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EXTENDED TIME-DEPENDENT GINZBURG-LANDAU EQUATIONS FOR ROTATING TWO-FLAVOR COLOR SUPERCONDUCTORS

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We discuss an extension of the time-dependent Ginzburg-Landau equations for rotating two-flavor color superconducting quark matter derived earlier. The extension treats the coefficient of the time-dependent term in the Ginzburg-Landau equation as complex number, whose imaginary part describes non-dissipative effects. We derive time-dependent London type equation for the color-electric potential which obtains an additional time-dependent contribution from this imaginary part. This additional term describes non-dissipative propagation effects. In addition we derive general expressions for the energy flux and the dissipative function of the system.

Key words: *neutron stars: Ginzburg-Landau theory: quark matter: superdconductivity*

1. *Introduction*. The Ginzburg-Landau (hereafter GL) theory of superconductivity has been very successful in describing semi-phenomenologically the properties of a variety of condensed matter systems, such as the electronic superconductors, liquid ^{3}He phases, etc. It has been applied to color superconducting matter both in the early stages of development [1] and more recently in Refs. [2-6]. The latter references discuss in detail the equilibrium GL theories for key, robust color superconducting phases, such as the two-flavor color superconducting phase (2SC) and the three flavor color-flavor-locked (CFL) phase. The 2SC phase features a condensate of quarks paired in color and flavor anti-triplet state, i.e., the red up quarks pair with green down quarks, etc., whereas the blue quarks remain unpaired.

In our previous work [7] we have developed a non-equilibrium Ginzburg-Landau approach to the rotating 2SC color superconducting matter. We have introduced a time-dependent term in the GL equations, which takes into account the relaxation of the order parameter of the condensate. This term has a universal form, but it contains a real phenomenological coefficient, which encodes the microphysics of transport and dissipation in the condensate under consideration. We have estimated the value of this term in the case of the 2SC condensate, using the analogy between this condensate and the Bardeen-Cooper-Schrieffer (BCS) condensate of the superconducting electrons. This analogy is natural because pairing in both cases occurs in a state of zero total angular momentum and even parity. In the 2SC phase the blue quarks are assumed to be unpaired, so that they can scatter Cooper pairs inelastically.

In the case where the condensate is rotating we have to incorporate in the GL equations the homogeneous magnetic field (London moment) [8], which is due to Coriolis force and the inhomogeneous electric field [9], which balances the centrifugal force. The time-dependent treatment of the problem requires explicit consideration of the electric fields in a superconductor, which need to be included into time-dependent GL formalism.

In this work we will include additional contribution of the centrifugal force in the time-dependent Ginzburg-Landau (TDGL) equation of the 2SC condensate. In rotating superconductors the centrifugal force induces the scalar potential of electric field and renormalizes the chemical potential, which in turn changes the critical temperature and the coefficient α in the GL equation. In order to preserve gauge invariance the correction of α must be added in the time-dependent term. Therefore the phenomenological coefficient of the time-dependent term acquires an imaginary part. It is this modification that we address in this work, where we modify the time-dependent Ginzburg-Landau (TDGL) equation by considering the constant in front of the time-dependent term to be complex number. The imaginary part of this coefficient is due to non-dissipative forces, therefore in modified TDGL equation appears an propagating term, whereas the dissipative function remains unaffected.

This paper is structured as follows. In Sec. 2 we review the time-dependent GL equations for non-relativistic rotating electronic superconductors. In Sec. 3 we derive explicit equations for the energy flow and the dissipative function using time-dependent GL equations. The equations of the previous sections are generalized to the case of the 2SC color superconductor in Sec. 4. Our conclusions are summarized in Sec. 5.

