АСТРОФИЗИКА

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COVARIANT FORMULATION OF THE DYNAMICAL EQUATIONS OF QUANTUM VORTICES IN TYPE II SUPERCONDUCTORS

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We have derived the closed system of covariant equations which describe the motion of quantum vortices regarded as a two-dimensional polarized liquid. We have obtained the covariant expressions of the forces acting on the vortices; from the equilibrium condition of these forces we have deduced the equation satisfied by the velocity field of the fluid. It is shown that this velocity field depends respectively on the friction coefficient, the density of vortices and the superconducting current. From this closed system of equations we derived the relaxation equation when a variable magnetic field is applied.

Key words: *superconductors: dynamical equations*

1. *Introduction.* It is a well known fact that quantum vortices are generated in type II superconductors ($\lambda/\xi > 1/\sqrt{2}$) when the intensity of the applied magnetic field H exceeds that of the critical field H_{c1} [1]. Such a system of vortices is generated in the core of neutron stars where protons are superconducting and the ratio $\lambda/\xi > 1$; vortices are also generated in a rotating neutron superfluid [2,3]. The dynamical equations describing the motions of vortices in rotating superfluids and type II superconductors, within the framework of the Newtonian theory, have been examined in refs. [3,4]. The relativistic generalization of the corresponding equations for a rotating superfluid has been presented in ref. [5]. The purpose of this paper is to derive the equations of motion of vortices for type II superconductors in General relativity. We shall see that these equations connect the magnetic field to the density of vortices and permit the study of: 1) relaxation process of vortices in a time varying magnetic field, 2) the behavior of magnetic fields in the core of neutron stars. In sec. 2 we derive the dynamical equations of quantum vortices. Adopting Synge's approach of electromagnetism in general relativity the relation between magnetic flux and number of vortices is given. In sec. 3, we present a covariant formulation of the forces acting on vortices, i.e. the friction force and the force due to the supercurrent. In sec. 4, from the equilibrium condition of the acting forces the dependence of the vortex velocity field on the supercurrent is derived. In sec. 5 we derive the relaxation equation for superconducting current and

obtain the formula for the relaxation time. In sec. 6 we summarize our results and note that detailed investigation of relaxation process require the specification of the superconducting medium under consideration. In the Appendix, following London, we briefly sketch the derivation of Eq.(10) by considering the motion of a superparticle subject only to the action of the Lorentz and Magnus forces.

2. *The dynamical equations of quantum vortices in type II superconductors.* Consider a static universe with metric of the form

$$ds^2 = g_{ij}dx^i dx^j + g_{44}\left(dx^4\right)^2 , \tag{1}$$

where the g 's are independent of the time coordinate x^4. In this universe we have a type II superconductor with world lines of the normal part along the x^4-lines; consequently

$$u^i(n) = 0 , \quad g_{44}\left(u^4(n)\right)^2 = -1 , \quad u^4(n) = \frac{1}{\sqrt{-g_{44}}} . \tag{2}$$

As it is well known, when the intensity of the applied magnetic field H is less than the critical value H_{c1} for the creation of quantum vortices, the equations which relate the supercurrent to the electromagnetic field are the London equations which, in covariant form, read

$$\nabla_{[\mu} M_{\nu]} \equiv s_{\mu\nu} = 0 , \tag{3}$$

where ∇ is the operator of covariant derivation and

$$M_\nu = \frac{mc}{e^2 n(s)} j_\nu + A_\nu , \tag{4}$$

with e $(e < 0)$, m and $n(s)$ denoting respectively the charge, mass and number density of superelectrons. j_ν is the 4-current defined by

$$j_\nu = en(s) u_\nu(s) , \tag{5}$$

and A_ν the 4-potential connected to the electromagnetic field tensor by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \tag{6}$$

