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EQUILIBRIUM STRUCTURE OF ROTATIONALLY AND TIDALLY DISTORTED PRASAD MODEL INCLUDING THE EFFECT OF MASS VARIATION INSIDE THE STAR

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In this paper we propose suitable modifications in the concept of Roche equipotentials to account for the effect of mass distribution inside the star on its equipotential surfaces and use this in conjunction with the approach of Kippenhahn and Thomas, in a manner earlier used by Prasad and Mohan to incorporate the effects of rotational and tidal forces in the equations of stellar structure parameters. The proposed method has been used to compute the structure parameters of rotationally and tidally distorted Prasad model of the star.

Key words: Roche equipotentials: equilibrium structure of stars: Prasad model: rotating stars: stars in binary systems

1. Introduction. Observations show that most of the binary stars are rotating as well as revolving around their common center of mass. Rotational forces as well as tidal distortion of the companion star have their effects on the equilibrium structure, shape and other observable physical parameters of binary stars. Prasad [1] introduced a model (called Prasad model thereafter) in which density ρ inside the star varies according to the law $\rho = \rho_c (1-x^2)$, ρ_c being the density of fluid at the centre and x a non-dimensional measure of the distance of a fluid element from its centre. Since then, several authors such as Gurm [2], Kippenhahn and Thomas [3], Prasad and Mohan [4], Agarwal [5], Sharma [6], Lal et. al. [7] etc. have addressed themselves to these types of problems.

The mathematical problem of determining the effects of rotation and tidal forces on the equilibrium structure of a star is quite complex. Approximate methods have therefore often been used in literature to study such problems. In some such approximations, Kopal [8,9], Mohan and Singh [10,11], Mohan and Saxena [12], Mohan et al. [13,14], the actual equipotential surfaces of a rotationally and tidally distorted star are approximated by equivalent Roche equipotentials, assuming both stars in the binary system to be point masses. This approximation is valid for highly centrally condensed types of stars. Lal et. al. [15] have checked the validity of series expansion being used for the

position of a point on a Roche equipotential for such approximations. However, in the case of stars in which the central condensation is not too large, this approximation is not very justified. Therefore, it will be of interest to analyze the effect of incorporating suitable modifications in the Roche equipotentials to account for the mass distribution inside the star. Baur [16] recently modified the Roche potentials to account for mechanical effects of the mutual irradiation of the binary components. In a somewhat similar manner we have suitably modified the Roche equipotentials to account for the mass distribution inside the primary component (e.g. Mohan et. al. [17]).

The paper is organized as follows: expressions for the modified Roche equipotentials of a rotationally and tidally distorted star are obtained in Section 2. This modified concept of Roche equipotentials is next used in Section 3, to obtain the equilibrium structure of a rotationally and tidally distorted Prasad model of a star. Computational results for the inner structure, shapes and certain other physical parameters of rotationally and tidally distorted Prasad model including mass variation inside the star is next obtained in Section 4 and compared with corresponding results earlier obtained by some other authors.

2. Modified Roche equipotentials. In order to investigate the equilibrium structures and stability of binary stars, the concept of Roche equipotentials has been frequently used in literature (Kopal [8]). However, while computing the Roche equipotential surfaces, the whole mass of the stars (the primary component as well as the secondary) is assumed to be concentrated at their centers. This approximation, though reasonably valid for highly centrally condensed stars, is not reasonably true for stars, such as main sequence and pre-main sequence stars which are not very highly centrally condensed. The concept of Roche equipotentials, therefore, needs to be suitably modified to incorporate the effects of mass distribution inside the star so that it can provide a better approximation for the structure of a rotationally and tidally distorted star which is not very highly centrally condensed.

In this section we suitably modify the mathematical expression for Roche equipotentials to reasonably account for the mass distribution inside the primary star (in whose inner structure we are primarily interested), assuming, as earlier, that the secondary star is still a point mass.

Let M_0 and M_1 be the masses of the primary and the secondary components of a binary system of stars in which the primary is assumed to be much massive than the secondary $(M_0 >> M_1)$. Let $M_0(r)$ represent the mass interior to a sphere of radius r inside the primary component. Let D be the mutual separation between the centers of the two stars. Further suppose that the position of the two components of this binary system is referred to a rectangular system of Cartesian coordinates having the origin at the center of gravity of the primary of mass M_0 , the x - axis along the line joining the centers of the two components of the binary and z axis perpendicular to the plane of the orbit of the two components (Fig.1).



