming two different volumetric expansions.

In the exponential model, we observe that universe is dominated by phantom fluid at large time and in power law model, we observe that the energy density (ρ) at an early epoch is large and at late time, it decreases. Also we conclude that initially the EoS parameter is $w=-1$, which is equivalent to the cosmological constant (Λ), after that for short period $w<-1$ (i.e. the universe was phantom fluid dominated) and at late time, it is evolving $w>-1$ (i.e. at the present time). The earlier phantom fluid later on converted to the quintessence dominated phase of universe. The range of EoS parameter $-1.06<w<-0.79$ which is consistent with the observational data. Some important cosmological physical parameters are investigated for the solutions such as mean Hubble parameter, Anisotropy parameter in both models.

This study will throw some light on the structure formation of the universe, which has astrophysical significance. It is interesting to note that in absence of magnetic field and scalar field tends to 1, our results resembles to the investigated results of Akarsu et al. [51] in general relativity.

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КОСМОЛОГИЧЕСКАЯ МОДЕЛЬ КАЛУЗА-КЛАЙНА ДЛЯ АНИЗОТРОПНОЙ НАМАГНИЧЕННОЙ ТЕМНОЙ МАТЕРИИ В ТЕОРИИ ГРАВИТАЦИИ БРАНСА-ДИКЕ

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Рассматривается пространственно-однородная космологическая модель Калуза-Клайна для намагниченной анизотропной жидкости в скалярной тензорной теории гравитации, предложенной Бранс-Дике [1]. Точные решения модели получены в виде разложения по волнометрическим экспонентам и степенного разложения. Физическое поведение моделей обсуждается с помощью нескольких физических величин.

Ключевые слова: *модель Калуза-Клайна: электромагнитное поле: темная энергия: теория Бранс-Дике*

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