2. ***Time-dependent GL equations for a non-relativistic electronic superconductor.*** Before discussing the case of 2SC superconductors, we would like to start with a discussion of the modifications needed for a more familiar system - the electronic superconductor. The case of the rotating superconductor and the corresponding TDGL equations were derived and discussed in Ref. [7]. The requirement of gauge invariance of TDGL equations for a non-rotating electronic superconductor leads to substitution of $\hbar\partial/\partial t$ by a term of the form [10] $\hbar\partial/\partial t+2ie\tilde{\phi}(\mathbf{r},t)$, where $\tilde{\phi}=\phi+\mu/e$. Here ϕ is scalar potential of electric field, μ is the local chemical potential of the superconductor, e is the electron charge, $e<0$. $\tilde{\phi}$ may be identified as the electrochemical potential divided by the electronic charge e. The effects of magnetic field are included by replacing $\hbar\nabla$ by $\hbar\nabla-(2ie/c)\mathbf{A}(\mathbf{r},t)$, where $\mathbf{A}(\mathbf{r},t)$ is vector potential of magnetic field. The time dependence of the order parameter $\Delta(\mathbf{r},t)$ in equilibrium is given by

$$\Delta(\mathbf{r},t)=\Delta(\mathbf{r})\exp(-2i\mu t/\hbar). \quad (1)$$

A gauge transformation consists in the simultaneous substitutions

$$\Delta \to \Delta\exp[(2ie/\hbar c)\varphi], \quad \mathbf{A}\to\mathbf{A}+\nabla\varphi, \quad \tilde{\phi}\to\tilde{\phi}-\frac{1}{c}\frac{\partial\varphi}{\partial t}, \quad (2)$$

where φ is an arbitrary scalar function. Therefore, the gauge-invariant TDGL equation for the order parameter of a non-relativistic and non-rotating electronic dirty superconductor have the following form [10]

$$-\Gamma\left(\frac{\partial}{\partial t}+\frac{2ie}{\hbar}\tilde{\phi}\right)\Delta=\alpha\Delta+\beta|\Delta|^2\Delta-\gamma_d\left(\hbar\nabla-\frac{2ie}{c}\mathbf{A}\right)^2\Delta, \quad (3)$$

where the coefficients α, β and γ_d of time independent Gl theory are [11,12]

$$\alpha=\frac{T-T_c}{T_c}\nu, \quad \beta=\frac{7\zeta(3)}{8(\pi k_B T_c)^2}\nu, \quad \gamma_d=\frac{\pi\nu D}{8\hbar k_B T_c}. \quad (4)$$

Here $D=\upsilon_{Fe}^2\tau/3$ is the diffusion coefficient, υ_{Fe} is the electron Fermi velocity, τ is the electronic mean free time due to scattering by impurities, $\nu=m_e p_{Fe}/2\pi^2\hbar^3$ is the density of states at the Fermi surface, m_e is the electron mass, p_{Fe} is the Fermi momentum, T is the temperature and T_c is the critical temperature, k_B is Boltzmann constant, $\zeta(3)$ is Riemann zeta function. The relaxation parameter Γ in systems with the strong inelastic scattering has been derived from the microscopic theory in Ref. [10] as

$$\Gamma=\frac{\pi\hbar\nu}{8k_B T_c}. \quad (5)$$

The total current $\mathbf{j}$, is the sum of the supercurrent defined as

$$\mathbf{j}_s=2ie\hbar\gamma_d(\Delta\nabla\Delta^*-\Delta^*\nabla\Delta)-\frac{8\gamma_d e^2}{c}|\Delta|^2\mathbf{A}, \quad (6)$$

and the normal current $\mathbf{j}_n=\sigma_n\tilde{\mathbf{E}}$, where $\sigma_n=2De^2\nu$ is the electrical conductivity of normal electrons in the dirty limit. The normal current is induced by the effective electric field $\tilde{\mathbf{E}}$ [10]

$$\tilde{\mathbf{E}}(\mathbf{r},t)=-\frac{1}{c}\frac{\partial\mathbf{A}(\mathbf{r},t)}{\partial t}-\nabla\tilde{\phi}(\mathbf{r},t). \quad (7)$$