Fig.1. Axes of reference of rotating binary system.

Total potential ψ due to the gravitational, rotational and other disturbing forces acting at an arbitrary point P(x, y, z) distant r from the centre of the primary and r, from the centre of the secondary star may be expressed as

$$\psi = G \frac{M^*}{r} + G \frac{M_1}{r_1} + \frac{1}{2} \Omega^2 \left[\left(x - \frac{M_1 D}{M_0 + M_1} \right)^2 + y^2 \right], \quad (1)$$

where $M^* = \begin{cases} M_0 & \text{if } r > R \\ M_0(r) & \text{if } r < R \end{cases}$, Ω is the angular velocity of rotation and G the gravitational constant. The three terms on the right hand side of (1) are, respectively, the potential arising from the mass of the primary star, the disturbing potential of its companion of mass M_1 and the potential arising from the centrifugal force. In the first term on the right hand side of (1), M_0 normally used in the definition of Roche potential, has been replaced by M^* to account for the mass distribution inside the primary star. This has been done keeping in view the fact that in the case of a sphere, the gravitational potential at a point outside the sphere is GM(r)/r, M(r) being the mass contained inside a sphere of radius r concentric with the centre of stars of mass M.

Equation (1) in non-dimensional form can be expressed as

$$\psi^{*} = \frac{z}{r^{*}} + q \left[\frac{1}{\sqrt{1 - 2\lambda r^{*} + r^{*2}}} - \lambda r^{*} \right] + nr^{*2} (1 - \upsilon^{2}), \qquad (2)$$

where

$$\psi^{\bullet} = \frac{D\psi}{GM_0} - \frac{M_1^2}{2M_0(M_0 + M_1)}$$

and

	$\frac{M_0(r)}{M_0},$	if	r < R		
2-	1,	if	r > R.		

Also $r^* = r/D$ is a non-dimensional measure of the distance and $\lambda = \sin\theta \cos\phi$, $\mu = \sin\theta \sin\phi$, $\upsilon = \cos\theta$, r, θ, ϕ being the spherical polar coordinates of the point *P*. Obviously, *z* is a non-dimensional parameter, which becomes zero at the center of the primary component of mass M_0 and one at the points on and outside the surface of the primary component. It will have a value between 0 and 1 at points inside the primary star. Also $q = M_1/M_0$ is a nondimensional parameter representing the ratio of the mass of the secondary star over the mass of the primary component (q < 1), while 2n represents the square of the normalized angular velocity Ω . If q = 0, equation (2) reduces to the potential of a rotating spherical model rotating with angular velocity Ω and if n=0, it reduces to the potential of a spherical model distorted only by the tidal effects of the companion. For a binary system in synchronous rotation, $\Omega^2 = G(M_0 + M_1)/D^3$. In terms of the non-dimensional variables as defined above, becomes n = (q+1)/2.

Equipotential surfaces represented by $\psi^* = \text{const}$ as given in (2) are the modified Roche equipotential surfaces of the primary component of a rotationally and tidally distorted binary system which reasonably account for the mass distribution in the interior of the primary star. This, however, modifies potential at the points inside the primary star only. On substituting z = 1 in (2) or $M_0(r) = M_0$ in (1), it reduces to the expression for the Roche equipotential which has been earlier used by Kopal [8] and other authors.

Following Kopal [8], it can be shown that the values of r, θ, ϕ on the surfaces of the Roche equipotentials as given by (2) are connected through the relation

$$r^{*} = r_{0} \left[1 + a_{0}r_{0}^{3} + \frac{qP_{3}}{z}r_{0}^{4} + \frac{qP_{4}}{z}r_{0}^{5} + \left\{ \frac{qP_{5}}{z} + \frac{3a_{0}^{2}}{z^{2}} \right\}r_{0}^{6} + \left\{ \frac{qP_{6}}{z} + \frac{7qa_{0}P_{3}}{z} \right\}r_{0}^{7} + \left\{ \frac{qP_{7}}{z} + \frac{8qa_{0}P_{4}}{z} + \frac{4q^{2}P_{3}^{2}}{z^{2}} \right\}r_{0}^{8} + \left\{ \frac{qP_{8}}{z} + \frac{9qa_{0}P_{5}}{z} + \frac{9q^{2}P_{3}P_{4}}{z^{2}} \right\}r_{0}^{9} + \left\{ \frac{qP_{9}}{z} + \frac{10qa_{0}P_{6}}{z} + \frac{5q^{2}}{z^{2}} \left(P_{4}^{2} + 2P_{3}P_{5}\right)\right]r_{0}^{10} + \cdots \right],$$
(3)

where $r_0 = \frac{z}{\psi - q}$, $a_0 = \frac{q P_2}{z} + \frac{n(1 - v^2)}{z}$, $P_j = P_j(\lambda)$ are Legendre polynomials and

