АСТРОФИЗИКА

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THE EFFECT OF A DENSITY DEPENDENT BAG CONSTANT ON THE STRUCTURE OF HOT NEUTRON STAR WITH A QUARK CORE

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As we go from center toward the surface of a neutron star, the state of baryonic matter changes from the de-confined quark-gluon to a mixed phase of quark and hadronic matter and a thin crust of hadronic matter. For the quark matter, within MIT bag model, the total energy density of the system is the kinetic energy for non-interacting quarks plus a bag constant. In this article first we have considered a density dependent bag constant obtained using the recent experimental results of CERN SPS on the formation of a quark-gluon plasma. For calculations of the hadron phase, we use the lowest order constrained variational method. The equation of state of mixed phase has been determined using Gibbs conditions. Finally, we have calculated the structure of a hot neutron star with quark core employing TOV equation. Our results show that a density dependent bag constant leads to a higher mass and lower radius for the hot neutron star with respect to the case in which we use a fixed bag constant.

Key words: *Neutron star:quark core:quark matter:bag constant:structure*

1. *Introduction*. Neutron stars (NS) are among the densest of massive objects in the universe. A hot neutron star is born following the gravitational collapse of the core of a massive star just after the supernova explosion. The interior temperature of a neutron star at its birth is of the order 20 - 50 MeV [1]. As we go from the surface to the center of a neutron star, at suffciently high densities, the matter is expected to undergo a transition from hadronic matter where the quarks are confined inside the hadrons to a state of deconfined quarks, with up, down and strange quarks in the quark matter. Other quarks have higher masses and do not appear in this state. Glendenning has shown that a proper construction of the hadron-quark phase transition inside the neutron stars implies the coexistence of nucleonic matter and quark matter over a finite range of pressure. Therefore, a mixed hadron-quark phase exists in the neutron star where its energy is lower than that of the quark matter and the nucleonic matter over a finite rang of Pressure [2]. This shows that we can consider a neutron star to be composed of a hadronic matter layer, a mixed phase of quarks and hadrons and, in the core, quark matter.

The deconfined quark phase is treated within the popular MIT bag model [3]. In this model, the total energy density is the sum of a nonperturbative energy shift $\mathcal{B}$ (the bag constant) and the kinetic energy for noninteracting quarks. The bag constant $\mathcal{B}$ can be interpreted as the difference between the energy densities of the perturbative vacuum and physical ones, which has a constant value such as $\mathcal{B}=55$ and $90\ \mathrm{MeV/fm^3}$ in the initial model of MIT, recently it is constrained to be compatible with the recent experimental results obtained at CERN on the formation of a quark-gluon plasma. The resulting picture is the following: during the early stages of the heavy-ion collision, a very hot and dense state (fireball) is formed whose energy materializes in the form of quarks and gluons strongly interacting with each other, exhibiting features consistent with expectations from a plasma of deconfined quarks and gluons [4,5]. In general, it is not obvious if the informations on the nuclear EOS from high energy heavy ion collisions can be related to the physics of neutron stars interior. The possible quark-gluon plasma produced in heavy ion collision is expected to be characterized by small baryon density and high temperature, while the possible quark phase in neutron stars appears at high baryon density and low temperature. However, if one adopts for the hadronic phase a noninteracting gas model of nucleons, antinucleons and pions, the original MIT bag model predicts that the deconfined phase occurs at an almost constant value of the quark-gluon energy density, irrespective of the thermodynamical conditions of the system [6].

Burgio et al. [7,8] have investigated the structure of neutron stars with a quark core at zero and finite temperatures, using the Brueckner-Bethe-Goldstone formalism to determine the equation of state of the hadronic matter. We have calculated the structure properties of the cold neutron star by considering a quark phase at its core [9] and compared the results with our previous calculations for the neutron star without the quark core [10]. In these works, we have employed the lowest order constrained variational (LOCV) method for the hadronic matter calculations. Recently, we have calculated the structure of a hot neutron star with a quark core with a fixed bag constant ($\mathcal{B}=90\ \mathrm{MeV/fm^3}$) [11]. In the present paper, we intend to extend these calculations to hot neutron stars with a quark cores by considering a density dependent bag constant.

2. *Equation of State*. In this section we calculate the equation of state of a neutron star composed of a hadronic matter, a mixed phase of quarks and hadrons and a quark core as follows.