Consider now a rotating superconductor. In the rotating reference frame chemical potential μ_r is defined as [13]

$$\mu_r=\mu-\frac{m_e\Omega^2 r^2}{2}. \quad (8)$$

The Coriolis force does not influence the thermodinamic properties of the rotating superconductors. Therefore in the rotating reference frame the electrochemical potential is renormalized as follows [7]

$$\tilde{\phi}\to\tilde{\phi}_r=\tilde{\phi}-\frac{m_e\Omega^2 r^2}{2e}, \quad (9)$$

and vector potential is renormalized in the following way [14,15]

$$\mathbf{A} \rightarrow \mathbf{A}_r = \mathbf{A} + \frac{m_e c}{e}[\boldsymbol{\Omega} \times \mathbf{r}]. \tag{10}$$

Therefore the TDGL equations for a uniformly rotating dirty superconductor will become [7]

$$-\Gamma\left[\frac{\partial}{\partial t} + \frac{2ie}{\hbar}\left(\tilde{\phi} - \frac{m_e \Omega^2 r^2}{2e}\right)\right]\Delta = \alpha\Delta + \beta|\Delta|^2\Delta - \gamma_d\left\{\hbar\nabla - \frac{2ie}{\hbar}\left[\mathbf{A} + \frac{m_e c}{e}[\boldsymbol{\Omega}\times\mathbf{r}]\right]\right\}^2\Delta \tag{11}$$

$$\mathbf{j}_d = 2ie\hbar\gamma_d\left(\Delta\nabla\Delta^* - \Delta^*\nabla\Delta\right) - \frac{8\gamma_d e^2}{c}|\Delta|^2\left[\mathbf{A} + \frac{m_e c}{e}[\boldsymbol{\Omega}\times\mathbf{r}]\right] - \\ -\sigma_n\left(\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \nabla\tilde{\phi} - \frac{m_e \Omega^2}{e}\mathbf{r}\right). \tag{12}$$

The explicit treatment of the electric field in the moving superconductors requires a modification of TDGL equations, namely, the relaxation parameter Γ is modified from purely real to a complex number [16,17]

$$\Gamma = \Gamma' + i\Gamma''. \tag{13}$$

The origin of an imaginary part of Γ is as follows [18]. It is known that a moving vortex induces a scalar potential proportional to vortex velocity. The induced potential adds to chemical potential of the superconductor:

$$\mu = \mu_0 - e\phi. \tag{14}$$

On the other hand, the critical temperature depends on the chemical potential, therefore the coefficient α becomes

$$\alpha = \alpha_0 - \frac{\partial \alpha}{\partial \mu}e\phi = -\nu\left(1 - \frac{T}{T_c}\right) + \frac{\nu}{T_c}\frac{\partial T_c}{\partial \mu}e\phi. \tag{15}$$

The critical temperature in BCS theory is $T_c = \Omega_{BCS}\exp(-1/\lambda)$, where Ω_{BCS} is the cut-off energy of paiting interaction, $\lambda = |g|\nu$, g is coupling constant. Therefore correction to α becomes

$$\delta\alpha = \frac{1}{\lambda}\frac{\partial \nu}{\partial \mu}e\phi. \tag{16}$$

In order to preserve a gauge invariance the correction should be written in the form

$$\delta\alpha\Delta = -i\frac{\hbar}{2\lambda}\frac{\partial \nu}{\partial \mu}\left(\frac{\partial \Delta}{\partial t} + \frac{2ie}{\hbar}\phi\Delta\right), \tag{17}$$

which implies that the coefficient Γ (13) has an imaginary part

$$\Gamma'' = -\frac{\hbar}{2\lambda}\frac{\partial \nu}{\partial \mu}. \tag{18}$$

In the rotating frame of reference the chemical potential of superconductor (8) may be written in the form (14). Therefore the TDGL equation of the rotating superconductor will also be modified.

