АСТРОФИЗИКА

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FINITE TEMPERATURE CALCULATIONS FOR THE BULK PROPERTIES OF STRANGE STAR USING A MANY-BODY APPROACH

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We have considered a hot strange star matter, just after the collapse of a supernova, as a composition of strange, up and down quarks to calculate the bulk properties of this system at finite temperature with the density dependent bag constant. To parameterize the density dependent bag constant, we use our results for the lowest order constrained variational (LOCV) calculations of asymmetric nuclear matter. Our calculations for the structure properties of the strange star at different temperatures indicate that its maximum mass decreases by increasing the temperature. We have also compared our results with those of a fixed value of the bag constant. It can be seen that the density dependent bag constant leads to higher values of the maximum mass and radius for the strange star.

Key words: *strange star:equation of state.structure*

1. *Introduction.* Strange stars are those which are built mainly from self bound quark matter. The surface density of strange star is equal to the density of strange quark matter at zero pressure ($\sim 10^{15}$ g/cm^3), which is fourteen orders of magnitude greater than the surface density of a normal neutron star. The central density of these stars is about five times greater than the surface density [1-3]. The existence of strange stars which are made of strange quark matter was first proposed by Itoh [4] even before the full developments of QCD. Later Bodmer [5] discussed the fate of an astronomical object collapsing to such a state of matter. In 1970s, after the formulation of QCD, the perturbative calculations of the equation of state of the strange quark matter was developed, but the region of validity of these calculations was restricted to very high densities [6]. The existence of strange stars was also discussed by Witten [7]. He conjectured that a first order QCD phase transition in the early universe could concentrate most of the quark excess in dense quark nuggets. He suggested that the true state of matter was strange quark matter. Based on theoretical works of Witten on cosmic separation of phases, the transition temperature is approximately 100 MeV, an acceptable QCD temperature [7]. Witten proposal was that the strange quark matter composed of light quarks is more stable than nuclei, therefore strange quark matter can be considered as the ground state of matter. The strange quark matter would be the bulk quark matter phase

consisting of almost equal numbers of up, down and strange quarks plus a small number of electrons to ensure the charge neutrality. A typical electron fraction is less than 10^{-3} and it decreases from the surface to the center of strange star [1-3]. Strange quark matter would have a lower charge to baryon ratio compared to the nuclear matter and can show itself in the form of strange stars [7-10].

Just after the collapse of a supernova, a hot strange star may be formed. A strange star may be also formed from a neutron star and is denser than the neutron star. If sufficient additional matter is added to a strange star, it will collapse into a black hole. Neutron stars with masses of $1.5-1.8\,M_{\odot}$ with rapid spins are theoretically the best candidates for conversion to the strange stars. An extrapolation based on this indicates that up to two quark-novae occur in the observable universe each day. Besides, recent Chandra observations indicate that objects RX J185635-3754 and 3C58 may be bare strange stars [11].

In this article, we consider a hot strange star born just after the collapse of a supernova. Here we ignore the effects of the presence of electrons, and consider a strange star purely made up of the quark matter consisting of the up, down and strange quarks. The energy of quark matter is calculated at finite temperature, and then its equation of state is derived. Finally using the equation of state of quark matter, the structure of strange star at diffrent temperatures is computed by integrating the Tolman-Oppenheimer-Volkoff (TOV) equations.

2. *Calculation of Quark Matter Equation of State.*

2.1. *Density Dependent Bag Constant.* Different models have been used for deriving the equation of state of quark matter. Therefore there is a great variety of the equations of state for this system. The model which we use is the MIT bag model which was developed to take into account the non perturbative effcts of quark confinement by introducing the bag constant. In this model, the energy per volume for the quark matter is equal to the kinetic energy of the free quarks plus a bag constant $\mathcal{B}$ [12]. The bag constant $\mathcal{B}$ can be interpreted as the difference between the energy densities of the noninteracting quarks and the interacting ones. Dynamically it acts as a pressure that keeps the quark gas in constant density and potential. This constant is shown to have different values which are 55 and 90 MeV/fm^3 in the initial MIT bag model. Since the density of strange quark matter increases from surface to the core of the strange star, it is more appropriate to use a density dependent bag constant rather than a fixed bag constant.