A consequence of Eq.(3) is the vanishing of the flux of magnetic field across a spacelike V_2. Indeed, let $t^\alpha_{(1)}$, $t^\alpha_{(2)}$, $t^\alpha_{(3)}$ and $t^\alpha_{(4)}$ be an orthogonal tetrad of unit vectors defined at each point of the 2-dimensional spacelike region V_2 bounded by a closed curve V_1. Our t-tetrad has the same orientation as that of the parametric lines of the cylindrical coordinates $\left(x^1, x^2, x^3, x^4\right) = (r, \varphi, z, ct)$. $t^\alpha_{(1)}$ and $t^\alpha_{(2)}$ are chosen tangent to V_2 and are oriented along the radial and azimuthal parametric lines, $t^\alpha_{(3)}$ and $t^\alpha_{(4)}$ are of course normal to V_2 and respectively tangent to the $x^3 = z$ and $x^4 = ct$ coordinate lines. Using Eqs.(4) and (6), integration of Eq.(3) on the spacelike element V_2 bounded by the curve V_1 yields [6]:

$$\frac{mc}{e^2 n(s)} \int_{V_2} \nabla_{[\mu} j_{\nu]} t^\mu_{(1)} t^\nu_{(2)} dS + \int_{V_2} F_{\mu\nu} t^\mu_{(1)} t^\nu_{(2)} dS = 0 , \tag{7}$$

where dS is the invariant element of area on V_2. According to Stokes's theorem the first integral is equal to the circulation of j_ν round the closed circuit V_1 bounding V_2. Assuming that the region of V_2 near the boundary curve V_1 is free from vortices, the supercurrent is then equal to zero and Eq.(7) reduces to

$$\int_{V_2} F_{\mu\nu} t^{\mu}_{(1)} t^{\nu}_{(2)} dS = 0\,, \tag{8}$$

which physically can be interpreted as the vanishing of the flux of magnetic field across V_2 as measured by an observer with 4-velocity equal to $t^{\xi}_{(4)}$ i.e. at rest with respect to the normal component of the superconductor. In the presence of vortices, the flux of magnetic field across V_2 being different from zero, the right-hand side of Eq.(3) must be set equal to a non zero antisymmetric tensor characterizing the system of vortices. This tensor may be written in the form [5]

$$D^{\rho\sigma} = -u^{\rho}(L)\nu^{\sigma}(L) + u^{\sigma}(L)\nu^{\rho}(L)\,, \tag{9}$$

where $u^{\rho}(L)$ and $\nu^{\rho}(L)$ are respectively the unit 4-velocity of the vortex and the unit spacelike vector defining the direction of the vortex; $\nu^{\sigma}(L)$ being normal to V_2 has, of course, the direction of $t^{\nu}_{(3)}$. Accordingly, in the presence of vortices Eq.(3) must be replaced by (see Appendix)

$$\nabla_{[\mu} M_{\nu]} \equiv s_{\mu\nu} = \frac{1}{2}\eta_{\mu\nu\rho\sigma} D^{\rho\sigma}\,. \tag{10}$$

Eq.(10) may also be exhibits in the form

$$\eta^{\mu\nu\rho\sigma}\nabla_{[\rho} M_{\sigma]} = -2\,sD^{\mu\nu}\,, \tag{11}$$

where

$$\eta^{\mu\nu\rho\sigma} = \frac{1}{\sqrt{-g}}\varepsilon_{\mu\nu\rho\sigma}\,, \quad \eta_{\mu\nu\rho\sigma} = -\sqrt{-g}\,\varepsilon_{\mu\nu\rho\sigma}\,, \tag{12}$$

$\varepsilon_{\mu\nu\rho\sigma}$ being the usual permutation symbol and

$$s^2 = \frac{1}{2}s^{\mu\nu}s_{\mu\nu}\,. \tag{13}$$

Vectors A_ν and j_ν have to satisfy the Lorentz and continuity conditions

$$\nabla_\nu A^\nu = 0\,, \quad \nabla_\nu j^\nu = 0\,. \tag{14}$$

Eq.(11) is the fundamental equation describing the dynamics of the system of vortices. Let us note that from Eq.(11) one easily derive the equation of conservation of vortex number

$$\nabla_\mu\left(sD^{\mu\nu}\right) = 0\,. \tag{15}$$

The physical interpretation of the scalar s is obtained by considering the integral form of the dynamical equation (11):

$$\frac{mc}{e^2 n(s)}\int_{V_2}\eta^{\mu\nu\rho\sigma}\nabla_{[\rho}\, J_{\sigma]}t_{(3)\mu}t_{(4)\nu}\,\varepsilon_3\varepsilon_4\, ds+\int_{V_2}\eta^{\mu\nu\rho\sigma}\, F_{\rho\sigma}t_{(3)\mu}t_{(4)\nu}ds =$$
$$= \int_{V_2} 2\, sD^{\mu\nu}t_{(3)\mu}t_{(4)\nu}\,\varepsilon_3\varepsilon_4\, ds\,, \tag{16}$$

with $\varepsilon_3 = 1$ and $\varepsilon_4 = -1$ being respectively the indicators of $t_{(3)}$ and $t_{(4)}$.

