АСТРОФИЗИКА

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A MORE EXACT EXPRESSION FOR THE GRAVITATIONAL DEFLECTION OF LIGHT, DERIVED USING MATERIAL MEDIUM APPROACH

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The deflection of a ray of light passing close to a gravitational mass, is generally calculated from the null geodesic which the light ray (photon) follows. However, there is an alternate approach, where the effect of gravitation on the ray of light is estimated by considering the ray to be passing through a material medium. Calculations have been done in this paper, following the latter approach, to estimate the amount of deflection due to a static non-rotating mass. The refractive index of such a material medium, has been calculated in a more rigorous manner in the present work and the final expression for the amount of deflection calculated here is claimed to be more exact than all other expressions derived so far, using material medium approach. Based on this expression, the amount of deflection for a sun grazing ray has been also calculated. The exact amount of deflection can be performed in a number of ways, without the material medium approach. However, the method presented here using material medium approach and without any weak field approximation, is believed to be original.

Key words: *gravitational deflection of light:material medium*

1. *Introduction*. The gravitational deflection of light is one of the important predictions of the General Theory of Relativity (GTR) proposed by Einstein, which plays a key role in understanding problems related to Astronomy, Cosmology, Gravitational Physics and other related branches.

Newtons theory of universal gravitation had already predicted that the path of any material particle moving at a finite speed is affected by the pull of gravity. By the late 18th century, it was possible to apply Newtons law to compute the deflection of light by gravity. Cavendish commented briefly on the gravitational deflection of light in the late 1700s and Soldner gave a detailed derivation in 1801.

The idea of bending of light was revived by Einstein in 1911 and the quantitative prediction for the amount of deflection of light passing near a large mass (M) was found identical to the old Newtonian prediction, $d = 2GM/\left(c^2 r_\odot\right)$, where $r_\odot$ is the closet distance of approach and in this case approximately the solar radius. It was not until late in 1915, as Einstein completed the general theory, he realized his earlier prediction was incorrect and the angular deflection should actually be twice the size he predicted in 1911. This was subsequently confirmed by Eddington in 1919 through an experiment performed during the solar eclipse.

The exact amount of defleection for a ray of light passing close to a gravitational mass can be worked out from the null geodesic, which a ray of light follows [1,2,3]. Such expressions for bending generally involve Elliptical Integrals and were first given by Darwin in 1959 [4].

The deflection of a light ray passing close to a gravitational mass can be alternately calculated by following an approach, where the effect of gravitation on the light ray is estimated by considering the light ray to be passing through a material medium. However, the value of the refractive index of that medium is decided by the strength of gravitational field [5].

The concept of this equivalent material medium was discussed by Balazs [6] as early as in 1958, to calculate the effect of a rotating body, on the polarization of an electromagnetic wave passing close to it. Plebanski [7] had also utilized this concept in 1960, to study the scattering of a plane electromagnetic wave by gravitational field. The author also mentioned that this concept of equivalent material medium was first pointed out by Tamm [8] in 1924. Atkinson [9] investigated the allowed trajectories of light rays near a massive star and obtained an expression for velocity of light at an arbitrary point for cases, when the light ray is travelling both radially and tangentially to the field. This phenomena admits a splitting of a light ray into two rays in an anisotropic inhomogenous medium with spherical symmetry [10]. A general procedure for utilizing this concept, for deflection calculation has been worked out by Felice [11]. Later this concept was also used by Mashoon [12,13], to calculate the deflection and polarization due to the Schwarzschild and Kerr black holes. Fischbach and Freeman [14], derived the effective refractive index of the material medium and calculated the second order contribution to the gravitational deflection. In a similar way Sereno [15] has used this idea for gravitational lensing calculation by drawing the trajectory of the ray by Fermat's principle. More recently Ye and Lin [16], emphasized the simplicity of this approach and calculated the gravitational time delay and the effect of lensing.

On the other hand, the calculation of higher order deflection terms, due to Schwarzschild Black hole, from the null geodesic, has been performed recently by Iyer and Petters ([17] and references their in). Using null geodesics, gravitational lensing calculations have been done by a number of authors in past [18,19]. More recently, for strong gravitational field, lensing calculations have been done by Bisnovatyi-Kogan and Tsupko [20].