According to the analysis of the experimental data obtained at CERN, the quark-hadron transition takes place at about seven times the normal nuclear matter energy density (156 MeV/fm^3) [13,14]. Recently, a density dependent form has been also considered for $\mathcal{B}$ [15-18]. The density dependence of $\mathcal{B}$ is highly model dependent. In this article, the density dependence of $\mathcal{B}$ will

be parameterized, and we make the asymptotic value of $\mathcal{B}$ approach a finite value $\mathcal{B}_\infty$ [18],

$$\mathcal{B}(n) = \mathcal{B}_\infty + (\mathcal{B}_0 - \mathcal{B}_\infty) e^{-\gamma(n/n_0)^2} . \tag{1}$$

The parameter $\mathcal{B}_0 = \mathcal{B}$ $(n=0)$ has constant value which is assumed to be $\mathcal{B}_0 = 400$ MeV/fm^3 in this work, and γ is the numerical parameter which is usually equal to $n_0 \approx 0.17$ fm^{-3}, the normal nuclear matter density. $\mathcal{B}_\infty$ depends only on the free parameter $\mathcal{B}_0$. We know that the value of the bag constant $\mathcal{B}$ should be compatible with experimental data. The experimental results at CERN-SPS confirms a proton fraction $x_{pl} = 0.4$ (data is from experiment on accelerated Pb nuclei) [13,18]. Therefore, in order to evaluate $\mathcal{B}_\infty$, we use the equation of state of the asymmetric nuclear matter. The calculations regarding this can be found in the next section.

2.2. *Computation of $\mathcal{B}_\infty$ using the asymmetric nuclear matter calculations.* We use the equation of state of the asymmetric nuclear matter to calculate $\mathcal{B}_\infty$. For calculating the equation of state of asymmetric nuclear matter, we employ the lowest order constrained variational many-body method based on the cluster expansion of the energy as follows [19-27].

The asymmetric nuclear matter is defined as a system consisting of Z protons (pl) and N neutrons (nt) with the total number density $n = n_{pl} + n_{nt}$ and proton fraction $x_{pl} = n_{pl}/n$, where n_{pl} and n_{nt} are the number densities of protons and neutrons, respectively. For this system we consider a trial wave function as follows,

$$\psi = F\phi , \tag{2}$$

where ϕ is the slater determinant of the single-particle wave functions and F is the A-body correlation operator $(A = Z + N)$ which is taken to be

$$F = S\prod_{i>j} f(ij) \tag{3}$$

and S is a symmetrizing operator. For the asymmetric nuclear matter, the energy per nucleon up to the two-body term in the cluster expansion is

$$E(f) = \frac{1}{A}\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} = E_1 + E_2 . \tag{4}$$

The one-body energy, E_1, is

$$E_1 = \sum_{i=1}^{2}\sum_{k_i}\frac{\hbar^2 k_i^2}{2m} , \tag{5}$$

where labels 1 and 2 are used for proton and neutron respectively, and k_i is the momentum of particle i. The two-body energy, E_2, is

$$E_2 = \frac{1}{2A}\sum_{ij}\langle ij|v(12)|ij - ji\rangle , \tag{6}$$

where

$$v(12) = -\frac{\hbar^2}{2m}\left[f(12), \left[\nabla_{12}^2, f(12)\right]\right] + f(12)V(12)f(12). \quad (7)$$

In the above equation, $f(12)$ and $V(12)$ are the two-body correlation and nucleon-nucleon potential, respectively. In our calculations, we use $UV_{14} + TNI$ nucleon-nucleon potential [28]. Now, we minimize the two-body energy with respect to the variations in the correlation functions subject to the normalization constraint. From the minimization of the two-body energy, we obtain a set of differential equations. We can calculate the correlation functions by numerically solving these differential equations. Using these correlation functions, the two-body energy is obtained and then we can compute the energy of asymmetric nuclear matter. The procedure of these calculations has been fully discussed in reference [20].