The authors of Ref. [17] have derived modified TDGL equations within the BCS model of superconductivity for the case of weak pair breaking $\tau k_B T_c >> \hbar$, where τ is the characteristic pair-breaking time. This time can be identified with the inelastic electron-phonon collision time or the spin-flip time. For their equations to be valid the condition $\tau\Delta << \hbar$ must also be fulfilled. This corresponds to the case of dirty superconductors or the superconductors with small concentration of paramagnetic impurities. The modified TDGL equation has the form of equation (3), where complex parameter Γ is represented by the expression (13). This equation has been used to explain the sign change in the Hall effect in both conventional and high temperature superconductors. The coefficient Γ' is given by formula (5), while the imaginary part Γ'' is defined as [17]

$$\Gamma'' = -\frac{\hbar}{2}\frac{(1+\lambda)}{\lambda}\frac{\partial\nu}{\partial\xi_p}, \tag{19}$$

where $\xi_p = \varepsilon_p - \varepsilon_F$, ε_p is the quasiparticle spectrum and ε_F is the Fermi energy. For the case of parabolic spectrum the imaginary part becomes

$$\Gamma'' = -\frac{m^2}{4\pi^2\hbar^2\, p_F}\left(\frac{\pi\hbar}{2 p_F |a|} + 1\right), \tag{20}$$

where $a = m|g|/4\pi\hbar^2$ is the s-wave scattering length. In the BCS model the first term in brackets is much larger than unity. The modified TDGL equation was derived in Ref. [19], where imaginary part contains only the second term of (20). The same work also discusses the crossover from BCS superconductivity to Bose-Einstein condensation. In closing we note that the ratio of the imaginary to the real part of Γ

$$\zeta = -\frac{\Gamma''}{\Gamma'} = \frac{2 k_B T_c}{\pi\varepsilon_F}\left(\frac{\pi\hbar}{2 p_F |a|} + 1\right) \tag{21}$$

is a small quantity for the model discussed above.

3. *The energy balance*. In this section we consider the balance of the free energy of a superconductor in an electromagnetic field. The free energy F of the superconductor consists of the free energy F_{em} of the electromagnetic field and the free energy difference F_{sn} between superconducting and normal state. The free energy F_{em} of the electromagnetic field is given by

$$F_{em} = \frac{1}{8\pi}\int \left(H^2 + E^2\right) dV. \tag{22}$$

The free energy difference F_{sn} is given by the expression of Ginzburg and Landau [20]

$$F_{sn}=\int\left[\alpha|\Delta|^2+\frac{\beta}{2}|\Delta|^4+\gamma_d\left|\left(-i\hbar\nabla-\frac{2e}{c}\mathbf{A}\right)\Delta\right|^2\right]dV, \tag{23}$$

where coefficients are defined by (4). The modified TDGL equation for the order parameter has the same form as the original TDGL equations, however the coefficient Γ is now a complex function

$$-\Gamma\left(\frac{\partial\Delta}{\partial t}+\frac{2ie}{\hbar}\tilde{\phi}\Delta\right)=\frac{\delta F_{sn}}{\delta\Delta^*}, \tag{24}$$

whereas its conjugate equation reads

$$-\Gamma^*\left(\frac{\partial\Delta^*}{\partial t}-\frac{2ie}{\hbar}\tilde{\phi}\Delta^*\right)=\frac{\delta F_{sn}}{\delta\Delta}. \tag{25}$$

The supercurrent $\mathbf{j}_s$ is defined as follows

$$\frac{\mathbf{j}_s}{c}=-\frac{\delta F_{sn}}{\partial\mathbf{A}}. \tag{26}$$

We will need the following two non-stationary Maxwell equations in the followings

$$\mathrm{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t},\qquad \mathrm{rot}\mathbf{H}=\frac{4\pi}{c}\mathbf{j}+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}, \tag{27}$$

where

$$\mathbf{H}=\mathrm{rot}\mathbf{A},\qquad \mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}. \tag{28}$$

Next we calculate the time-derivatives of the free energy of the system; the derivative of the free energy of the electromagnetic field is

$$\frac{dF_{em}}{dt}=\int(-\mathrm{div}\mathbf{S}-\mathbf{j}\cdot\mathbf{E})dV, \tag{29}$$

where $\mathbf{S}=(c/4\pi)[\mathbf{E}\times\mathbf{H}]$ is the Pointing vector. Furthermore,

