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HIGHER-DIMENSIONAL ANISOTROPIC MODIFIED HOLOGRAPHIC RICCI DARK ENERGY COSMOLOGICAL MODEL IN LYRA MANIFOLD

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We used modified holographic Ricci dark energy to find anisotropic LRS Bianchi type I cosmological model in five-dimensions based on Lyra geometry. The exact solutions of the Einstein field equations were obtained by using hybrid expansion law (HEL). We have investigated the interacting and non-interacting dark energy and dark matter. It is found that at late times the equation of state parameter (EOS) for non-interacting case behaves like a cosmological constant whereas it behaves like phantom dark energy for interacting model. Some cosmological parameters and stability of the models are discussed. The physical and geometric aspects of the models have been analysed. They are consistent with the recent observational results.

Keywords: *LRS Bianchi type I space-time*: *Lyra geometry*: *modified holographic Ricci dark energy*: *hybrid expansion law*

1. *Introduction*. Supernova observations (SNeIa) point towards an accelerated expansion of the universe and also various astrophysical observational evidences have decided the fact that the universe experiences an early inflation as well as late-time accelerated expansion [1–4]. In modern cosmology it believed that this is caused by a mysterious form of energy termed as dark energy (DE) [5] with positive energy density and negative pressure. The simplest candidate for dark energy is the cosmological constant Λ with the equation of state $\omega = -1$. This includes the hurdle of fine-tunning and cosmic coincidence [6,7]. Further investigations reveal that there are other types of DE, such as quintessence [8], phantom [9,10], tachyon [11], dilation [12] with interacting dark energy models like holographic [13] and agegraphic [14] models. Also, the cosmic viscosity is successful in achieving to play a role of dark energy candidate causing the acceleration of the universe [15-17]. From the observational fact of Wilkinson Microwave Anisotropy Probe (WMAP) it is found that the dark energy occupies 68.3%, the dark matter occupies 26.8% of the total energy of the universe and the rest 4.9% energy is baryonic matter [18].

The exact physical situation at very early stages of the formation of our Universe is still unknown. The investigation of higher-dimensional space-time is important because it is believed that the cosmos at the early stage of evolution of the universe might have had a higher dimensional era. A fifth dimension is introduced by Kaluza and Klein [19,20] to unify gravity with electromagnetic interaction. In view of Chodos and Detweller's [21] investigations, the present four-dimensional stage of the universe could have been preceded by a higher-dimensional stage, which becomes four-dimensional in the sense that extra dimensions contract to unobserved planckian length scale due to dynamical contraction. This contraction of the extra dimension is a result of cosmological evolution. In view of the development of superstring theory and supergravity theory, higher dimensional physics acquires a new degree of emphasis. A good number of interior and exterior solutions of Einstein's equation in higher dimensions have been derived by Yoshimura [22], Koikawa [23], Myers and Perry [24] and Krori et al. [25]. Reddy and Venkateswara [26], Adhav et al. [27], Reddy [28] and a host of authors have contribution to the study of higher dimensional cosmological models in general relativity and also in other alternate theory of gravitation. The higher-dimensional anisotropic DE cosmological models play an important role in the study of early stages of evolution of the universe.

Holographic dark energy model is emerging from the holographic principle which was first presented by 't Hooft [29]. This principle states that the entropy of a system scales not with its volume, but also with its surface area. The energy density of holographic dark energy is $\rho_{HDE} = 3c^2 M_{pl}^2 L^{-2}$, where L is the infrared (IR) cut off radius, $M_{pl}^2 = 1/8\pi G$ is the Planck mass and c is constant according to Li et al. [30]. Gao et al. [31] obtained a holographic dark energy model. In this model the future event horizon is replaced by the inverse of the Ricci scalar curvature. This model is "Ricci Dark Energy" (RDE) model. A new holographic Ricci dark energy was proposed by Granda and Oliveros [32,33] with the density of dark energy as $\rho_{HDE} = 3M_{pl}^2\left(\eta_1 H^2 + \eta_2 \dot{H}\right)$. In 2009, Chen and Jing [34] modified this model as $\rho_{MHRDE} = 3M_{pl}^2\left(\eta_1 H^2 + \eta_2 \dot{H} + \eta_3 \ddot{H}H^{-1}\right)$. Katore et al. [35], Kumar and Yadav [36] and a considerable number of researchers have studied cosmological models with anisotropic dark energy.

Several researchers were inspired by Einstein's geometrization of gravitation in his theory of general relativity to geometrize other physical fields. Weyl [37] proposed a unified theory to geometrize gravitation and electromagnetism. But this theory was not considered as it was depended on non-integrability of length transfer. Lyra [38] suggested a modification by introducing a gauge function into the structure less manifold which removes the non-integrability condition of the length of a vector under parallel transport. Sen [39] and Sen and Dunn [40] suggested a new scalar-tensor theory of gravitation and constructed an analogue of the Einstein field equations based on Lyra's geometry, which in normal gauge may be written as

$$R_{ij}-\frac{1}{2}g_{ij}R+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}g_{ij}\,\phi_k\phi^k=-\chi T_{ij}\,, \tag{1}$$