2.1. *Hadron Phase*. In our calculations the equation of state of hot nucleonic matter is computed using the lowest order constrained variational (LOCV) method [12-19]. The details of our calculations for the hadronic phase has been fully discussed in [11].

2.2. *Quark Phase*. We use the MIT bag model for the quark matter. In this model, the energy density is the sum of kinetic energy of quarks and a bag constant $\mathcal{B}$ which is interpreted as the difference between energy densities of non interacting quarks and interacting ones [20],

$$\varepsilon_{tot} = \varepsilon_u + \varepsilon_d + \varepsilon_s + \mathcal{B}, \tag{1}$$

where ε_i is the kinetic energy per volume of particle i, the kinetic energy of quarks has been discussed in [11] and the bag constant $\mathcal{B}$, can be interpreted as the difference between the energy densities of the noninteracting quarks and interacting ones, which has constant values such as $\mathcal{B} = 55$, 90 and 220 MeV in the initial model of MIT. But the density of quark matter is not uniform, therefor we can consider a density dependent $\mathcal{B}$. We try to determine a range of possible values for $\mathcal{B}$ by exploiting the experimental data obtained at the CERN SPS, where several experiments using high-energy beams of Pb nuclei reported (indirect) evidence for the formation of a quark-gluon plasma [21,22]. According to the analysis of those experiments, the quark-hadron transition takes place at about seven times normal nuclear matter energy density ($\varepsilon_0 = 156$ MeV/fm^3). In the literature, there are attempts to understand the density dependence of $\mathcal{B}$ [23,24]. However, currently the results are highly model dependent and no definite picture has come out yet. Therefore, we attempt to provide effective parametrization for this density dependence. Our parametrization are constructed in such a way that at asymptotic densities $\mathcal{B}$ approaches a finite value $\mathcal{B}_\infty$. For the bag constant $\mathcal{B}$, we use a density dependent Gaussian parametrization [7,25],

$$\mathcal{B}(n) = \mathcal{B}_\infty + (\mathcal{B}_0 - \mathcal{B}_\infty)\exp\left[-\beta(n/n_0)^2\right] \tag{2}$$

with $\mathcal{B}_0 = \mathcal{B}(n=0) = 400$ MeV/fm^3 and $\beta = 0.17$. We know that the value of the bag constant $\mathcal{B}$ should be compatible with empirical results. The experimental results at CERN-SPS show a proton fraction $x_p = 0.4$ [26]. Therefore, for calculation of $\mathcal{B}_\infty$, we employ the equation of state of the asymmetric nuclear matter as follows. First, we use the equation of state of asymmetric hadronic matter characterized by a proton fraction $x_p = 0.4$ and the UV14+TNI potential. By assuming that the hadron-quark transition takes place at the energy density $\varepsilon = 1100$ MeV/fm^3, we find that the hadronic matter baryonic density is n_t (transition density) and at values lower than n_t, the quark matter energy density is higher than that of nuclear matter, while by increasing the baryonic density, the two energy densities become equal at this density, and after that, the nuclear matter energy density remains always higher. Energy density equation of quark matter with two flavors u and d reduces to

$$\varepsilon_Q = \varepsilon_u + \varepsilon_d + \mathcal{B}. \tag{3}$$

Beta-equilibrium and charge neutrality conditions lead to the following relation for the number density of quarks,

$$n_B = 2n_u = n_d\,. \tag{4}$$

We determine $\mathcal{B}_\infty$ by putting quark energy density and hadronic energy density equal to each other at any temperature. Later we can calculate the energy density and determine pressure and the equation of state for quark phase such as determined in [11].

2.3. *Mixed phase.* For mixed phase, where the fraction of space occupied by quark matter smoothly increase from zero to unity, we have a mixture of hadrons, quarks and electrons. According to the Gibss equilibrium condition, the temperatures, pressures and chemical potentials of both phases are equal [2]. Calculation of equation of state of mixed phase has been fully discussed in [11].