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terms up to second order of smallness in n and q are retained. This relation can be used to obtain the shapes of Roche equipotential $\psi = \text{const}$. Whereas on account of inclusion of mass distribution inside the primary star, these will get modified at points inside the primary star due to the presence of z, outside the primary star (where z=1), these will be same as earlier obtained by Kopal [8]. Following Kopal [8] and Mohan et. al. [17], modified expressions for the volume enclosed V_{ψ} , and the surface area S_{ψ} of an equipotential surface can be obtained if desired in series form.

3. Equilibrium structures of rotationally and tidally distorted Prasad model. If we assume that the primary component of binary system behaves as Prasad model and rotating about its axis then its equilibrium structure will be distorted by rotation as well as the tidal effects of the companion. In order to determine the equilibrium structure of this rotationally and tidally distorted stellar model, we may follow the approach of Mohan and Saxena [12], it is assumed that the rotational velocity and the mass of the secondary star as compared to the primary are suitably small.

Let r_{ψ} denote the radius of the topologically equivalent spherical model which corresponds to an equipotential surface $\psi = \text{const}$ of this rotationally and tidally distorted Prasad model. Also, let R_{ψ} be the value of r_{ψ} on the equipotential surface $\psi = \text{const}$ of this rotationally and tidally distorted model. Following the approach as discussed by Saini [18], r_{ψ} and R_{ψ} are given as:

$$r_{\psi} = Dr_0 \left[1 + \frac{4 n r_0^3}{3z} + \left(\frac{4 q^2 r_0^6}{4z^2} + \frac{8 n q}{15z^2} + \frac{76 n^2}{45z^2} \right) r_0^6 + \frac{5 q^2 r_0^8}{7z^2} + \frac{2 q^2 r_0^{10}}{3z^2} + \dots \right], \quad (4)$$

$$R_{\psi} = Dr_{0s} \left[1 + \frac{4nr_{0s}^{3}}{3z} + \left(\frac{4q^{2}r_{0}^{6}}{4z^{2}} + \frac{8nq}{15z^{2}} + \frac{76n^{2}}{45z^{2}} \right) r_{0s}^{6} + \frac{5q^{2}r_{0s}^{8}}{7z^{2}} + \frac{2q^{2}r_{0s}^{10}}{3z^{2}} + \dots \right],$$
(5)

and

$$z = \frac{5}{2} \left(\frac{r_{\psi}}{R_{\psi}} \right)^3 - \frac{3}{2} \left(\frac{r_{\psi}}{R_{\psi}} \right)^5,$$

where

$$r_0 = \frac{z}{\psi - q}.$$
 (6)

Further let ρ_{ψ} denote the value of density on an equipotential $\psi = \text{const}$. The density distribution law of rotationally and tidally distorted Prasad model is given as

$$\rho_{\Psi} = \rho_c \left(1 - \frac{r_{\Psi}^2}{R_{\Psi}^2} \right). \tag{7}$$

On substituting the value of r_{ψ} and R_{ψ} from equation (4) and (5) in equation

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(7) we get

$$\rho_{\psi} = \rho_{c} \left[1 - \frac{D^{2} r_{0}^{2}}{R_{\psi}^{2}} \left\{ 1 + \frac{4nr_{0}^{3}}{3z} + \left(\frac{8q^{2} r_{0}^{6}}{5z^{2}} + \frac{16nq}{15z^{2}} + \frac{172r^{2}}{45z^{2}} \right) r_{0}^{6} + \frac{10q^{2} r_{0}^{8}}{7z^{2}} + \frac{4q^{2} r_{0}^{10}}{3z^{2}} + \dots \right] \right] . (8)$$