where ϕ_i is the displacement vector, $c=1$ and $8\pi G=\chi$ and other symbols have their usual meaning in the Riemannian geometry. A brief note on Lyra's geometry is given by Beesham [41] and Singh and Singh [42]. According to Halford [43], ϕ_i the displacement vector in Lyra geometry plays the role of cosmological constant in the normal general relativistic treatment. Rahman et al. [44,45] presented cosmological models in Lyra geometry. Singh and Desikan [46] presented the exact solutions for FRW cosmological model in Lyra's geometry with constant deceleration parameter. These theories are the modified theories of the gravitation or alternate theory of gravitation. The accelerating expansion of the universe can be explained in the context of these modified theories of gravitation. Motivated by the above investigationswe study here a higher dimensional (5D) cosmological model in one of these modified theories of gravitation, i.e. Lyra geometry. It is of great significance at the early stage of the universe.

The paper is organized as follows: The metric and field equations are given in Section 2. The solution of the field equations are presented in Section 3. In Section 4, some physical and geometrical representations of the model are discussed. The stability of models, the cosmic parameter and statefinder diagnostic parameters are discussed respectively in Section 5, Section 6 and Section 7. In Section 8, results and discussions of various parameters are discussed. The paper is devoted to some concluding remarks in Section 9.

2. *The metric and field equations*. The 5-D Bianchi type I metric is given by

$$ds^2=dt^2-X^2\,dx^2-Y^2\left(dy^2+dz^2\right)-Z^2\,d\psi^2\,, \tag{2}$$

where X, Y and Z are functions of cosmic time t.

The field equations based on Lyra manifold in normal gauge as proposed by Sen [39] and Sen and Dunn [40] is

$$R_{ij}-\frac{1}{2}g_{ij}\,R+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}g_{ij}\,\phi_k\phi^k=-\left(T_{ij}+\overline{T_{ij}}\right), \tag{3}$$

where ϕ_i is the displacement vector and is defined as

$$\phi_i=\left(\beta(t),0,0,0,0\right) \tag{4}$$

and in geometrised unit $8\pi G=1$, $c=1$, R_{ij} is the Ricci tensor, R is the Ricci scalar, T_{ij} and $\overline{T_{ij}}$ are the energy momentum tensors for dark matter (pressure less) and MHRDE. T_{ij} and $\overline{T_{ij}}$ defined as follows

$$T_j^i=\mathrm{diag}\left[\rho_{MA},0,0,0,0\right], \tag{5}$$

$$\overline{T_j^i} = \text{diag}\left[\rho_{MHRDE}, -p_{MHRDE_x}, -p_{MHRDE_y}, -p_{MHRDE_z}, -p_{MHRDE_\psi}\right]$$
$$= \text{diag}\left[1, -\omega_x, -\omega_y, -\omega_z, -\omega_\psi\right]\rho_{MHRDE} \tag{6}$$
$$= \text{diag}\left[1, -\omega_{MHRDE}, -(\omega_{MHRDE}+\delta), -(\omega_{MHRDE}+\delta), -\omega_{MHRDE}\right]\rho_{MHRDE},$$

where ρ_{MA} is the energy density of the dark matter and ρ_{MHRDE} is the energy density of the MHRDE and p_{MHRDE_x}, p_{MHRDE_y}, p_{MHRDE_z}, p_{MHRDE_ψ}, are the pressures on the x, y, z and ψ respectively. The skewness parameter δ is the deviations from ω_{MHRDE} on y and z axes. Here EOS parameter ω_{MHRDE} and skewness parameter δ can be functions of t.

The field equation (3) using Eqs. (4), (5) and (6) takes the form

$$2\frac{\dot{Y}\dot{Z}}{YZ} + 2\frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{Y}^2}{Y^2} + \frac{3}{4}\beta^2 = -\omega_{MHRDE}\rho_{MHRDE} \tag{7}$$

$$\frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + \frac{\ddot{Z}}{Z} + \frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{Z}\dot{X}}{ZX} + \frac{3}{4}\beta^2 = -(\omega_{MHRDE}+\delta)\rho_{MHRDE} \tag{8}$$

$$\frac{\ddot{X}}{X} + 2\frac{\ddot{Y}}{Y} + 2\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}^2}{Y^2} + \frac{3}{4}\beta^2 = -\omega_{MHRDE}\rho_{MHRDE} \tag{9}$$

$$2\frac{\dot{X}\dot{Y}}{XY} + 2\frac{\dot{Y}\dot{Z}}{YZ} + \frac{\dot{X}\dot{Z}}{XZ} + \frac{\dot{Y}^2}{Y^2} - \frac{3}{4}\beta^2 = \rho_{MA} + \rho_{MHRDE}\,. \tag{10}$$

The energy conservation equation is

$$\dot{\rho}_{MA} + \left(\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right)\rho_{MA} + \dot{\rho}_{MHRDE} + $$
$$+ (1+\omega_{MHRDE})\rho_{MHRDE}\left(\frac{\dot{X}}{X} + \frac{2\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right) + 2\frac{\dot{Y}}{Y}\delta\rho_{MHRDE} = 0\,, \tag{11}$$

where dot $(\cdot)$ denotes derivative with respect to the cosmic time t.