As it was mentioned in the previous section, the experimental results at CERN-SPS confirms a proton fraction $x_p = 0.4$ [13,18], therefore to compute $\mathcal{B}_\infty$, we proceed in the following manner:

- Firstly, we use our results of the previous section for the asymmetric nuclear matter characterized by a proton fraction $x_p = 0.4$. By assuming that the hadron-quark transition takes place at the energy density equal to 1100 MeV/fm^3 [13,18], we find that the baryonic density of the nuclear matter is $n_B = 0.98$ fm^{-3} (transition density). At densities lower than this value the energy density of the quark matter is higher than that of the nuclear matter. With increasing the baryonic density these two energy densities become equal at the transition density and above this value the nuclear matter energy density remains always higher.

- Secondly, we determine $\mathcal{B}_\infty = 8.99$ MeV/fm^3 by putting the energy density of the quark matter and that of the nuclear matter equal to each other.

2.3. *Calculations for the energy of quark matter at finite temperature*. To calculate the energy of quark matter, we need to know the density of quarks in terms of the baryonic density. We do this by considering two conditions of beta equilibrium and charge neutrality. This leads to the following relations

$$\mu_d = \mu_u - \mu_e , \quad (8)$$

$$\mu_s = \mu_u - \mu_e , \quad (9)$$

$$\mu_s = \mu_d , \quad (10)$$

$$2/3\, n_u - 1/3\, n_s - 1/3\, n_d - n_e = 0 , \quad (11)$$

where μ_i and n_i are the chemical potential and the number density of particle i, respectively. As mentioned, we consider the system as pure quark matter ($n_e = 0$) [8,29-31]. Thus according to relation (11), we have

$$n_u = 1/2(n_s + n_d). \tag{12}$$

The chemical potential, μ_i, at any adopted values of the temperature T and the number density n_i is determined by applying the following constraint,

$$n_i = \frac{g}{2\pi^2}\int_0^\infty f(n_i, k, T)k^2 dk, \tag{13}$$

where

$$f(n_i, k, T) = \frac{1}{\exp\left\{\beta\left(\left(m_i^2c^4 + \hbar^2 k^2 c^2\right)^{1/2} - \mu_i\right)\right\} + 1} \tag{14}$$

is the Fermi-Dirac distribution function [32]. In the above equation, $\beta = 1/k_B T$ and g is the degeneracy number of the system.

As it is previously mentioned, we consider the total energy of the quark matter as the sum of the kinetic energy of the free quarks and the bag constant $\mathcal{B}$. Therefore, the total energy per volume of the quark matter ε_{tot} can be obtained using the following relation,

$$\varepsilon_{tot} = \varepsilon_u + \varepsilon_d + \varepsilon_s + \mathcal{B}, \tag{15}$$

where ε_i is the kinetic energy per volume of particle i,

$$\varepsilon_i = \frac{g}{2\pi^2}\int_0^\infty \left(m_i^2c^4 + \hbar^2 k^2 c^2\right)^{1/2} f(n_i, k, T)k^2 dk. \tag{16}$$

After calculating the energy, we can determine the other thermodynamic properties of the system. The entropy of the quark matter S_{tot} can be derived as follows

$$S_{tot} = S_u + S_d + S_s, \tag{17}$$

where S_i is the entropy of particle i,

$$\begin{aligned} S_i(n_i, T) = -\frac{3}{\pi^2}k_B \int_0^\infty [f(n_i, k, T)\ln(f(n_i, k, T)) + \\ + (1 - f(n_i, k, T))\ln(1 - f(n_i, k, T))]k^2 dk. \end{aligned} \tag{18}$$

The Helmholtz free energy per volume $\mathcal{F}$ is given by

$$\mathcal{F} = \varepsilon_{tot} - TS_{tot}. \tag{19}$$

The entropy per particle of the quark matter as a function of the baryonic density for two cases of the constant and density dependent $\mathcal{B}$ at different temperatures are plotted in Figs.1 and 2. For a fixed temperature, we see that the entropy per particle decreases by increasing the baryonic density and for all relevant densities, it is seen that the entropy increases by increasing the temperature.