On substituting value of ρ_{ψ} from (8) and using the approach used by Mohan and Saxena [12], integrating with respect to r_0 and using the fact that $M_{\psi} = 0$ at centre $r_0 = 0$, we get

$$M_{\psi} = \frac{4\pi\rho_{c}D^{3}r_{0}^{3}}{3} \left[1 - \frac{3D^{2}}{5R_{\psi}^{2}}r_{0}^{2} + \frac{2nr_{0}^{3}}{z} - \frac{2nR^{2}r_{0}^{5}}{zR_{\psi}^{2}} + \left(\frac{12q^{2}r_{0}^{6}}{5z^{2}} + \frac{8nq}{15z^{2}} + \frac{32n^{2}}{5z^{2}}\right)r_{0}^{6} + \left(\frac{15q^{2}}{7z^{2}} - \frac{12q^{2}D^{2}}{5z^{2}R_{\psi}^{2}} - \frac{8nqD^{2}}{5z^{2}R_{\psi}^{2}} - \frac{116n^{2}}{15z^{2}}\right)r_{0}^{8} + \left(\frac{2q^{2}}{5z^{2}} - \frac{15q^{2}R^{2}}{7z^{2}R_{\psi}^{2}}\right)r_{0}^{10} + \dots\right].$$
(9)

Similarly, on substituting ρ_{ψ} from (8) and M_{ψ} from (9) and using the approach used by Mohan and Saxena [12] and integrating with respect to r_0 , we get

$$P_{\psi} = \frac{2\pi G \rho_{\psi}^2 D^2}{3} \left[K - r_0^2 + \frac{4 D^2 r_0^4}{5 R_{\psi}^2} - \frac{4 n r_0^5}{5 z} - \frac{D^4 r_0^2}{5 R_{\psi}^2} + \frac{32 n D^2 r_0^7}{21 z R_{\psi}^2} - \left(\frac{q^2 r_0^8}{2 z^2} + \frac{n q}{3 z^2} + \frac{4 n^2}{3 z^2} \right) r_0^8 - \frac{28 n D^4 r_0^9}{45 z R_{\psi}^4} + \left(10 \right) + \left(-\frac{3 q^2}{10 z^2} + \frac{144 D^2 q^2}{125 z^2 R_{\psi}^2} + \frac{96 D^2 n q}{125 z^2 R_{\psi}^2} + \frac{4225 n^2}{1125 z^2} \right) r_0^{10} + \dots \right],$$

where K is a constant of integration whose value may be calculated by using boundary condition say $P_{\psi} = 0$ at $r_0 = r_{0s}$. This yields

$$K = r_{0s}^{2} - \frac{4D^{2}r_{0s}^{4}}{5R_{\psi}^{2}} + \frac{4nr_{0s}^{5}}{5z} + \frac{D^{4}r_{0s}^{6}}{5R_{\psi}^{2}} - \frac{32nD^{2}r_{0s}^{7}}{21zR_{\psi}^{2}} + \left(\frac{q^{2}r_{0s}^{8}}{2z^{2}} + \frac{nq}{3z^{2}} + \frac{4n^{2}}{3z^{2}}\right)r_{0s}^{8} + \frac{28nD^{4}r_{0s}^{9}}{45zR_{\psi}^{4}} - \left(\frac{3q^{2}}{10z^{2}} + \frac{144D^{2}q^{2}}{125z^{2}R_{\psi}^{2}} + \frac{96D^{2}nq}{125z^{2}R_{\psi}^{2}} + \frac{4225n^{2}}{1125z^{2}}\right)r_{0s}^{10} + \dots$$
(11)

Similarly the volume V_{ψ} , surface area S_{ψ} , g^{-} and g^{-1} of rotationally and tidally distorted Prasad model are obtained as

$$V_{\psi} = \frac{4\pi Dr_0^3}{3} \left[1 + \frac{2nr_0^3}{z} + \left(\frac{12q^2}{5z^2} + \frac{8nq}{5z} + \frac{32n^2}{5z^2} \right) r_0^6 + \frac{15q^2r_0^8}{7z^2} + \frac{2q^2r_0^{10}}{z^2} + \dots \right], \quad (12)$$

$$S_{\psi} = 4\pi r_0^2 D^2 \left[1 + \frac{4\pi r_0^3}{3z} + \left(\frac{7q^2}{5z^2} + \frac{14\pi q}{15z^2} + \frac{56\pi^2}{15z^2} \right) r_0^6 + \frac{9q^2 r_0^8}{7z^2} + \frac{11q^2 r_0^{10}}{9z^2} + \dots \right], \quad (13)$$

$$\overline{g} = \frac{zGM_{\psi}}{r_0^2 D^2} \left[1 - \frac{8nr_0^3}{3z} - \left(\frac{3q^2}{z^2} + \frac{2nq}{z^2} + \frac{40n^2}{9z^2} \right) r_0^6 - \frac{51q^2r_0^8}{14z^2} - \frac{13q^2r_0^{10}}{3z^2} + \dots \right], \quad (14)$$