3. *Solutions of the field equations*. The spatial volume V is given by

$$V = XY^2Z = R^4\,, \tag{12}$$

where R is the average scale factor. The Hubble's parameter H is given by

$$H = \frac{1}{4}\frac{\dot{V}}{V} = \frac{1}{4}\left(H_x + H_y + H_z + H_\psi\right) = \frac{1}{4}\left(\frac{\dot{X}}{X} + \frac{2\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right), \tag{13}$$

where H_x, H_y, H_z and H_ψ are the directional Hubble parameters in the directions of x, y, z and ψ axes respectively.

The anisotropy parameter A_p is given by

$$A_p = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 = \frac{1}{4H^2}\left[\left\{\frac{\dot{X}}{X} - \frac{1}{4}\left(\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right)\right\}^2 + \right.$$
$$\left. + 2\left\{\frac{\dot{Y}}{Y} - \frac{1}{4}\left(\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right)\right\}^2 + \left\{\frac{\dot{Z}}{Z} - \frac{1}{4}\left(\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right)\right\}^2\right]. \tag{14}$$

Eq. (7) and Eq. (8) together yield

$$\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y} = \frac{d_1}{V}\exp - \left(\int \frac{XY\,\delta\rho_{MHRDE}}{\dot{X}Y - X\dot{Y}}\,dt\right), \tag{15}$$

where d_1 is a constant of integration. To solve the Eq. (15), we take (according to Adhav [47])

$$\delta\rho_{MHRDE} = \frac{\dot{X}}{X} - \frac{\dot{Y}}{Y}. \tag{16}$$

Using Eq. (16), Eq. (15) takes the form

$$\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y} = \frac{d_1}{V}e^{-t}. \tag{17}$$

Eqs. (7)-(10) are four field equations with seven unknowns X, Y, Z, β, ω_{MHRDE}, ρ_{MHRDE}, and ρ_{MA}. So, in order to solve the system completely we need extra three relations. We take the following relations (i), (ii) and (iii):

(i) Chen and Jing [34] proposed the MHRDE density as

$$\rho_{MHRDE} = 3\left(\eta_1 H^2 + \eta_2 \dot{H} + \eta_3 \ddot{H}H^{-1}\right), \tag{18}$$

where η_1, η_2 and η_3 are constants and $M_{pl}^2 = 8\pi G = 1$.

(ii) The average scale factor $R(t)$ which is a combination of power law and exponential law as proposed by Akarsu et al. [48] is given by

$$R(t) = R_p\left(\frac{t}{t_p}\right)^{a_1} e^{a_2\left(t/t_p - 1\right)}, \tag{19}$$

where a_1 and a_2 are non-negative constants and R_p and t_p represents the present value of scale factor and age of the universe.

The relation (19) is a combination of power and exponential law which is commonly known as Hybrid Expansion Law (HEL). And also

(iii) $$Z = \frac{1}{t^n}, \tag{20}$$

where $n > 0$ is a positive constant.

Eq. (12), on using Eq. (19) gives the spatial volume V of the model as

$$V = XY^2Z = R^4 = R_p^4\left(\frac{t}{t_p}\right)^{4a_1} e^{4a_2\left(t/t_p-1\right)}. \tag{21}$$

Eq. (20) and Eq. (21) yield

$$XY^2 = R_p^4\left(\frac{t}{t_p}\right)^{4a_1} t^n e^{4a_2\left(t/t_p-1\right)}. \tag{22}$$

Eq. (17) gives

$$X = Yd_2 \exp\left[\frac{d_1}{R_p^4}\int \frac{e^{-t}t_p^{4a_1}}{t^{4a_1}e^{4a_2\left(t/t_p-1\right)}}dt\right], \tag{23}$$

where d_2 is a constant of integration.

Eqs. (22) and (23) yield

$$Y = R_p^{4/3}\left(\frac{t}{t_p}\right)^{4a_1/3} e^{4a_2\left(t/t_p-1\right)/3} t^{n/3} d_2^{-1/3}\left\{\exp\left[\frac{d_1}{R_p^4}\int \frac{e^{-t}t_p^{4a_1}}{t^{4a_1}e^{4a_2\left(t/t_p-1\right)}}dt\right]\right\}^{-1/3} \tag{24}$$

$$X = R_p^{4/3}\left(\frac{t}{t_p}\right)^{4a_1/3} e^{4a_2\left(t/t_p-1\right)/3} t^{n/3} d_2^{2/3}\left\{\exp\left[\frac{d_1}{R_p^4}\int \frac{e^{-t}t_p^{4a_1}}{t^{4a_1}e^{4a_2\left(t/t_p-1\right)}}dt\right]\right\}^{2/3} \tag{25}$$

$$H_x = \frac{\dot{X}}{X} = \frac{4}{3}\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)+\frac{n}{3t}+\frac{2}{3}\frac{d_1}{R_p^4}\left(\frac{t}{t_p}\right)^{-4a_1} e^{-\left\{4a_2\left(t/t_p-1\right)+t\right\}} \tag{26}$$

$$H_y = H_z = \frac{\dot{Y}}{Y} = \frac{4}{3}\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)+\frac{n}{3t}-\frac{1}{3}\frac{d_1}{R_p^4}\left(\frac{t}{t_p}\right)^{-4a_1} e^{-\left\{4a_2\left(t/t_p-1\right)+t\right\}} \tag{27}$$

$$H_\psi = \frac{\dot{Z}}{Z} = -\frac{n}{t} \tag{28}$$

$$H = \frac{\dot{R}}{R} = \left(\frac{a_1}{t}+\frac{a_2}{t_p}\right). \tag{29}$$

The deceleration parameter q is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -1 + a_1 t_p^2\left(a_1 t_p + a_2 t\right)^{-2}. \tag{30}$$

3.1. *Non-interacting dark energy and dark matter*. In this section we have assumed that there is no interaction between Dark Energy and Dark Matter.