In Figs.3 and 4, the free energy per volume of the quark matter versus the baryonic density for two cases of the constant and density dependent $\mathcal{B}$ are presented at diffrent temperatures. We can see that the free energy of the

quark matter has positive values for all densities and temperatures. For all densities, it is seen that the free energy decreases by increasing the temperature.

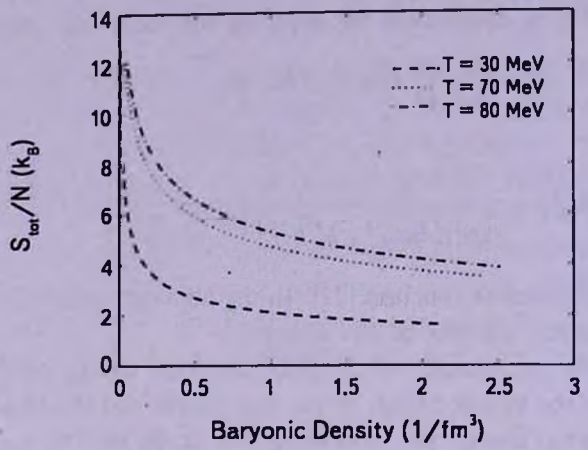

Fig.1. The entropy per particle of the quark matter versus the baryonic density at different temperatures for $\mathcal{B} = 90\ MeV/fm^3$.

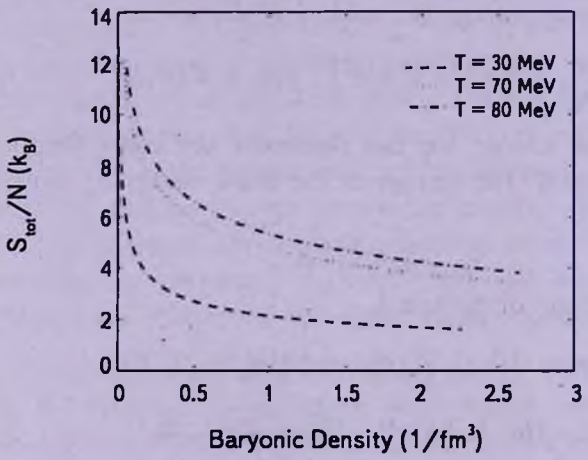

Fig.2. As Fig.1 but for the density dependent $\mathcal{B}$.

To obtain the structure of the strange star, the equation of state of the quark matter is needed. For deriving the equation of state, the following equation is used,

$$P(n,T) = \sum_i n_i \frac{\partial \mathcal{F}_i}{\partial n_i} - \mathcal{F}_i \,, \tag{20}$$

where P is the pressure. The pressure of the quark matter versus the baryonic density for two cases of the constant and density dependent $\mathcal{B}$ are plotted in Figs.5 and 6. It is seen that by increasing both density and temperature, the

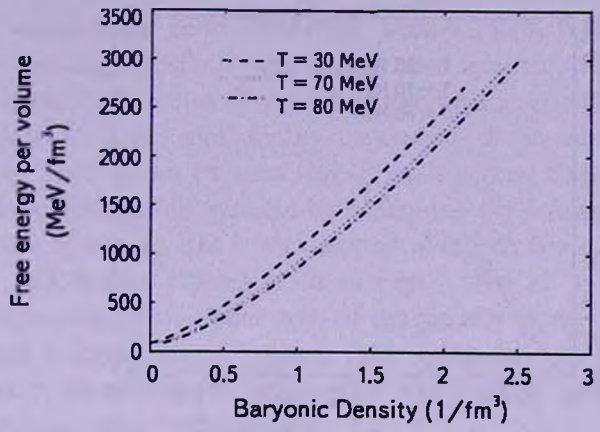

Fig.3. The free energy per volume of the quark matter versus the baryonic density at different temperatures for $\mathcal{B} = 90\,\mathrm{MeV/fm^3}$.