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$$\overline{g^{-1}} = \frac{r_0^2 D^2}{zGM_{\psi}} \left[1 + \frac{8 n r_0^3}{3 z} + \left(\frac{31q^2}{5 z^2} + \frac{62 n q}{15 z^2} + \frac{584 n^2}{45 z^2} \right) r_0^6 + \frac{101q^2 r_0^8}{14 z^2} + \frac{75 q^2 r_0^{10}}{9 z^2} + \dots \right]. (15)$$

4. Numerical evaluation of structure for Prasad model and analysis of results. For a better appreciation of the effects of rotation and tidal distortions on the values of density, mass and pressure at various points inside the star, we have used equations (8), (9) and (10) to numerically compute the values of ρ_{Ψ} , M_{Ψ} and P_{Ψ} at various points inside Prasad model.

The results presented in Tables 1, 2, 3 and 4 (see also Fig.2 and Fig.3) give the values of certain structures parameters and related observable quantities

Table 1

STRUCTURE PARAMETERS OF UNDISTORTED STARS FOR PRASAD MODEL (ψ =5.0, n=0.0, q=0.0)

x	V.	S _w	ρ _ψ	М.,	Pw	σ	3	T_{e}/T_{e}	L,/L,
0.1	0.00001	0.00041	0.99000	0.00248	0.00004	0.00000	0.00000	0.14142	1.00000
0.2	0.00006	0.00161	0.96000	0.01951	0.00028	0.00000	0.00000	0.20005	1.00000
0.3	0.00022	0.00362	0.91000	0.06384	0.00081	0.00000	0.00000	0.24497	1.00000
0.4	0.00052	0.00642	0.84000	0.14464	0.00150	0.00000	0.00000	0.28285	1.00000
0.5	0.00102	0.01040	0.75000	0.26562	0.00209	0.00000	0.00000	0.31626	1.00000
0.6	0.00171	0.01443	0.64000	0.42336	0.00228	0.00000	0.00000	0.34648	1.00000
0.7	0.00272	0.01967	0.51000	0.60539	0.00190	0.00000	0.00000	0.37417	1.00000
0.8	0.00402	0.02565	0.36000	0.78848	0.00111	0.00000	0.00000	0.40009	1.00000
0.9	0.00585	0.03246	0.19000	0.93676	0.00032	0.00000	0.00000	0.42425	1.00000
1.0	0.00800	0.04000	0.00000	1.00006	0.00000	0.00000	0.00000	0.44728	1.00000



Fig.2. Behavior of some structure parameters of undistorted Prasad Models of stars ($\psi = 5.0$, n = 0.0, q = 0.0).

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Table 2

STRUCTURE PARAMETERS OF ROTATIONALLY DISTORTED STARS FOR PRASAD MODEL (ψ =5.0, n = 0.1, q = 0.0)

r	V	S	ρ	Mw	P _w	σ	E	T_e/T_p	L_e/L_p
x 0.1 0.2 0.3 0.4	0.00001 0.00006 0.00022 0.00051	5 _♥ 0.00040 0.00160 0.00360 0.00640 0.01040	0.99000 0.96002 0.91000 0.84009 0.75014	0.00242 0.01950 0.06379 0.14452 0.26543	0.00024 0.01404 0.01259 0.01037 0.00787	0.00032 0.00032 0.00033 0.00035 0.00037	0.00032 0.00032 0.00033 0.00035 0.00037	0.14139 0.19995 0.24490 0.28279 0.31616	0.99872 0.99865 0.99862 0.99855 0.99848
0.5 0.6 0.7 0.8 0.9 1.0	0.00173 0.00276 0.00403 0.00582 0.00800	0.01440 0.01960 0.02561 0.03242 0.04000	0.64018 0.5102 0.36002 0.19019 0.00000	0.42308 0.60509 0.78822 0.93664 1.0000	0.00537 0.00314 0.00141 0.00032 0.00000	0.00040 0.00045 0.00052 0.00062 0.0008	0.00040 0.00045 0.00051 0.00062 0.00079	0.34632 0.37408 0.39989 0.42413 0.44700	0.99830 0.99812 0.99796 0.99757 0.99607