Eq. (11) yields the conservation equation for matter and MHRDE as

$$\dot{\rho}_{MA} + \left(\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right)\rho_{MA} = 0 \tag{31}$$

$$\dot{\rho}_{MHRDE} + (1 + \omega_{MHRDE})\rho_{MHRDE}\left(\frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y} + \frac{\dot{Z}}{Z}\right) + 2\frac{\dot{Y}}{Y}\delta\rho_{MHRDE} = 0. \tag{32}$$

Eq. (31) implies $\dot{\rho}_{MA} + 4H\rho_{MA} = 0$

$$\rho_{MA} = d_3 t^{-4a_1} e^{-4a_2 t/t_p}, \tag{33}$$

where d_3 is an integrating constant. The expression for EOS parameter ω_{MHRDE} for MHRDE is obtained from Eq. (32) as

$$4H\rho_{MHRDE}\omega_{MHRDE} = -\left[\dot{\rho}_{MHRDE} + 4H\rho_{MHRDE} + 2\frac{\dot{Y}}{Y}\left(\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y}\right)\right]. \tag{34}$$

Eqs. (18) and (29) lead to

$$\rho_{MHRDE} = 3t^{-2}\left[\eta_1 (a_1 t_p + a_2 t)^2 t_p^{-2} - a_1\eta_2 + 2a_1\eta_3 t_p (a_1 t_p + a_2 t)^{-1}\right]. \tag{35}$$

The skewness parameter δ can be obtained from Eq. (16) using Eqs. (24) and (25) as

$$\delta = \frac{d_1}{R_p^4}\left(\frac{t}{t_p}\right)^{-4a_1} e^{-\{4a_2(t/t_p - 1) + t\}} \frac{1}{\rho_{MHRDE}}. \tag{36}$$

Eq. (34) on using the values of X, Y, Z, ρ_{MHRDE}, H yields,

$$4H\rho_{MHRDE}\omega_{MHRDE} = \left(\frac{a_1}{t} + \frac{a_2}{t_p}\right)\left\{\frac{6a_1\eta_1}{t^2} + \frac{12\eta_2 a_1}{t^2} - \frac{8d_1}{3R_p^4} e^{-[t + 4a_2(t/t_p - 1)]}\left(\frac{t}{t_p}\right)^{-4a_1}\right\}$$
$$- \frac{6a_1\eta_2}{t^3} + \frac{6a_1\eta_3}{t^6}\left(2ta_1 + 3t^2\frac{a_2}{t_p}\right)\left(\frac{a_1}{t} + \frac{a_2}{t_p}\right)^{-2} - 12\eta_1\left(\frac{a_1}{t} + \frac{a_2}{t_p}\right)^3 - \frac{24a_1\eta_3}{t^3} \tag{37}$$
$$+ \frac{2}{3}\left\{\frac{d_1}{R_p^4} e^{-[t + 4a_2(t/t_p - 1)]}\left(\frac{t}{t_p}\right)^{-4a_1}\right\}^2 - \frac{2nd_1}{3tR_p^4}\left(\frac{t}{t_p}\right)^{-4a_1} e^{-[t + 4a_2(t/t_p - 1)]}.$$

The displacement field vector β obtained from Eq. (9) on using Eqs. (26), (27), (35) and (37) is given by

$$\frac{3}{4}\beta^2 = \frac{4a_1+n}{t^2} - \left\{\frac{4}{3}\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)+\frac{n}{3t}+\frac{2d_1}{3R_p^4}e^{-\left[t+4a_2\left(t/t_p-1\right)\right]}\left(\frac{t}{t_p}\right)^{-4a_1}\right\}^2 -$$
$$2\left\{\frac{4}{3}\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)+\frac{n}{3t}-\frac{d_1}{3R_p^4}e^{-\left[t+4a_2\left(t/t_p-1\right)\right]}\left(\frac{t}{t_p}\right)^{-4a_1}\right\}\times$$
$$\left\{\frac{8}{3}\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)+\frac{2n}{3t}+\frac{d_1}{3R_p^4}e^{-\left[t+4a_2\left(t/t_p-1\right)\right]}\left(\frac{t}{t_p}\right)^{-4a_1}\right\}-$$
$$\omega_{MHRDE}\rho_{MHRDE} - \left[\frac{4}{3}\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)+\frac{n}{3t}-\frac{d_1}{3R_p^4}\left(\frac{t}{t_p}\right)^{-4a_1} e^{-\left[t+4a_2\left(t/t_p-1\right)\right]}\right]^2 . \tag{38}$$