With the above background, in the present work, we follow the material medium approach, to calculate a more accurate expression for the deflection term due to a non-rotating sphere (Schwarzschild geometry). It is claimed that the present expression will be more accurate than all other expression calculated in past, using material medium concept.

2. *The effective refractive index and the trajectory of light ray.* As discussed earlier, the gravitational field influences the propagation of electromagnetic radiation by imparting to the space an effective index of refraction $n(r)$ [4].

For a static and spherically symmetric gravitational field, the solution of Einstein's Field Equation was given by K.Schwarzschild in 1961, which is as follows [4]:

$$ds^2 = \left(1 - \frac{r_g}{r}\right)c^2 dt^2 - r^2\left(\sin^2\theta d\phi^2 + d\theta^2\right) - \frac{dr^2}{\left(1 - \frac{r_g}{r}\right)}, \tag{1}$$

where $r_g = 2km/c^2$ called Schwarzschild Radius, which completely defines the gravitational field in vacuum produced by any centrally-symmetric distribution of masses. The above line element can be expressed in an isotropic form by introducing a new radius co-ordinate (ρ) with the following transformation equation [4]

$$\rho = \frac{1}{2}\left[\left(r - \frac{r_g}{2}\right) + r^{1/2}\left(r - r_g\right)^{1/2}\right] \tag{2}$$

or

$$r = \rho\left(1 + \frac{r_g}{4\rho}\right)^2. \tag{3}$$

The resulting isotropic form of Schwarzschild equation will be now:

$$ds^2 = \left(\frac{1 - r_g/4\rho}{1 + r_g/4\rho}\right)^2 c^2 dt^2 - \left(1 + \frac{r_g}{4\rho}\right)^4 \left(d\rho^2 + \rho^2\left(\sin^2\theta d\phi^2 + d\theta^2\right)\right). \tag{4}$$

Now in spherical co-ordinate system the quantity $(d\rho^2 + \rho^2(\sin^2\theta d\phi^2 + d\theta^2))$ has the dimension of square of infinitesimal length vector $d\vec{\rho}$.

By setting $ds = 0$, the velocity of light can be identified from the expression of the form $ds^2 = f(\rho)dt^2 - d\vec{\rho}^2$, as $\upsilon(\rho) = \sqrt{f(\rho)}$. Therefore the velocity of light in the present case (characterized by Schwarzschild radius r_g) can be expressed as:

$$\upsilon(\rho) = \frac{(1 - r_g/4\rho)\, c}{(1 + r_g/4\rho)^3}. \tag{5}$$

But this above expression of velocity of light is in the unit of length ρ per unit time. We therefore write

$$\upsilon(r) = \upsilon(\rho)\frac{dr}{d\rho} = \upsilon(\rho)\left[\left(1 + \frac{r_g}{4\rho}\right)^2 - \frac{r_g}{2\rho}\left(1 + \frac{r_g}{4\rho}\right)\right] = \left(\frac{r_g - 4\rho}{r_g + 4\rho}\right)^2 c. \tag{6}$$

Substituting the value of ρ from Eqn. (2) in Eqn. (6), we get:

$$\upsilon(r)=\left(\frac{r_g/2-2\rho}{r_g/2+2\rho}\right)^2 c=\left(\frac{r_g/2-\left(\left(r-r_g/2\right)+r^{1/2}\left(r-r_g\right)^{1/2}\right)}{r_g/2+\left(\left(r-r_g/2\right)+r^{1/2}\left(r-r_g\right)^{1/2}\right)}\right)^2 c=$$
$$=\left(\frac{r_g-r-r^{1/2}\left(r-r_g\right)^{1/2}}{r+r^{1/2}\left(r-r_g\right)^{1/2}}\right)^2 c=\frac{c\left(r-r_g\right)}{r}. \tag{7}$$

Therefore the refractive index $n(r)$ at a point with spherical polar coordinate (r), can be expressed by the relation:

$$n(r)=\frac{c}{\upsilon(r)}=\frac{r}{r-r_g}. \tag{8}$$

It is evident that at a very large distance from the spherical distribution of mass where space is Euclidian, this value of the refractive index reduces to 1 and at the Schwarzschild radius becomes infinity. This is a physically consistent situation. Here we note that the values of refractive index derived following either Atkinson [9] or Fischback and Freeman [14] lead to an expression in terms of some infinite converging series. Fischback and Freeman [14] estimated light deflection by a massive object by truncating the series at some stage, whereas no deflection values were calculated with Atkinson's refrcative index values [9].