Table 3

STRUCTURE PARAMETERS OF TIDALLY DISTORTED STARS FOR PRASAD MODEL (ψ =5.0, n=0.0, q=0.1)

x	V.,	S,	ρ _w	M _w	P _w	σ	3	T_e/T_p	L_{e}/L_{p}
01	0.00001	0.00042	0.95000	0.00248	0.00003	0.00034	0.00034	0.14283	0.99823
0.1	0.00006	0.00166	0.96000	0.01952	0.00023	0.00036	0.00036	0.20195	0.99815
0.2	0.00022	0.00375	0.91000	0.06385	0.00084	0.00038	0.00038	0.24736	0.99806
0.5	0.00054	0.00664	0.84000	0.14464	0.00156	0.00040	0.00040	0.28566	0.99797
0.5	0.00105	0.01041	0.75000	0.26562	0.00218	0.00044	0.00044	0.31934	0.99772
0.6	0.00182	0.01497	0.64000	0.42336	0.00237	0.00048	0.00048	0.34988	0.99744
0.7	0.00291	0.02040	0.51000	0.60539	0.00198	0.00055	0.00055	0.37785	0.99710
0.8	0.00435	0.02665	0.36000	0.78848	0.00116	0.00064	0.00064	0.40392	0.99668
0.9	0.00618	0.03372	0.19000	0.93676	0.00033	0.00079	0.00078	0.42830	0.99586
1.0	0.00850	0.04165	0.00000	1.00000	0.00000	0.00103	0.0010	0.45141	0.99446

Table 4

STRUCTURE PARAMETERS OF ROTATIONALLY AND DISTORTED STARS FOR PRASAD MODEL (ψ =5.0, n=0.1, q=0.1)

x	V.,	S,	ρΨ	M,	₽ _₩	σ	6	T_{e}/T_{p}	L,/L,
0.1	0.00001	0.00041	0.99000	0.00248	0.00005	0.00069	0.00069	0.14282	0.99687
0.2	0.00006	0.00166	0.96000	0.01950	0.00028	0.00071	0.00071	0.20193	0.99677
0.3	0.00023	0.00375	0.91000	0.06379	0.00083	0.00074	0.00074	0.24736	0.99662
0.4	0.00054	0.00666	0.84010	0.14451	0.00156	0.00078	0.00078	0.28556	0.99642
0.5	0.00106	0.01041	0.75015	0.26541	0.00218	0.00084	0.00084	0.31928	0.99613
0.6	0.00183	0.01499	0.64020	0.42307	0.00237	0.00092	0.00092	0.34972	0.99574
0.7	0.00292	0.02041	0.51024	0.60507	0.00198	0.00103	0.00103	0.37770	0.99520
0.8	0.00436	0.02667	0.36025	0.78820	0.00116	0.00120	0.00120	0.40387	0.99441
0.9	0.00620	0.03376	0.19020	0.93663	0.00032	0.00145	0.00145	0.42824	0.99319
1.0	0.00851	0.04166	0.00000	1.00000	0.00000	0.00188	0.00188	0.45121	0.99110

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of undistorted, rotationally distorted, tidally distorted and rotationally and tidally distorted Prasad model for $\psi = 5.0$. Results show that there are no major changes on the internal structure and the behavior of stars due to the effect of tidal distortion, differential rotation and the account for mass variation inside



Fig.3. Behavior of some structure parameters of rotationally and tidally distorted Prasad Models of stars ($\psi = 5.0$, n = 0.1, q = 0.1).

the star on its equipotential surfaces. However, it has been observed that due to the effect of tidal distortion and differential rotation, the volume of the core at the middle of the Prasad model increases slightly.

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РАВНОВЕСНАЯ СТРУКТУРА ВРАЩАТЕЛЬНО- И ПРИЛИВОВОЗМУЩЕННОЙ МОДЕЛИ ПРАСАДА, ВКЛЮЧАЮЩЕЙ ЭФФЕКТ ИЗМЕНЕНИЯ МАССЫ ВНУТРИ ЗВЕЗДЫ

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В данной работе предлагается соответствующая модификация понятия эквипотенциалов Роша, учитывающая влияние распределения массы внутри звезды на ее эквипотенциальные поверхности. Она используется в сочетании с подходом Kippenhahn-а и Thomas-а так же, как это ранее было использовано со стороны Mohan и другими для включения влияния вращательных и приливных сил в уравнения структурных параметров звезд. Предложенный метод испоьзуется для вычисления структурных параметров вращательно- и приливовозмущенной модели звезд Прасада.

Ключевые слова: эквипотенциалы Роша: равновесная структура звезд: модель Прасада: вращающиеся звезды: звезды в двойных системах

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