3.2. *Interacting dark energy and dark matter*. We have considered that there is an interaction between Dark Energy and Dark Matter. The Eq. (11) yields the conservation equation for matter and MHRDE as

$$\dot{\rho}_{MA} + \left(\frac{\dot{X}}{X}+2\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right)\rho_{MA} = Q' \tag{39}$$

$$\dot{\rho}_{MHRDE} + \left(1+\omega_{MHRDE}\right)\rho_{MHRDE}\left(\frac{\dot{X}}{X}+2\frac{\dot{Y}}{Y}+\frac{\dot{Z}}{Z}\right)+2\frac{\dot{Y}}{Y}\delta\rho_{MHRDE} = -Q', \tag{40}$$

where Q' is the interaction between Dark Energy (DE) and Dark Mater (DM). Here $Q'>0$ means that the energy flows from DE to DM, $Q'<0$ means that the energy flows in opposite direction and $Q'=0$ means that there is no interaction between DE and DM. In general, Q' is inversely proportional of time. Wei and Cai [49] proposed

$$Q' = 4H\sigma\rho_{MA} \tag{41}$$

where $\sigma>0$ is a coupling constant.

Eqs. (39) and (41) implies

$$\rho_{MA} = d_4 R^{4(\sigma-1)}, \quad \sigma>1 \tag{42}$$

where d_4 is a constant of integration.

Eqs. (40) and (41) on using the values of *X*, *Y*, *Z*, ρ_{MHRDE} , *H* yields,

$$4H\rho_{MHRDE}\,\omega_{MHRDE}=-Q'+\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)\left\{\frac{6a_1\eta_1}{t^2}+\frac{12\eta_2 a_1}{t^2}-\frac{8d_1}{3R_p^4}e^{-\left[t+4a_2\left(t/t_p-1\right)\right]}\left(\frac{t}{t_p}\right)^{-4a_1}\right\}$$
$$-\frac{6a_1\eta_2}{t^3}+\frac{6a_1\eta_3}{t^6}\left(2ta_1+3t^2\frac{a_2}{t_p}\right)\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)^{-2}-12\eta_1\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)^3-\frac{24a_1\eta_3}{t^3}$$
$$+\frac{2}{3}\left\{\frac{d_1}{R_p^4}e^{-\left[t+4a_2\left(t/t_p-1\right)\right]}\left(\frac{t}{t_p}\right)^{-4a_1}\right\}^2-\frac{2nd_1}{3tR_p^4}\left(\frac{t}{t_p}\right)^{-4a_1}e^{-\left[t+4a_2\left(t/t_p-1\right)\right]}. \quad (43)$$

4. *Some physical and geometrical representation*. The graphical representations of various cosmological parameters for both the interacting case and non-interacting case are discussed here. The numerical values used in the graphs are

$$R_p=1,\quad t_p=13.7,\quad a_1=0.1,\quad a_2=1.55,\quad n=2,\quad d_1=0.12,\quad d_2=0.001,$$
$$d_3=0.01,\quad \eta_1=1,\quad \eta_2=0.5,\quad \eta_3=0.4,\quad \sigma=2,\quad d_4=0.01,\quad q_0=-0.77 \quad (44)$$

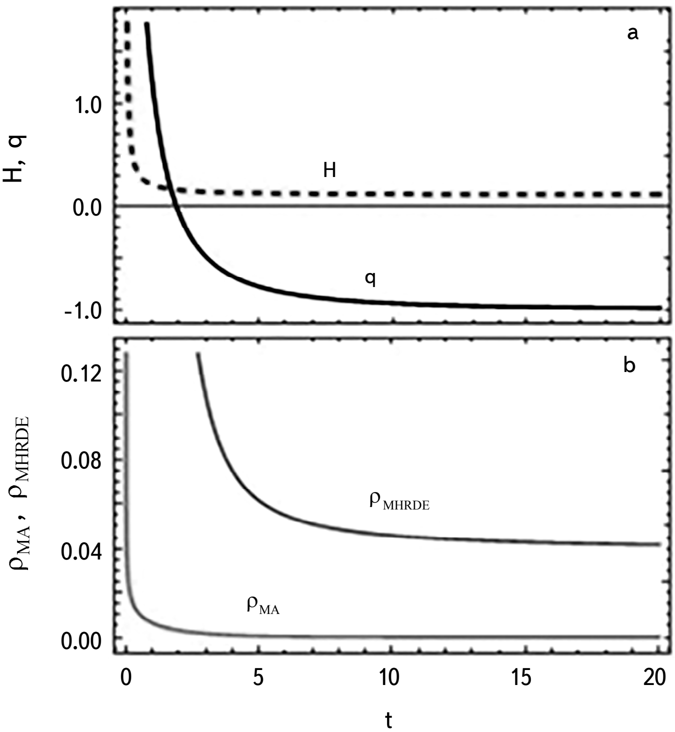

Fig.1. a) The plots of H (dashed line) and q (solid line) versus cosmic time t. b) The plots of ρ_{MA} and ρ_{MHRDE} versus cosmic time t.