In this paper we consider the problem of calculation of gravitational bending to be a problem of geometrical optics, where we have to find the trajectory of a light ray travelling in a medium, whose refractive index has spherical symmetry. The expression for refractive index obtained here (as in Eqn.(8)) is simple. Here the trajectory of the light ray and the center of mass (source of gravitational potential) together define a plane. The equation of such a ray in a plane polar co-ordinate system (r,θ) can be written as [21]:

$$\theta=A\int_{r_\odot}^{\infty}\frac{dr}{r\sqrt{n^2r^2-A^2}}. \tag{9}$$

The trajectory is such that $n(r)d$ always remains a constant, where d is the perpendicular distance between the trajectory of the light ray from the origin and the constant is taken here as A [21]. In our present problem the light is approaching from asymptotic infinity $(r=-\infty)$ to the gravitational mass, which is placed at the origin and characterized by Schwarzschild radius r_g. The closest distance of approach for the approaching ray is b and the ray goes to $r=\infty$, after undergoing certain amount of deflection $(\Delta\phi)$, by the presence of Schwarzschild mass.

Here, the parameter b can be replaced by solar radius $r_\odot$. When the light ray passes through the closest distance of approach (ie $r=b$ or $r_\odot$), the tangent to the trajectory becomes perpendicular to the vector $\vec{r}$ (which is $\vec{r}_\odot$). Therefore, we can write $A=n(r_\odot)r_\odot$. The trajectory of the light ray had been already constructed before like this, by Ye and Lin [16] and the value of deflection ($\Delta\phi$), can be written as:

$$\Delta\phi = 2\int_{r_\odot}^{\infty} \frac{dr}{r\sqrt{\left(\frac{n(r)r}{n(r_\odot)r_\odot}\right)^2 - 1}} - \pi . \tag{10}$$

However, Ye and Lin [16], had in our opinion used a value of refractive index $n(r)$ which was approximated and somewhat ad hoc. Fischbach and Freeman [14] also in their attempt to calculate a more accurate value of deflection, considered terms only up to second order in the expression for refractive index. However, in our attempt to do so we shall avoid making any such approximation in the following. We denote the above integral in Eqn. (10) by I and write

$$I = \int_{r_\odot}^{\infty} \frac{dr}{r\sqrt{\left(\frac{n(r)r}{n(r_\odot)r_\odot}\right)^2 - 1}} = n(r_\odot)r_\odot \int_{r_\odot}^{\infty} \frac{dr}{r\sqrt{(n(r)r)^2 - (n(r_\odot)r_\odot)^2}} =$$
$$= n(r_\odot)r_\odot \int_{r_\odot}^{\infty} \frac{dr}{r\sqrt{\frac{r^4}{(r-r_g)^2} - \frac{r_\odot^4}{(r_\odot - r_g)^2}}} = n(r_\odot)r_\odot \int_{r_\odot}^{\infty} \frac{dr}{r^2\sqrt{\frac{1}{(1-r_g/r)^2} - \frac{r_\odot^2 r^{-2}}{(1-r_g/r_\odot)^2}}} . \tag{11}$$

Now we change the variable to $x = r_g/r$ and introduce a quantity $a = r_g/r_\odot$. We also denote $n(r_\odot)$ by $n_\odot$. Accordingly we write:

$$I = n_\odot r_\odot \int_a^0 \frac{-x^{-2}r_g dx}{r^2\sqrt{\frac{1}{(1-x^2)} - \frac{x^2}{(a(1-a))^2}}} = n_\odot r_\odot \int_a^0 \frac{-x^{-2}r_g dx}{xr^2\sqrt{\frac{1}{(x(1-x))^2} - \frac{1}{(a(1-a))^2}}} =$$
$$= \frac{n_\odot r_\odot}{r_g} \int_0^a \frac{dx}{x\sqrt{\frac{1}{(x(1-x))^2} - \frac{1}{(a(1-a))^2}}} = \frac{n_\odot r_\odot}{r_g} \int_0^a \frac{(1-x)dx}{\sqrt{1 - \frac{(x(1-x))^2}{(a(1-a))^2}}} . \tag{12}$$