2.4. *Results.* Our results for the energy densities corresponding to different phases i.e. hadronic matter, pure quark matter and a mixed phase of quarks and hadrons, are given in Fig.1 at two temperatures. It can be seen that at

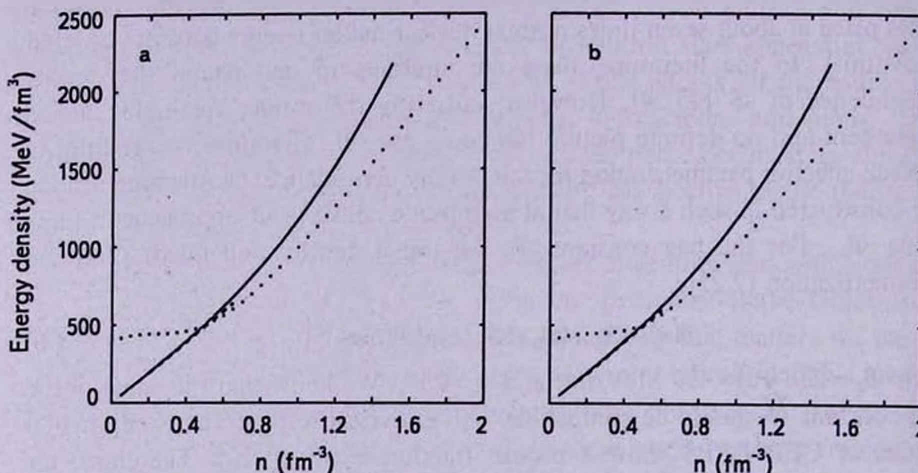

Fig.1. Energy density versus the baryon density for the hadron phase (solid line), mixed phase (dashed line) and quark phase (dotted line) at $T = 10$ (a) and 20 MeV (b).

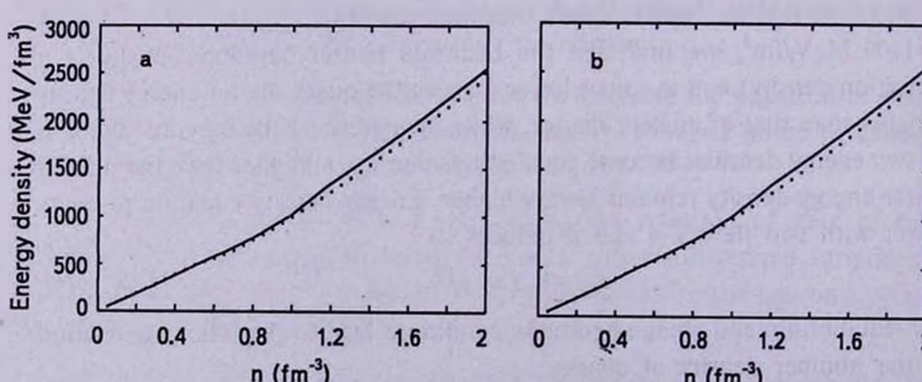

Fig.2. Energy density versus the baryon density for the neutron star with the quark core with density dependent (dotted line) and independent (solid line) bag constant at $T = 10$ (a) and 20 MeV (b).

low densities there is a pure hadronic phase. We have found that a mixed phase exists at higher densities up to $n \sim 0.5\,\text{fm}^{-3}$. It is obvious that a pure quark phase emerges by increasing the density. The energy density for neutron star with density dependent and density independent $\mathcal{B}$ are shown in Fig.2. It is clear that for a density dependent bag constant, the energy density is smaller than that of the independent one. We have calculated the equation of state (the pressure versus baryon density) for the neutron star with the quark core using the density dependent and independent bag constants. Fig.3 shows the relevant results at two temperatures. These equations of state are used as an input into the general relativistic equation of hydrostatic equilibrium.

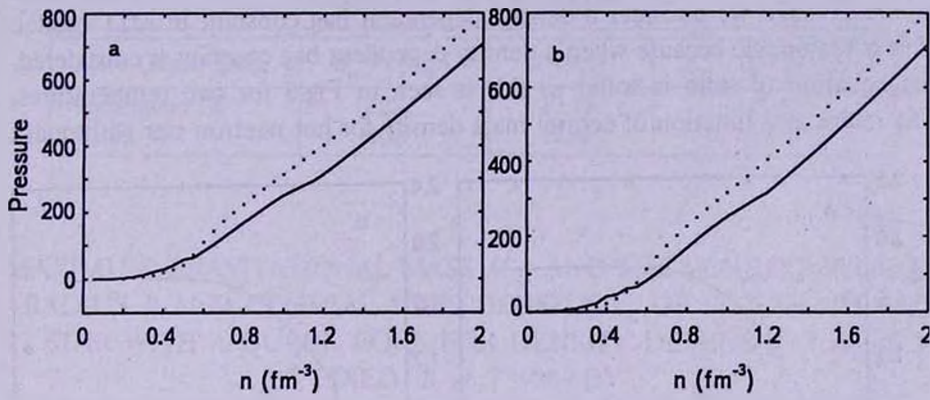

Fig.3 Pressure versus the baryon density for the neutron star with the quark core with density dependent (dotted line) and independent (solid line) bag constant at $T = 10$ (a) and 20 MeV (b).