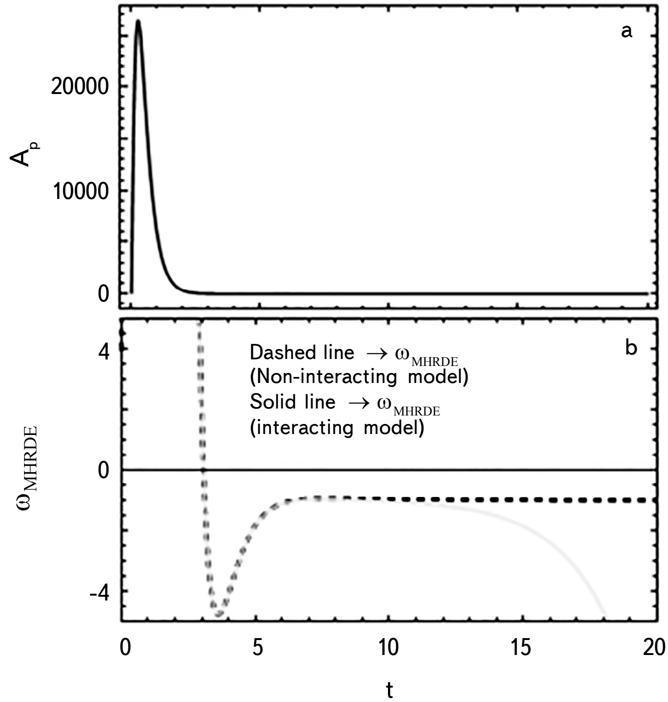

Fig.2. a) The plot of A_p versus cosmic time t. b) The plot of ω_{MHRDE} versus cosmic time t.

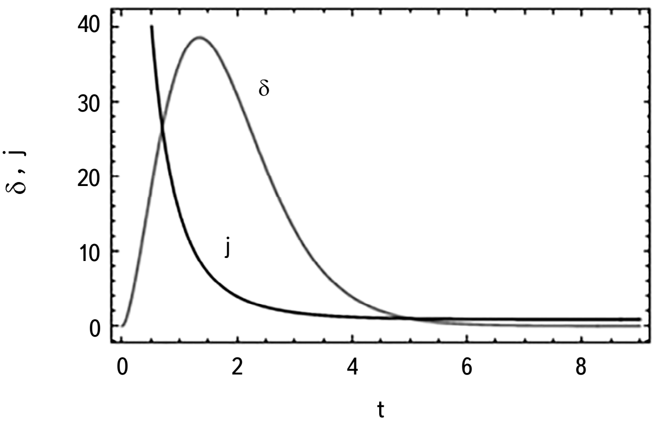

Fig.3. The plots of δ and j versus cosmic time t.

5. *Stability analysis*. The stability conditions can be determined by testing the sound speed. The square sound speed for any fluid is given as $\upsilon_{sq}^2 = \dot{p}_{MHRDE}/\dot{\rho}_{MHRDE}$, $p_{MHRDE} = \omega_{MHRDE}\rho_{MHRDE}$. The positive value of υ_{sq}^2 implies that the model is stable whereas the negative value implies that the model is

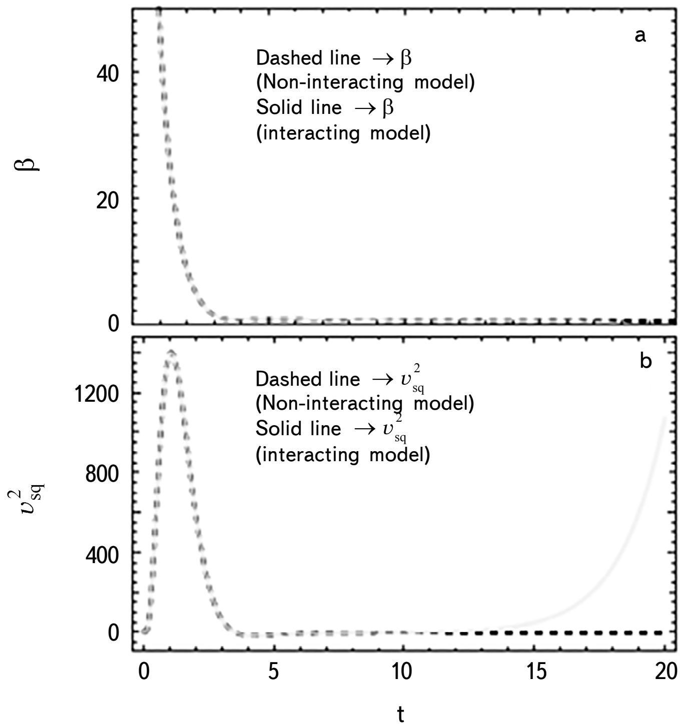

Fig.4. a) The plots of β versus cosmic time t. b) The plots of υ^2_{sq} versus cosmic time.

unstable. Also, the casualty condition must be satisfied. It means that the sound speed is less than the speed of light. The Energy conditions, i.e. Weak Energy Conditions (WEC), Dominant Energy Conditions (DEC) and Strong Energy Conditions (SEC) are respectively given by

(I) $\rho_{MHRDE} \geq 0$ (II) $\rho_{MHRDE} + p_{MHRDE} \geq 0$ (III) $\rho_{MHRDE} + 4\,p_{MHRDE} \geq 0$