For our convenience we can denote the quantity $1/(a(1-a))$ by D. This also implies

$$D = \frac{r_\odot^2}{r_g(r_\odot - r_g)} . \tag{13}$$

However, the Integral I is not simple to integrate into know form. We split the above Integral, as a sum of two Integrals and proceed as follows:

$$I = \left(\frac{n_{\odot} r_{\odot}}{r_g}\right)\left[\int_0^a \frac{(1-2x)dx}{\sqrt{1-D^2x^2(1-x)^2}} + \int_0^a \frac{xdx}{\sqrt{1-D^2x^2(1-x)^2}}\right] =$$

$$= \left(\frac{n_{\odot} r_{\odot}}{r_g}\right)\int_0^a \frac{(1-2x)dx}{\sqrt{1-D^2x^2(1-x)^2}} + \left(\frac{n_{\odot} r_{\odot}}{r_g}\right)\int_0^a \frac{xdx}{\sqrt{1-D^2x^2(1-x)^2}} = \tag{14}$$

$$= \left(\frac{n_{\odot} r_{\odot}}{r_g}\right) I_1 + \left(\frac{n_{\odot} r_{\odot}}{r_g}\right) I_2 ,$$

where $I_1 = \int_0^a \frac{(1-2x)dx}{\sqrt{1-D^2x^2(1-x)^2}}$ and $I_2 = \int_0^a \frac{xdx}{\sqrt{1-D^2x^2(1-x)^2}}$ are two integrals respectively. Now it is simple to identify

$$\frac{n_{\odot} r_{\odot}}{r_g} = \frac{1}{1-a} \cdot \frac{1}{a} = \frac{1}{a(1-a)} = D$$

Changing the variable from x to $y = Dx(1-x)$, we can write $D(1-2x)dx = dy$. Accordingly the upper and lower limits $x=0$ and $x=a$ change to $y=0$ and $y = Da\left(1 - \frac{r_g}{r_{\odot}}\right) = \frac{1}{a(1-a)} a(1-a) = 1$. Therefore for the first part in Eqn. (14) we can write:

$$\left(\frac{n_{\odot} r_{\odot}}{r_g}\right) I_1 = \int_0^a \frac{D(1-2x)dx}{\sqrt{1-D^2x^2(1-x)^2}} = \int_0^1 \frac{dy}{\sqrt{1-y^2}} = \left[\sin^{-1} y\right]_0^1 = \frac{\pi}{2}. \tag{15}$$

Therefore, from Eqn. (10), one may write the amount of deflection as:

$$\Delta\phi = 2\int_{r_{\odot}}^{\infty} \frac{dr}{r\sqrt{\left(\frac{n(r)r}{n(r_{\odot})r_{\odot}}\right)^2 - 1}} - \pi = 2\left(\frac{n_{\odot} r_{\odot}}{r_g}\right) I_1 + 2\left(\frac{n_{\odot} r_{\odot}}{r_g}\right) I_2 - \pi =$$

$$= \pi + 2\left(\frac{n_{\odot} r_{\odot}}{r_g}\right) I_2 - \pi = \left(\frac{2 n_{\odot} r_{\odot}}{r_g}\right)\int_0^a \frac{xdx}{\sqrt{1-D^2x^2(1-x)^2}}. \tag{16}$$

Thus the gravitational bending for a ray of light grazing the static gravitational mass (with Schwarzschild radius r_g) with the closest distance of approach $r_{\odot}$ can be expressed as:

$$\Delta\phi = 2D\int_0^a \frac{xdx}{\sqrt{1-D^2x^2(1-x)^2}} \tag{17}$$

The above expression for gravitational deflection has been obtained from the Schwarzschild Equation (Eqn(1)), without applying any approximation at any stage. Owing to this, it is claimed that this expression of bending is more exact as compared to all other expressions derived till today, using equivalent material medium concept. However,the integration of the quantity in Eqn.