$$\frac{dF_{sn}}{dt}=\int\left[\frac{\partial\Delta}{\partial t}\frac{\delta F_{sn}}{\delta\Delta}+\frac{\partial\Delta^*}{\partial t}\frac{\delta F_{sn}}{\delta\Delta^*}+\frac{\delta\mathbf{A}}{\delta t}\frac{\delta F_{sn}}{\delta\mathbf{A}}\right]dV+\int\mathrm{div}\,\mathbf{j}_F\,dV, \tag{30}$$

where $\mathbf{j}_F$ is given by

$$\mathbf{j}_F=\gamma_d\hbar^2\left[\frac{\partial\Delta}{\partial t}\left(\nabla+\frac{2ie}{\hbar c}\mathbf{A}\right)\Delta^*+\frac{\partial\Delta^*}{\partial t}\left(\nabla-\frac{2ie}{\hbar c}\mathbf{A}\right)\Delta\right]. \tag{31}$$

Using Eqs. (24), (25), (26) and the relation

$$\frac{2ie}{\hbar}\left(\Delta^*\frac{\delta F_{sn}}{\delta\Delta^*}-\Delta\frac{\delta F_{sn}}{\delta\Delta}\right)=\mathrm{div}\,\mathbf{j}_s, \tag{32}$$

we transform the expression of Eq. (30) to the following form

$$\frac{dF_{sn}}{dt}=\int\left\{-\mathbf{j}_s\left(\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\right)-2\Gamma'\left|\frac{\partial\Delta}{\partial t}+\frac{2ie}{\hbar}\tilde{\phi}\Delta\right|^2+\tilde{\phi}\,\mathrm{div}\,\mathbf{j}_s\right\}dV+\int\mathrm{div}\,\mathbf{j}_F\,dV. \tag{33}$$

Finally, adding eqs. (29) and (33), we obtain the energy balance equation

$$\frac{dF}{dt} = -\int W dV - \int \operatorname{div} \mathbf{j}_E \, dV , \tag{34}$$

where the free energy current is

$$\mathbf{j}_E = -\mathbf{j}_F - \tilde{\phi}\mathbf{j}_s + \mathbf{S} , \tag{35}$$

and the dissipation function is

$$W = 2\Gamma' \left| \frac{\partial \Delta}{\partial t} + \frac{2ie}{\hbar}\tilde{\phi}\Delta \right|^2 + \sigma_n \left| \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} + \nabla\tilde{\phi} \right|^2 , \tag{36}$$

where the first term describes the energy loss due to relaxation of the order parameter while the second term is associated with the Joule heating by the normal currents. Eq. (36) shows that only the real part of coefficient Γ is relevant for the dissipative function. Its imaginary part appears enters the non-dissipative forces. To demonstrate this, we write the order parameter in the form $\Delta = |\Delta| \exp(i\chi)$ and calculate the variation of the free energy with respect to the phase of the order parameter

$$\frac{\partial F_{sn}}{\partial \chi} = \frac{i}{2}\left(\Delta \frac{\delta F_{sn}}{\delta \Delta} - \Delta^* \frac{\delta F_{sn}}{\delta \Delta^*} \right) = -\frac{\hbar}{4e} \operatorname{div} \mathbf{j}_s . \tag{37}$$

Using Eqs. (24) and (25) we obtain from Eq. (37)

$$\frac{\partial F_{sn}}{\partial \chi} = -\frac{2e\Gamma' |\Delta|^2}{\hbar}\Phi - \frac{\Gamma''}{2}\frac{\partial |\Delta|^2}{\partial t} = \frac{\hbar}{4e}\sigma_n \operatorname{div} \tilde{\mathbf{E}} , \tag{38}$$

where gauge invariant scalar potential $\Phi = \tilde{\phi} + (\hbar/2e)(\partial\chi/\partial t)$ and in the last term we used the conservation of the net current. Then with the help of definition of effective electric field (7) we obtain following equation

$$\nabla^2\Phi - \frac{8e^2\Gamma' |\Delta|^2}{\hbar^2 \sigma_n}\Phi = -\frac{1}{c} \operatorname{div} \frac{\partial \mathbf{Q}}{\partial t} + \frac{2e\Gamma''}{\hbar\sigma_n}\frac{\partial |\Delta|^2}{\partial t} , \tag{39}$$