6. *Cosmic jerk parameter*. Cosmic jerk parameter is defined as the third order derivative of the average scale factor w.r. to the cosmic time. It is a dimensional quantity and it is given by Chiba and Nakamura [50]

$$j(t) = \frac{1}{H^3}\frac{\dddot{R}}{R} = q + 2q^2 - \frac{\dot{q}}{H}. \tag{45}$$

Using Eqs. (29) and (30) in Eq. (45), we get the expression of cosmic jerk parameter as

$$j(t) = 1 - 3a_1 t_p^2 \left(a_1 t_p + a_2 t\right)^{-2} + 2a_1^2 t_p^4 \left(a_1 t_p + a_2 t\right)^{-4} + \frac{2a_1 a_2 t t_p^3}{\left(a_1 t_p + a_2 t\right)^4}. \tag{46}$$

It is believed that the transition from the decelerating to the accelerating phase of the universe isdue to a cosmic jerk parameter. This transition of the universe

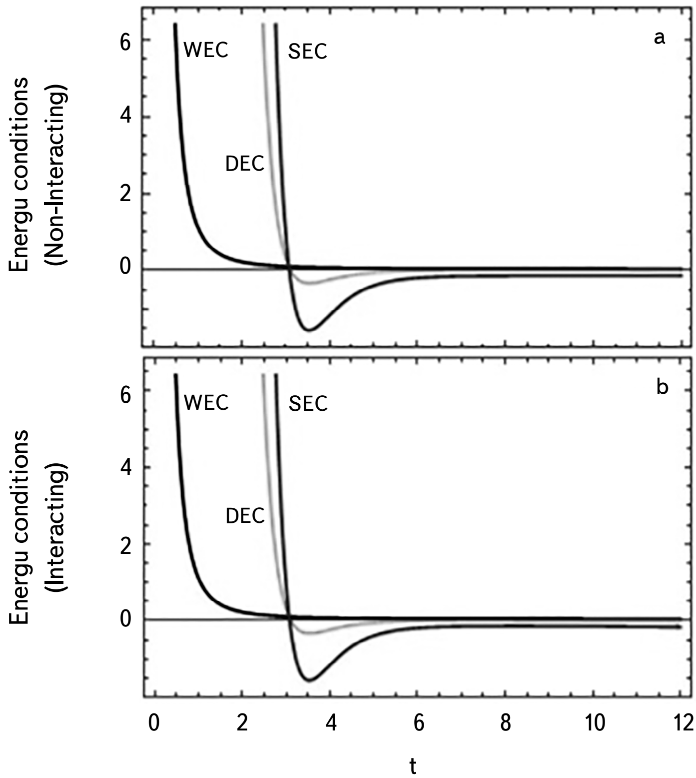

Fig.5. a) The plots of Energy conditions (for non-interacting case) versus cosmic time t. b) The figure shows the plot of Energy conditions (for interacting case) versus cosmic time.

occurs for different models with a positive value of the jerk parameter and the negative value of the deceleration parameter [51-53]. The ΛCDM model has a constant jerk $j = 1$.

7. *Statefinder parameters*. Sahni et al. [54] first introduced the statefinder parameters called {r, s} parameters to discriminate among the various DE models. The {r, s} parameters depends on the average scale factor. The important property of the {r, s} parameter is that it can explains the dynamics of the expansion of the universe by using the higher derivatives of R and the deceleration parameter q. The mathematical expression of {r, s} parameters are

$$r = \frac{\dddot{R}}{RH^3} \quad \text{and} \quad s = \frac{r-1}{3\left(q - \frac{1}{2}\right)}. \tag{47}$$

For our model, the {r, s} parameters take the form

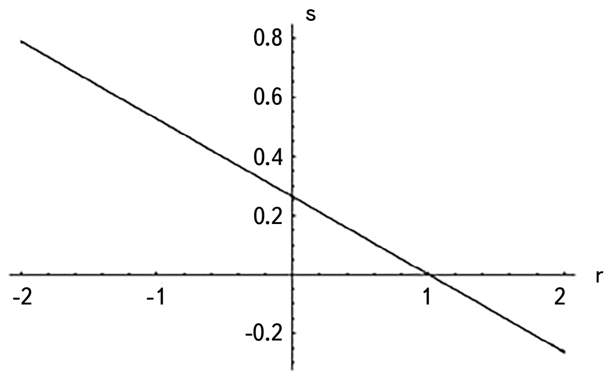

Fig.6. The plot of r versus s.

$$r=\frac{2a_1}{t^3\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)^3}-\frac{3a_1}{t^2\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)^2}+1 \tag{48}$$

$$3s=\frac{\dfrac{2a_1}{t^3\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)^3}-\dfrac{3a_1}{t^2\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)^2}}{\dfrac{a_1}{t^2\left(\frac{a_1}{t}+\frac{a_2}{t_p}\right)^2}-\dfrac{3}{2}}. \tag{49}$$