(17)), involves some complicated algebraic expressions containing elliptical functions. Using mathematica, we obtain the following expression after integration:

$$\int \frac{xdx}{\sqrt{1-D^2x^2(1-x)^2}} = 2\frac{\left(\sqrt{D}+\sqrt{D-4}\right)E-\left(2\sqrt{D-4}\right)F}{D\left(\sqrt{D+4}-\sqrt{D-4}\right)}, \tag{18}$$

where $E \equiv E\left(p, q^2\right)$ is the elliptic Integral of first kind and $F \equiv F\left(-q, p, q^2\right)$ is incomplete elliptic integral of Third kind. The arguments $p, q^2, -q, p, q^2$ are expressed by the following mathematical relations:

$$p = \arcsin\sqrt{\frac{\left(\sqrt{D-4}-\sqrt{D+4}\right)\left(\sqrt{D-4}+(2x-1)\sqrt{D}\right)}{\left(\sqrt{D-4}+\sqrt{D+4}\right)\left(\sqrt{D-4}-(2x-1)\sqrt{D}\right)}}, \tag{19}$$

$$q = \frac{\left(\sqrt{D-4}+\sqrt{D+4}\right)}{\left(\sqrt{D-4}-\sqrt{D+4}\right)}. \tag{20}$$

Finally we can write the expression for gravitational deflection ($\Delta\phi$) of the light ray, due to a static mass r_g with the closest distance of approach $r_\odot$ as:

$$\Delta\phi = 4\left\{\frac{\left(\sqrt{D}+\sqrt{D-4}\right)E-\left(2\sqrt{D-4}\right)F}{\left(\sqrt{D+4}-\sqrt{D-4}\right)}\right\}_{x=0}^{x=a}, \tag{21}$$

where the value of D is given by Eqn.(13) as $D = \dfrac{r_\odot^2}{r_g\left(r_\odot - r_g\right)}$ and $a = r_g/r_\odot$. Eqn. (21) is a general expression for bending of light, where $r_\odot$ can be replaced by the closest distance of approach of the light ray. This mathematical expression for deflection, derived here is claimed to be more accurate than all other expressions derived so far using material medium approach and it is equally valid for strong field. However, it may be noted that, the exact calculation of light ray deflection near the Sun can be performed in a number of ways, using well-known formulas and without the material medium approach. But the method presented here using material medium approach and without any weak field approximation, is believed to be original.

For a Sun grazing ray, as a test case, we can take the closest distance of approach as equal to solar radius which is $r_\odot = 695.500$ km and Schwarzschild radius corresponding to the mass of Sun as $r_g = 3$ km. We, therefore, get $a = \left(r_g/r_\odot\right) = 1/231.833$ and $D = 231.834$. Finally, we get a value of $\Delta\phi = 8.62690E10^{-6}$ radians or 1.77943 arc sec. However, by substituting more accurate values for $r_\odot$ and r_g, we can get more exact value for the amount of deflection for a sun grazing ray.

3. *Comparison with other expressions for deflections.* The expression obtained in the present work can be compared with the deflection

expressions obtained by other authors with material medium approach.

Fischback and Freeman [14] had calculated the second order terms in gravitational deflection of light, using material medium approach. The authors used an infinite convergent series $(1 + A/r + B/r^2 + \ldots)$ for refractive index $n(r)$, where $A = r_g$ (Schwarzschild radius) and B is some function of r_g. By considering only the first order term in $n(r)$ i.e. $n(r) = 1 + A/r$, the authors calculated the first order term in deflection, which is $2r_g/r_\odot^2$ or $4GM/(c^2 r_\odot)$.

In the present case, the refractive index $n(r)$ has been evaluated to be $r/(r - r_g)$ (cf. Eqn. (8)) and it can be expressed by the infinite converging series:

$$n(r) = 1 + a + a^2 + a^3 + \ldots \tag{22}$$

where $a = r_g/r_\odot$. This shows the refractive index expression used in Fischback and Freeman [14] and in the present work are the same in weak field limit.

As mentioned in Sec 1, Ye and Lin [16], had also calculated first order deflection term and the refractive index value they used was $n(r) = \exp(2GM/(rc^2))$. This expression for refractive index is also same as what has been derived in the present work, for weak field limit (considering terms up to first order). However, these authors did not derive higher order terms for gravitational deflections.