where gauge invariant vector potential is defined as $\mathbf{Q} = \mathbf{A} - (c\hbar/2e)\nabla\chi$. In the stationary limit $\partial\chi/\partial t = 0$, $\partial \mathbf{A}/\partial t = 0$, $\partial|\Delta|/\partial t = 0$ we obtain London type equation for scalar potential

$$l_E^2 \nabla^2 \tilde{\phi} = \frac{|\Delta|^2}{\Delta_0^2}\tilde{\phi} , \tag{40}$$

where we defined a new characteristic length

$$l_E = \left(\frac{\hbar^2 \sigma_n}{8e^2 \Gamma' \Delta_0^2} \right)^{1/2} , \tag{41}$$

which can be interpreted as the penetration depth of d.c. electric field into a superconductor. Here the order parameter for time-independent homogeneous state Δ_0 is

$$\Delta_0 = \left(-\frac{\alpha}{\beta}\right)^{1/2} = \left[\frac{8}{7\zeta(3)}\left(\frac{T_c - T}{T_c}\right)\right]^{1/2} \pi k_B T_c . \tag{42}$$

Therefore the modified TDGL equation for the order parameter for a uniformly rotating dirty superconductor takes the form of Eq. (11) with the complex parameter Γ defined by (5), (13) and (19). In the dissipation function (36) potentials $\tilde{\phi}$ and $\mathbf{A}$ must be replaced by the $\tilde{\phi}_r$ (9) and $\mathbf{A}_r$ (10), while in eq. (39) scalar potential Φ must be replaced by $\Phi_r = \Phi - m_e \Omega^2 r^2 / 2e$.

4. *2SC color superconductor*. We now establish the modified TDGL equations for the rotating 2SC color superconductor by using the analogy to the ordinary dirty superconductors [7]. Because a 2SC superconductor contains an admixture of blue quarks we have assumed that there will be some degree of inelastic scattering among quark Cooper pairs and unpaired blue quarks. Less important scattering is expected due to the electromagnetic interactions of the Cooper pairs with the normal electrons. Combined these interactions will be the source of destruction of the coherence among quark Cooper pairs [6]. The physical picture is analogous to the case where impurities act to destroy the coherence among the electronic Cooper pairs in dirty superconductors. Then modified equation for the order parameter of the uniformly rotating 2SC condensate is

$$-\Gamma\left[\frac{\partial}{\partial t} - \frac{iq}{\hbar}\phi_8'\right] d = \alpha d + \beta |d|^2 d - \gamma_d \left(\hbar\nabla + \frac{iq}{c}\mathbf{A}_x'\right)^2 d, \tag{43}$$

where the modified potentials are

$$\phi_8' = \phi_8 + \frac{\mu}{q} - \frac{\mu}{3qc^2}\Omega^2 r^2, \quad \mathbf{A}_x' = \mathbf{A}_x + \frac{2}{3}\frac{\mu}{cq}[\Omega \mathbf{r}], \tag{44}$$

where $d = |d| e^{-i\chi}$ is the order parameter, ϕ_8 is the scalar potential of the color electric field of the eight gluon, vector potentials of the "rotated" fields $\mathbf{A}_x$ and $\mathbf{A}_y$ are given by [21]

$$\mathbf{A}_x = -\sin\theta_M \mathbf{A} + \cos\theta_M \mathbf{A}_8 , \tag{45}$$

$$\mathbf{A}_y = \cos\theta_M \mathbf{A} + \sin\theta_M \mathbf{A}_8 . \tag{46}$$