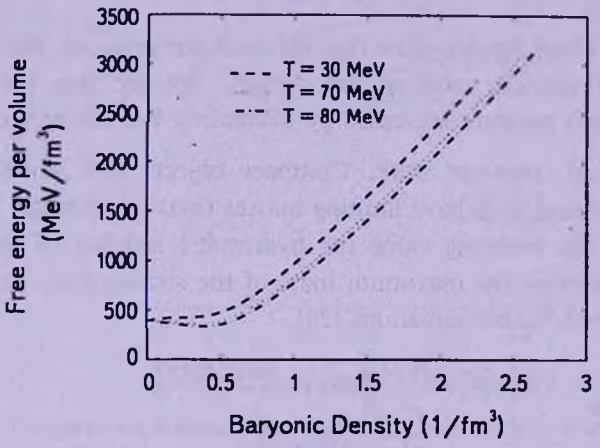

Fig.4. As Fig.3 but for the density dependent $\mathcal{B}$.

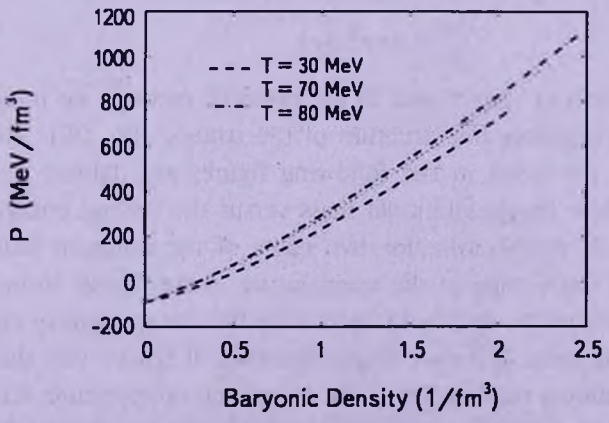

Fig.5. The pressure of the quark matter as a function of the baryonic density at different temperatures for $\mathcal{B} = 90\,\mathrm{MeV/fm^3}$.

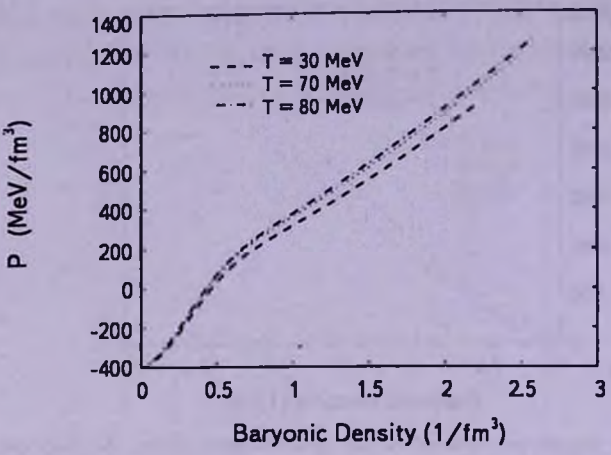

Fig.6. As Fig.5 but for the density dependent $\mathcal{B}$.

pressure increases. These figures show that for each temperature, the pressure becomes zero at a specific value of the density. We see that the density corresponding to zero pressure increases by decreasing the temperature.