Noting that

$$\delta^{\mu\nu}_{\rho\sigma}\, t^{\rho}_{(1)}t^{\sigma}_{(2)} = -\eta^{\mu\nu\rho\sigma}\, t_{(3)\rho}t_{(4)\sigma}\,\varepsilon_3\varepsilon_4\,, \tag{17}$$

the left hand side of Eq.(16) is seen to be equal to the right hand side of Eq.(7) accordingly, the left hand side of Eq.(16) represents the flux of magnetic field across V_2 as measured by an observer at rest with respect to the normal component of the superconductor, therefore s is proportional to the number density of vortices on V_2.

3. *Covariant formulation of the forces acting on vortices.* Let us assume that the vortices have the orientation of $t^{\alpha}_{(3)}$ i.e. normal to the spacelike V_2 containing $t^{\alpha}_{(1)}$ and $t^{\alpha}_{(2)}$. There are essentially two kinds of forces acting on vortices, the friction force and the force due to the supercurrent. We first consider the friction force acting on moving vortices. Let us note that friction is a consequence of the interaction between the normal matter in the core of a vortex and the normal component of the superconductor. From classical physics we know that this force is proportional to the velocity of the normal component relative to the vortex. By analogy, we adopt the following covariant expression of the friction force

$$F_{\rho} = \eta\, n(n)\tilde{u}_{\rho}(n, L), \tag{18}$$

where $\eta, n(n)$ and $\tilde{u}_{\rho}(n, L)$ are respectively the friction coefficient, the number density of the normal component and the velocity of the normal component relative to the vortex which, in accord with the requirement of relativity [7], may be written in the following covariant form

$$\tilde{u}_{\rho}(n, L) = \left(\delta^{\alpha}_{\rho} + u_{\rho}(L)u^{\alpha}(n)\right)u_{\alpha}(n). \tag{19}$$

Taking into account the orthogonality of $u^{\sigma}(L)$ and $\nu^{\sigma}(L)$ Eq.(19) may be exhibited in yet another form

$$\tilde{u}_{\rho}(n, L) = \perp_{\rho\sigma} u^{\sigma}(n) \tag{20}$$

where the projection tensor $\perp^{\sigma}_{\rho}$ is given by

$$\perp^{\sigma}_{\rho} = \delta^{\sigma}_{\rho} - \eta^{\sigma}_{\rho}\,, \quad \eta^{\sigma}_{\rho} = -u_{\rho}(L)u^{\sigma}(L) + \nu_{\rho}(L)\nu^{\sigma}(L). \tag{21}$$

Inserting Eq.(20) into (18) we finally obtain for the friction force the following expression

$$F_{\rho} = \eta\, n(n) \perp_{\rho\sigma} u^{\sigma}(n). \tag{22}$$

As already noted, the second force $\overline{F}_{\rho}$ acting on a quantum vortex is due to the superconducting current. By analogy with the classical case, $\overline{F}_\rho$ is taken proportional to the velocity of the superconducting component relative to the vortex i.e. $\widetilde{u}_\rho(s, L)$. $\overline{F}_\rho$ being respectively orthogonal to $v^\mu(L)$ and $u^\mu(L)$ may be written

$$\overline{F}_\rho = \frac{en(s)}{2c} s\, \eta_{\rho\mu\nu\sigma}\, \widetilde{u}^\mu(s, L) D^{\nu\sigma} . \tag{23}$$

According to the definition (23) of the relative velocity $u(s, L)$ the force $\overline{F}_\rho$ may be expressed in terms of the superconducting current $j^\sigma(s)$

$$\overline{F}_\rho = \frac{en(s)}{c} s_{\rho\mu} \perp^\mu_\sigma u^\sigma(s) = \frac{1}{c} j^\sigma(s) s_{\rho\sigma} . \tag{24}$$

Eqs. (22) and (24) may be regarded as the main forces acting on a vortex.