Therefore r is related to s by the following expression:

$$s=\frac{r-1}{3(q-1/2)},\quad q_0=-0.77. \tag{50}$$

8. *Results and discussions*. It is observed from Fig.1a that H is a decreasing function of t and vanishes for large values of t. From Fig.1a, it is seen that the deceleration parameter q is positive at early stage of the universe and is negative at later stage. This implies that the universe exhibits transition from the decelerating to accelerating phase. Initially the universe is decelerating and ultimately at late times it is accelerating. From Fig.2a, it is seen that A_p increases sharply at early stage of the universe and then decreases and ultimately tends to zero at late times. So, at early era the universe is anisotropic and at the late times the universe becomes isotropic. From Fig.1b, it is observed that ρ_{MA} is a

decreasing function of t and tends to zero at late-times. From Fig.1b, it is seen that ρ_{MHRDE} is a decreasing function of t and tends to small value at late-times. From Fig.2b, it is observed that ω_{MHRDE} tends to -1 at late times for non-interacting case, where dashed line represents the non-interacting model and interestingly it behaves like a cosmological constant [6]. Also, from Fig.2b, it is observed that $\omega_{MHRDE} < -1$ at late-times for interacting case. This depicts that the model behaves like phantom dark energy [8]. It is observed from Fig.3, that δ increases sharply at early stage of the universe and then decreases and ultimately tends to zero at late times. From Fig.3, it is seen that j tends to 1 at late times and is positive throughout the entire age of the universe. From Fig.4a, it is seen that β tends to zero at late times for both non-interacting (dashed line) case and interacting (solid line) case. From Fig.4b, for non-interacting model (dashed line) and interacting model (solid line), we see that υ_{sq}^2 is positive for both the cases. This confirms that our models are stable throughout the evolution of the universe. WEC is satisfied for both non-interacting and interacting models, but the DEC and SEC for both the models are violated which indicates that at late times our models proceed to accelerating expanded models of the universe Fig.5a, b. Hence our cosmological models are physically acceptable. From Fig.6, we observe that the value of s is negative when $r \geq 1$. Also, it is observed that universe starts from an Einstein static era ($r \to \propto, \ s \to \propto$) and goes to the ΛCDM model ($r = 1, \ s = 0$).

9. *Conclusions*. In this paper, we have studied interacting and non-interacting DE and DM in the anisotropic five-dimensional Bianchi type-I universe within the framework of Lyra geometry. The exact solutions of the Einstein field equations in Lyra geometry are obtained by making use of MHRDE proposed by Chen and Jing [34]. Also, we have used HEL which is a combination of power law and exponential law. The anisotropy of the universe ultimately tends to zero at later times and the universe becomes isotropic. The skewness parameter δ in this model also tends to zero at later age of the universe. Also, the equation of state parameter for MHRDE, ω_{MHRDE} approaches -1 at late times and it behaves like a cosmological constant for non-interacting model. But for interacting model $\omega_{MHRDE} < -1$ which indicates the model behaves like phantom dark energy at late times. The cosmic jerk parameter j approaches to 1 at late times of the evolution of the universe and it is positive throughout the age of the universe. Also, it is found that the sound speed is positive for non-interacting and interacting models and therefore our models are stable throughout the evolution of the universe. The statefinder diagnostic pair {r, s} is obtained. The trajectories in the {r, s} plane corresponds to the ΛCDM model (as shown in Fig.6). The displacement field vector β which plays an important role in the dynamics of the universe, tends to zero at later age of the universe. The constant displacement vector field β in

Lyra geometry plays the role of cosmological constant Λ in normal relativistic treatment [43]. Therefore, the displacement field behaves as a candidate for dark energy. Interestingly, within the frame work of Lyra geometry while investigating five-dimensional LRS Bianchi type-I model universe with time-dependent deceleration parameter our models (interacting and non-interacting) are dark energy models of the universe which are consistent with the observational findings. It is seen that all physical and geometrical aspects of the models are in good agreement with the recent scenario of modern cosmology. It is observed that our models may be useful for better understanding of higher-dimensional cosmological model in early stage of the universe with modified holographic Ricci dark energy based within the frame work of Lyra geometry.

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МНОГОМЕРНАЯ АНИЗОТРОПНАЯ КОСМОЛОГИЧЕСКАЯ МОДЕЛЬ МОДИФИЦИРОВАННОЙ ГОЛОГРАФИЧЕСКОЙ ТЕМНОЙ ЭНЕРГИИ РИЧЧИ В МНОГООБРАЗИИ ЛИРЫ

К.ДАС, Д.БХАРАЛИ

Чтобы найти анизотропную космологическую модель LRS Бьянки типа I в пяти измерениях на основе геометрии Лиры, использована модифицированная голографическая темная энергия Риччи. Точные решения уравнений поля Эйнштейна получены с помощью гибридного закона расширения (HEL). Задача рассмотрена как при наличии взаимодействия между темной энергией и темной материей, так и при ее отсутствии. Обнаружено, что на поздних стадиях расширения уравнение параметра состояния (EOS) для невзаимодейст-вующего случая ведет себя как космологическая постоянная, тогда как для взаимодействующей модели оно ведет себя как фантомная темная энергия. Обсуждаются некоторые космологические параметры и устойчивость моделей.

Физические и геометрические аспекты моделей были проанализированы и согласованы с недавними результатами наблюдений.

Ключевые слова: *LRS Бьянки: тип I: пространство-время: геометрия Лиры: модифицированная голографическая темная энергия Риччи: закон гибридного расширения*

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