3. *Structure of strange star.* Compact objects like white dwarfs, neutron stars and strange stars have limiting masses (maximum mass) and with a mass more than the limitting value the hydrostatic stability of the star is impossible. For obtaining the maximum mass of the strange star, we use the Tolman-Oppenheimer-Volkov equations [29],

$$\frac{dP}{dr} = -\frac{G\left[\varepsilon(r)+\frac{P(r)}{c^2}\right]\left[m(r)+\frac{4\pi r^3 P(r)}{c^2}\right]}{r^2\left[1-\frac{2Gm(r)}{rc^2}\right]}, \tag{21}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon(r). \tag{22}$$

By using the equation of state found in the previous section, we integrate the TOV equations to calculate the structure of the strange star [29]. The results of this calculation are given in the following figures and tables.

Figs.7 and 8 show the gravitational mass versus the central energy density at dfferent values of temperature for two cases of the constant and density dependent $\mathcal{B}$. For each value of the temperature, these figures show that the gravitational mass increases rapidly by increasing the energy density and finally reaches to a limiting value at higher energy densities. It is seen that the limiting value of the gravitational mass increases by decreasing temperature. Comparing Figs.7 and 8, one concludes that at all temperatures, for the density dependent bag constant, the rate of increasing mass with increasing the central density,

at lower values of the central densities, is substantially higher than that of the case for fixed bag constant, especially at zero temperature. In Figs.9 and 10, we have plotted the radius of strange star versus the central energy density for both $\mathcal{B} = 90$ MeV/fm^3 and density dependent $\mathcal{B}$ at different temperatures. From Figs.7-10, it can be seen that at each central density, both mass and the corresponding radius increase by decreasing the temperature. The gravitational mass of strange star is also plotted as a function of the radius for the constant and density dependent $\mathcal{B}$ in Figs.11 and 12. It is seen that for all temperatures, the gravitational mass of strange star increases by increasing the radius and it approaches a limiting value (maximum mass). Figs.11 and 12 show that by decreasing the temperature, the limiting values of mass and the corresponding radius both increase.

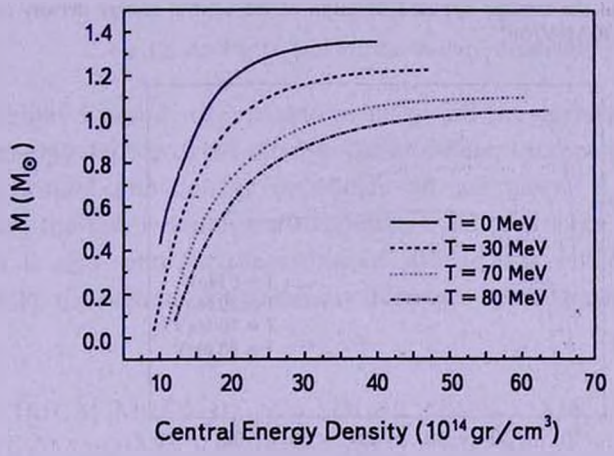

Fig.7. The gravitational mass of the strange star as a function of the central energy density at different temperatures for $\mathcal{B} = 90$ MeV/fm^3.

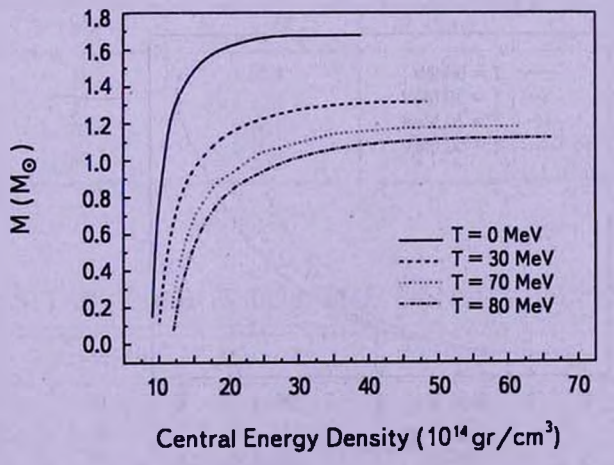

Fig.8. As Fig.7 but for the density dependent $\mathcal{B}$.