3. *Structure of the Hot Neutron Star with a Quark Core*. The equilibrium energy density distribution of slowly rotating spherical star is determined by the Tolman-Oppenheimer-Volkoff equation (TOV) [27-29],

$$\frac{dP}{dr} = -\frac{G\left[\varepsilon(r)+\frac{P(r)}{c^2}\right]\left[m(r)+\frac{4\pi r^3 P(r)}{c^2}\right]}{r^2\left[1-\frac{2Gm(r)}{rc^2}\right]}, \tag{5}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon(r). \tag{6}$$

P is the pressure and ε is the total energy density. For a given equation of state in the form $P(\varepsilon)$, the TOV equation yields the mass and radius of star as a function of central energy density.

In our calculation for the hot neutron star with quark core, we use the following equation of state: (i) Below the density $0.05\,\text{fm}^{-3}$, we use the equation of state calculated by Baym [30]. (ii) For the hadron phase, from the density of $0.05\,\text{fm}^{-3}$ up to the density where the mixed phase is started, we use the equation of state which is calculated in section 2.1. (iii) In the range of densities in mixed phase, we use the equation of state calculated in section 2.3. (iv) For quark phase, we use the equation of state calculated in section 2.2. Using the above equations of state, we integrate the TOV equation numerically and determine the structure of this star. All calculations are done for the density dependent bag constant $\mathcal{B}(n)$ at two different temperatures $T=10$ and 20 MeV. Our results are as follows.

The gravitational mass versus the central mass density for a hot neutron star with quark core for two different temperatures has been presented in Fig.4. We can see there is the limiting mass for hot neutron star and this mass increases when we consider a density dependent bag constant in MIT model. This is reasonable because when a density dependent bag constant is considered, the equation of state is softer as this is seen in Fig.3 for two temperatures. The radius as a function of central mass density for hot neutron star with quark

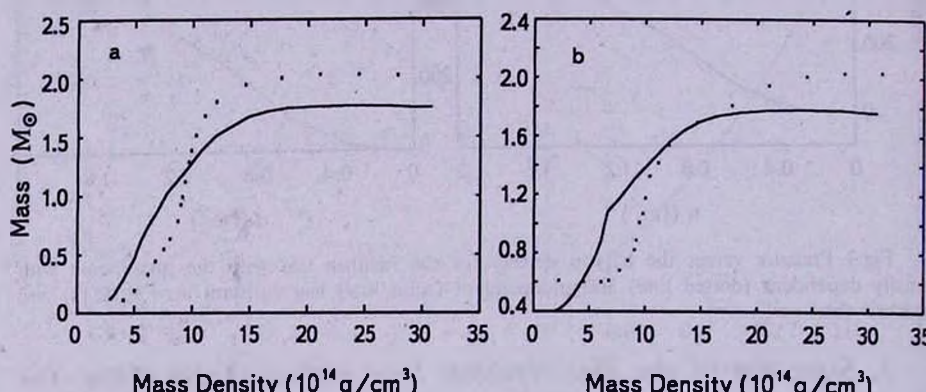

Fig.4. Gravitational mass versus the central mass density for the neutron star with the quark core with density dependent (dotted line) and independent (solid line) bag constant at $T=10$ (a) and 20 MeV (b).

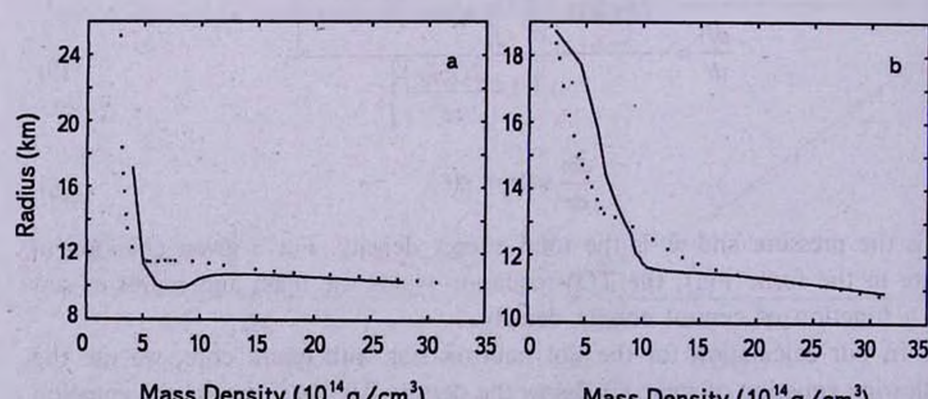

Fig.5. Radius versus the central mass density for the neutron star with the quark core with density dependent (dotted line) and independent (solid line) bag constant at $T=10$ (a) and 20 MeV (b).