In Eq. (44) we have used the fact that electric mixing angle θ_D in the 2SC phase vanishes [22]. The magnetic mixing angle is given in terms of electrical charge e and strong coupling constant g as

$$\cos\theta_M = \frac{\sqrt{3}\, g}{\sqrt{e^2 + 3g^2}}, \tag{47}$$

and $q = \sqrt{e^2 + 3g^2}/3$ is the charge of Cooper pair. Coefficients α, β and γ_d have their values given by Eq. (4), while the density of states and the diffusion coefficient in these equations are given by $\nu = \mu p_F / \pi^2 \hbar^3 c^2$ and $D = v_F^2 \tau_q / 3$;

where τ_q is the time-scale for the inelastic scattering of Cooper pairs with blue quarks and electrons. We expect that this time-scale should be roughly of the order of the momentum relaxation time-scale τ_s of the two counter-streaming quark matter beams [23]. The coefficient Γ is complex (13), where Γ' is defined by (5) with quark's density of states, while imaginary part Γ'' is given by

$$\Gamma'' = -\frac{p_F}{\pi^2\hbar^2 c^2}\left(\frac{\pi^2\hbar^3 c^2}{|g|\mu\, p_F}+1\right). \tag{48}$$

Here we have used Eq. (19) and $\xi = cp-\mu$; where $\mu = cp_F$ is chemical potential of ultrarelativistic quarks. The imaginary part Γ'' is small because the ratio of the imaginary and real parts of Γ is

$$\zeta = -\frac{\Gamma''}{\Gamma'} = \frac{8k_BT_c}{\pi\mu}\left(\frac{\pi^2\hbar^3 c^2}{|g|\mu\, p_F}+1\right). \tag{49}$$

The second TDGL equation for the net current is given by

$$\mathbf{j}_d = 2iq\hbar\gamma_d\left(d^*\nabla d - d\nabla d^*\right) - \frac{4\gamma_d q^2}{c}|d|^2\mathbf{A}'_x + \sigma_q\mathbf{E}', \tag{50}$$

where σ_q is the electrical conductivity of normal quark matter [23] and we have defined effective electric field $\mathbf{E}'$ as

$$\mathbf{E}' = -\frac{1}{c}\frac{\partial\mathbf{A}'_x}{\partial t} - \nabla\phi_8 - \frac{\nabla\mu}{q} + \frac{2\mu}{3qc^2}\Omega^2\mathbf{r}. \tag{51}$$

The modified TDGL equations define the relaxation times for the order parameter τ_d and for color magnetic field τ_j as

$$\tau_d = \frac{\Gamma}{|\alpha|}, \quad \tau_j = \frac{\sigma_q}{4\gamma_d q^2|d_0|^2}, \tag{52}$$

where d_0 is given by Eq. (42). The dissipative function, which corresponds to Eq. (43) is given [by the analogy with (36)] as

$$W = 2\Gamma'\left|\frac{\partial d}{\partial t} - \frac{iq}{\hbar}\phi'_8 d\right|^2 + \sigma_q\mathbf{E}'^2. \tag{53}$$

The first term in Eq. (53) describes the energy loss due to relaxation of order parameter, while the second is associated with Joule heating of normal currents. Upon writing the condensate order parameter in terms of its modulus and phase we rewrite the dissipation function in the form

$$W = 2\Gamma'\left[\left(\frac{\partial|d|}{\partial t}\right)^2 + |d|^2\left(\frac{\partial\chi}{\partial t} + \frac{q}{\hbar}\phi'_8\right)^2\right] + \sigma_q\mathbf{E}'^2, \tag{54}$$

which separates the contributions due to the relaxation of the magnitude and phase of the order parameter. Another interesting relation is obtained if we use the relations

$$\frac{2iq}{\hbar}\left(d\frac{\delta F}{\delta d}-d^*\frac{\delta F}{\delta d^*}\right)=\mathrm{div}\,\mathbf{j}_s\,, \tag{55}$$

where F the free energy difference between superconducting and normal state defined as

$$F=\int\left[2\alpha dd^*+\beta(dd^*)^2+2\gamma_d\left|\left(\hbar\nabla+i\frac{q}{c}\mathbf{A}_x\right)d\right|^2+\frac{1}{8\pi}(\mathrm{rot}\,\mathbf{A}_x)^2+\frac{1}{8\pi}(\mathrm{rot}\,\mathbf{A}_y)^2\right]dV. \tag{56}$$