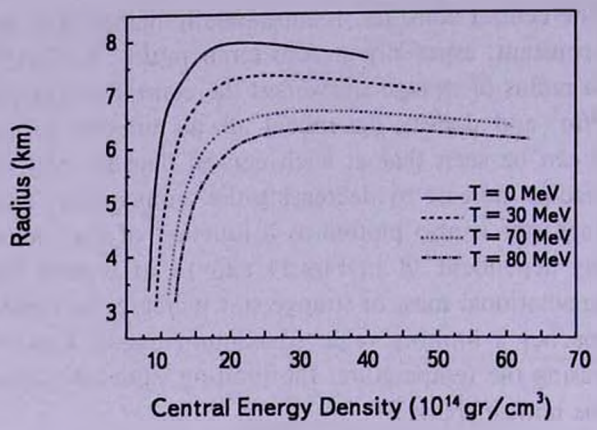

Fig.9. The radius of the strange star as a function of the central energy density at different temperatures for $\mathcal{B} = 90$ MeV/fm^3.

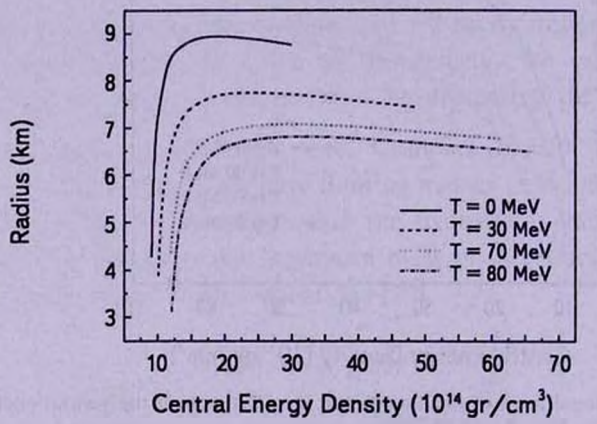

Fig.10. As Fig.9 but for the density dependent $\mathcal{B}$.

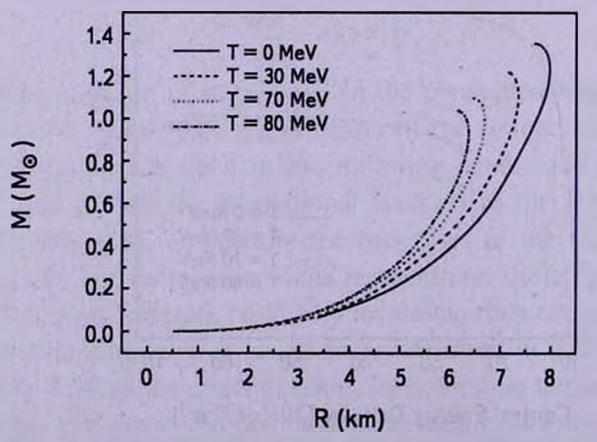

Fig.11. The gravitational mass of the strange star as a function of the radius at different temperatures for $\mathcal{B} = 90$ MeV/fm^3.

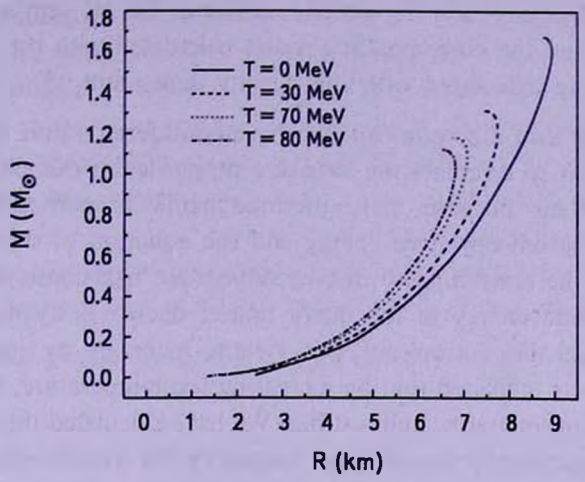

Fig.12. As Fig.11 but for the density dependent $\mathcal{B}$.