core for two different temperatures has been presented in Fig.5. The radius of star decreases when the bag constant is density dependent. Our results for the maximum gravitational mass of the hot neutron star with the quark core and the corresponding values of radius and central mass density have been given in Tables 1 and 2 for two different temperatures.

Table 1

MAXIMUM GRAVITATIONAL MASS M_{max} AND THE CORRESPONDING RADIUS R AND CENTRAL MASS DENSITY ε_c OF HOT NEUTRON STAR WITH A QUARK CORE FOR DENSITY DEPENDENT AND FIXED $\mathcal{B}$ at $T = 10$ MeV

NS + Quark Core	M_{max} ($M_\odot$)	R (km)	ε_c (10^{14} gr/cm^3)
Dependent $\mathcal{B}$	2.032	10.39	24.42
Fixed $\mathcal{B}$	1.76	10.45	27.38

Table 2

MAXIMUM GRAVITATIONAL MASS M_{max} AND THE CORRESPONDING RADIUS R AND CENTRAL MASS DENSITY ε_c OF HOT NEUTRON STAR WITH A QUARK CORE FOR DENSITY DEPENDENT AND FIXED $\mathcal{B}$ at $T = 20$ MeV

NS + Quark Core	M_{max} ($M_\odot$)	R (km)	ε_c (10^{14} gr/cm^3)
Dependent $\mathcal{B}$	2.033	10.9	24.43
Fixed $\mathcal{B}$	1.78	11	27.37

4. *Summary and Conclusion.* The structure of hot neutron star with a quark core using a density dependent bag constant has been investigated. From the surface toward the center of hot neutron star, a pure hadronic matter, a mixed phase of quarks and hadrons in a range of densities determined by employing the Gibbs conditions, and a pure quark matter in the core, have been considered. We have employed the LOCV method at finite temperature to get the equation of state of hot hadronic matter. The MIT bag model with the density dependent bag constant obtained by using the recent experimental results of CERN SPS on the formation of a quark-gluon plasma has been applied to compute the equation of state of hot quark matter. We have solved the TOV equation by a numerical method to determine the structural properties of hot neutron star with the quark core at $T = 10$ and 20 MeV. The results have been compared to those for the hot neutron star with $\mathcal{B} = 90$ MeV/fm^3. We have found that a density dependent bag constant leads to a higher mass and a lower radius for the neutron star in comparison to the case in which

the constant $\mathcal{B} = 90$ MeV/fm^3 has been used.

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ЭФФЕКТ, ЗАВИСЯЩИЙ ОТ ПЛОТНОСТИ ПОСТОЯННОЙ МЕШКА НА СТРУКТУРУ ГОРЯЧЕЙ НЕЙТРОННОЙ ЗВЕЗДЫ С КВАРКОВЫМ ЯДРОМ

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При движении от центра к поверхности нейтронной звезды состояние барионной материи изменяется от кварк-глюонов в состоянии деконфайнмента до смешанной фазы из кварковой и адронной материи, и до тонкой коры из адронной материи. Для кварковой материи в модели мешка МІТ полная плотность энергии системы состоит из кинетической не взаимодействующих кварков плюс постоянная мешка. В этой статье мы рассматриваем зависящую от плотности постоянную мешка, полученную с использованием современных экспериментальных данных по формированию кварк-глюонной плазмы на суперпротонном синхротроне CERN. Для вычислений в адронной фазе мы использовали вариационный условный метод низшего порядка. Уравнение состояния смешанной фазы получено с использованием условия Гиббса. Далее мы вычислили структуру горячей нейтронной звезды используя уравнение Толмена-Оппенгеймера-Волкова. Наши результаты показывают, что зависящая от плотности постоянная кваркового мешка приводит к большей массе и к меньшему радиусу горячей нейтронной звезды по сравнению со случаем, в котором мы используем фиксированную постоянную.

Ключевые слова: *нейтронная звезда:кварковая кора:кварковая материя: постоянная мешка:структура*

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