We now calculate the term in the left hand side of eq. (55) using modified equation for the order parameter (43) in the following form

$$-\Gamma\left(-\frac{iq}{\hbar}\phi_8' d\right)=\frac{\delta F}{\delta d^*}, \tag{57}$$

and its conjugate one

$$-\Gamma^*\left(\frac{\partial d^*}{\partial t}+\frac{iq}{\hbar}\phi_8' d^*\right)=\frac{\delta F}{\delta d}. \tag{58}$$

Writing again the condensate order parameter in terms of it modulus and phase we obtain following relation

$$2\Gamma'\left(\frac{\partial\chi}{\partial t}+\frac{q}{\hbar}\phi_8'\right)|d|^2=\mathrm{div}\,\mathbf{j}_s\,. \tag{59}$$

We next introduce the following gauge invariant potentials $\Phi_q=\phi_8'+(\hbar/q)(\partial\chi/\partial t)$ and $\mathbf{Q}_d=\mathbf{A}_x'-(c\hbar/q)\nabla\chi$ and use net current conservation to derive the following equation

$$\nabla^2\Phi_d-\frac{4\Gamma' q^2|d|^2}{\sigma_q\hbar^2}\Phi_d=-\frac{1}{c}\frac{\partial\mathbf{Q}_d}{\partial t}+\frac{2\Gamma'' q}{\sigma_q\hbar}\frac{\partial|d|^2}{\partial t}. \tag{60}$$

In the stationary limit $\partial\chi/\partial t=0$, $\partial\mathbf{A}/\partial t=0$, $\partial|d|/\partial t=0$ we obtain London type equation for scalar potential

$$\lambda_E^2\nabla^2\phi_8'-\frac{|d|^2}{d_0^2}\phi_8'=0, \tag{61}$$

where we defined the penetration depth of static color electric field into the 2SC phase as

$$\lambda_E=\left(\frac{\sigma_q\hbar^2}{4\Gamma' q^2 d_0^2}\right)^{1/2}. \tag{62}$$

We now estimate time-scales for which our modified TDGL equations are applicable. For the zero temperature energy gap of the order of 25 MeV critical temperature $T_c=0.57\Delta_0=14.2$ MeV. The condition of weak pair breaking [24] $\tau k_B T_c \gg \hbar$ is fulfilled if the inelastic quark-quark collision time $\tau>4.3\times10^{-19}$ s.

5. *Concluding remarks*. To summarize, we have modified the time-dependent Ginzburg-Landau equations for the rotating 2SC quark superconductors

derived earlier by us in Ref. [7]. The key idea is to account for possible imaginary part of the coefficient of the time-dependent term in the non-stationary GL equation. Such extension is motivated by the studies of electronic superconductors in metals. We have shown that the dissipative function of the system remains unchanged, whereas the non-stationary London equation for the color-electric field acquires an additional time-dependent term, which however vanishes in the stationary limit. We have derived only general expression without specifying the details of the problem, such as the geometry or the initial conditions. We anticipate that the formalism developed here can be applied to model the dynamics of the 2SC phase in massive compact stars, where it can occupy a substantial part of the star's volume (see e. g. [25]).

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РАСШИРЕННЫЕ ВРЕМЕННЫЕ УРАВНЕНИЯ ГИНЗБУРГА-ЛАНДАУ ДЛЯ ВРАЩАЮЩИХСЯ ДВУХ АРОМАТНЫХ ЦВЕТОВЫХ СВЕРХПРОВОДНИКОВ

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Мы обсуждаем расширение полученных ранее временных уравнений Гинзбурга-Ландау для вращающейся двухароматной цветовой сверхпроводящей кварковой материи. Это расширение рассматривает коэффициент в зависящем от времени слагаемом в уравнении Гинзбурга-Ландау как комплексную величину, мнимая часть которой описывает недиссипативные эффекты. Мы получияи временное уравнение типа уравнения Лондонов для цветового электрического потенциала, которое содержит добавочный вклад от этой мнимой части. Это добавочное слагаемое описывает недиссипативные эффекты распространения. Мы получили также общие выражения для потока энергии и диссипативной функции системы.

Ключевые слова: *нейтронные звезды: теория Гинзбурга-Ландау:кварковая материя:сверхпроводимость*

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