In Tables 1 and 2, the maximum mass and the corresponding radius and central energy density of the strange star at different temperatures for two cases of the constant and density dependent $\mathcal{B}$ are given. It is shown that by decreasing the temperature, the maximum mass of strange star increases. This behavior is also seen for the radius of strange star versus the temperature. Meanwhile, the central energy density decreases by decreasing the temperature.

Table 1

MAXIMUM MASS M_{max} IN SOLAR MASS UNIT $M_\odot$ AND THE CORRESPONDING RADIUS R AND CENTRAL ENERGY DENSITY ε_c OF THE STRANGE STAR AT DIFFERENT TEMPERATURES T FOR $\mathcal{B} = 90$ MeV/fm^3

T (MeV)	M_{max} ($M_\odot$)	R (km)	ε_c (10^{14} gr/cm^3)
0	1.354	7.698	38.24
30	1.228	7.073	47.54
70	1.101	6.416	60.60
80	1.039	6.142	63.65

Table 2

AS TABLE 1 BUT FOR THE DENSITY DEPENDENT $\mathcal{B}$

T (MeV)	M_{max} ($M_\odot$)	R (km)	ε_c (10^{14} gr/cm^3)
0	1.676	8.761	39.11
30	1.341	7.442	48.47
70	1.181	6.768	61.56
80	1.122	6.567	64.21

By comparing Tables 1 and 2, we can see that for all temperatures, the maximum mass and the corresponding radius calculated with the constant $\mathcal{B}$ are less than those calculated with the density dependent $\mathcal{B}$.

4. *Summary and Conclusion*. We have considered a pure quark matter for the strange star to calculate the structure properties of this object at finite temperature. For this purpose, some thermodynamic properties of the quark matter such as the entropy, free energy and the equation of state have been computed using the constant and density dependent bag constant $\mathcal{B}$. It was shown that the free energy of the quark matter decreases by increasing the temperature while the entropy of this system increases by increasing the temperature. It was indicated that by increasing the temperature, the equation of state of the quark matter becomes stiffer. We have calculated the gravitational mass of the strange star by numerically integrating the Tolman-Oppenheimer-Volkoff equations. Our results show that the gravitational mass of the strange star increases by increasing the central energy density. It was shown that this gravitational mass reaches a limiting value (maximum mass) at higher values of the central energy density. We have found that the maximum mass of the strange star decreases by increasing the temperature. It was also shown that the maximum mass and radius of the strange star in the case of density dependent $\mathcal{B}$ are higher than those in the case of constant $\mathcal{B}$.

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ВЫЧИСЛЕНИЕ ИНТЕГРАЛЬНЫХ ПАРАМЕТРОВ СТРАННОЙ ЗВЕЗДЫ С ИСПОЛЬЗОВАНИЕМ ФОРМАЛИЗМА ПРОБЛЕМЫ МНОГИХ ТЕЛ

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Мы рассмотрели сразу после коллапса сверхновой вещество горячей странной звезды, состоящее из странных, up and down кварков. и вычислили характеристики этой системы при конечной температуре с использованием зависящей от плотности постоянной кваркового мешка. Мы использовали

для параметризации зависящей от плотности постоянной кваркового мешка результаты, полученные нами при рассмотрении асимметричной ядерной материи в низшем порядке связанного вариационного метода. Наши вычисления структурных характеристик странной звезды при различных температурах показывают,что максимальная масса убывает с увеличением температуры. Мы сравнили также наши результаты с результатами, полученными в модели с неизменной постоянной кваркового мешка. Зависящяя от плотности постоянная кваркового мешка приводит к большим значениям максимальной массы и радиуса странной звезды.

Ключевые слова: *странная звезда:уравнение состояния:структура*

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