From the present work, it is clear that when one takes $n(r) = 1$, space is flat and there will be no deflection. This is true when we substitute $n(r) = 1$ in Eqn. (10), we get $\Delta\phi = 0$. The expression that has been derived in the present work, should always coincide to the well known general expression for bending in the weak field limit, which is $\Delta\phi = 4GM/(c^2 r_\odot)$. In the present work, the weak field value of refractive index is $n(r) = 1 + a$ and under weak field we can also write $1/D = a(1 - a) \sim a$ and $x(1 - x) \sim x$, as x, $a << 1$. By substituting, these weak field approximations, into the final integral expression of bending derived here (cf. Eqn. (17)), one gets:

$$\Delta\phi = 2D\int_0^a \frac{x\,dx}{\sqrt{1 - D^2x^2(1-x)^2}} \sim 2\int_0^a \frac{x\,dx}{\sqrt{a^2 - x^2}} = 2a = 2\frac{r_g}{r_\odot} = \frac{4GM}{c^2 r_\odot}. \tag{23}$$

This confirms the expression for deflection derived here for strong field coincides to the standard expression under weak field limit. In the present work, the quantity D was used (in the equations from (14) to (21)) as a substitution for $1/(a(1 - a))$. Now, D can be expressed by the converging series:

$$D = 1/a + 1 + a + a^2 + a^3 + \ldots \tag{24}$$

Therefore, in the weak field limit in order to calculate the value of deflection, we can substitute $D = 1/a$ in Eqn. (21), instead of $D = 1/(a(1 - a))$. Thus by making the above approximations $D \sim 1/a$ and $a << 1$, into Eqn. (21), as and when appropriate, we get:

$$\Delta\phi = 4\left\{\frac{\left(\sqrt{D} + \sqrt{D-4}\right)E - \left(2\sqrt{D-4}\right)F}{\left(\sqrt{D+4} - \sqrt{D-4}\right)}\right\}_{x=0}^{x=a} \sim (2/a)\{E - F\}_{x=0}^{x=a}. \tag{25}$$

With the same weak field approximation for the quantities in in Eqns. (19) and (20), we obtain $p \sim \arcsin\left(\sqrt{2a}\right)$ at $x=0$, $p \sim 0$ at $x=a$ and $q=-1/(2a)$. As a result we get under weak field limit:

$$\Delta\phi = -(2/a)\left(E\left(\arcsin\left(\sqrt{2}\,a\right), 1/\left(4a^2\right)\right) - F\left(1/2a\right), \arcsin\left(\sqrt{2}\,a\right), 1/\left(4a^2\right)\right), \quad (26)$$

where E is the elliptic Integral of first kind and F is Incomplete elliptic Integral of Third kind, as discussed earlier. Now by substituting the numerical value of $a = r_g/r_\odot$ we can get the weak field deflection value from Eqn. 26.

Thus the exact expression derived in the present work, matches exactly with the expressions derived by other earlier work under weak field limit.

As discussed in Section1, the higher order deflection terms have been also evaluated under strong field, by using methods other than material medium approach.

Darwin [22], had calculated first order strong field deflection term, using logarithmic series. Iyer and Petters [17] had calculated the deflection term in the strong field, with an expression containing complete and incomplete elliptical integrals. From this general expression, the authors could calculate the first order strong deflection term, calculated earlier by Darwin [22]. It may be worth to mention that, the strong field deflection term calculated by Iyer and Petters [17] and the one derived in the present work, both contain same linear combination of elliptical Integrals of same kind.

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БОЛЕЕ ТОЧНОЕ ВЫРАЖЕНИЕ ДЛЯ ГРАВИТАЦИОННОГО ОТКЛОНЕНИЯ СВЕТА, ПОЛУЧЕННОЕ С ИСПОЛЬЗОВАНИЕМ ПОДХОДА МАТЕРИАЛЬНОЙ СРЕДЫ

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Отклонение луча света при прохождении близко от гравитационной массы обычно вычислялось от нулевой геодезической линии, по которой распространяется световой луч (фотон). Однако имеется альтернативный подход, когда влияние гравитации на луч света оценивается, рассматривая прохождение луча через материальную среду. В настоящей статье вычисления по последнему методу проведены для оценки величины отклонения, вызванного невращающейся статической массой. Индекс рефракции такой материальной среды вычислен более строгим способом и полученное здесь окончательное выражение для величины отклонения претендует быть более точным, чем все другие величины, полученные до сих пор с использованием подхода материальной среды. На основании этого выражения вычислена также величина отклонения луча, падающего под скользящим углом к Солнцу. Точная величина отклонения может быть получена разными способами без применения подхода материальной среды. Однако представленный здесь метод с использованием подхода материальной среды, причем без какого-то приближения слабого поля, кажется оригинальным.

Ключевые слова: *гравитационное отклонение света:материальная среда*

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