4. *Determination of the vortex 4-velocity.* The 4-velocity of vortices can be obtained from the condition

$$F_\rho + \overline{F}_\rho = 0 , \tag{25}$$

or in explicit form

$$\frac{1}{c} j^\sigma(s) s_{\sigma\rho} = \eta\, n(n) \perp_{\rho\sigma} u^\sigma(n). \tag{26}$$

Considering the case of a vortex configuration of the maximally symmetric type, i.e. both stationary and cylindrical symmetry, we have three independent commuting Killing vectors k^μ, l^μ and m^μ, respectively parallel to $t^\mu_{(4)}$, $t^\mu_{(3)}$ and $t^\mu_{(2)}$, which are the generators of time translations, longitudinal space translations and axial rotations corresponding to the ignorable coordinates $x^4 = ct$, $x^3 = z$ and $x^2 = \varphi$. Since the supercurrent has no radial component and the normal component satisfies condition (II.2) the vectors $j^\rho(s)$ and $u^\rho(s)$ will be expressible in the form

$$j^\rho(s) = en(s)\gamma(s)\left(k^\rho + \Omega(s) m^\rho\right), \tag{27}$$

$$u^\rho(n) = \gamma(n) k^\rho , \tag{28}$$

where $\gamma(s) = \left(1 - \frac{\Omega^2(s) r^2}{c^2}\right)^{-1/2}$ is the Lorentz factor (resp. $\gamma(n) = 1$). Multiplying Eq.(26) by $s^{\lambda\tau}$ and taking into account that

$$s^{\rho\sigma} s_{\sigma\tau} = -s^2 \perp^\rho_\sigma , \quad s^\rho_\sigma \perp^\sigma_\tau = s^\rho_\tau , \tag{29}$$

we obtain another relation connecting $s_{\lambda\rho}$ and $\perp_{\lambda\rho}$

$$s^2 \perp_{\lambda\rho} j^\rho(s) = c\,\eta\, n(n) s_{\lambda\rho} u^\rho(n). \tag{30}$$

Insertion of Eqs.(27) and (28) into Eqs.(26) and (30) yields

$$en(s)\gamma(s)\left(k^\rho + \Omega(s) m^\rho\right) = c\,\eta\, n(n) \perp_{\rho\sigma} k^\rho , \tag{31}$$

$$en(s)\gamma(s) s^2 \left(k^\rho + \Omega(s) m^\rho\right) \perp_{\lambda\rho} = c\,\eta\, n(n) s_{\lambda\rho} k^\rho . \tag{32}$$

Multiplying respectively Eqs.(31) and (32) by m^σ and m^λ and summing with respect to σ and λ we obtain

$$m^\sigma k^\rho s_{\rho\sigma} = \frac{c\eta n(n)}{en(s)\gamma(s)} m^\sigma k^\rho \perp_{\sigma\rho} , \tag{33}$$

$$m^\lambda k^\rho \perp_{\lambda\rho} + \Omega(s) m^\rho m^\lambda \perp_{\lambda\rho} = \frac{c\eta n(n)}{en(s)\gamma(s)s^2} m^\lambda k^\rho s_{\lambda\rho} , \tag{34}$$

where we have taken into account that $m^\rho m^\sigma s_{\rho\sigma} = 0$. The solutions of Eqs.(33) and (34) may be exhibited in the form

$$\frac{1}{s}\frac{k^\rho m^\lambda s_{\lambda\rho}}{m^\rho m^\lambda \perp_{\lambda\rho}} = \frac{\kappa\Omega(s)}{1+\kappa^2} , \tag{35}$$

$$\frac{k^\rho m^\lambda \perp_{\lambda\rho}}{m^\rho m^\lambda \perp_{\lambda\rho}} = \frac{\kappa^2\Omega(s)}{1+\kappa^2} , \tag{36}$$

where

$$\kappa = \frac{en(s)\gamma(s)s}{c\eta n(n)} . \tag{37}$$

One easily verifies that the left hand side of Eqs.(35) and (36) correspond respectively to the radial and azimutal components of the 4-vector $u^\nu(L)$. Let us remark that, according to definition (27) of the supercurrent, solutions (35) and (36) express the dependence of the vortex velocity field on the azimuthal component of the 4-current $j^\rho(s)$.

We now derive the expression of these components making the following assumptions: (i) the presence of vortices does not modify the static gravitational field as given by Eq.(1), (ii) the homogeneous applied magnetic field is directed along the x^3-coordinate axis. Substitution of the tensor components

$$\begin{aligned} m^\rho \perp_{\rho\sigma} m^\sigma &= g_{22} + \eta_{22} = g_{22}\left(1 + u^2(L)u_2(L)\right), \\ m^\rho \perp_{\rho\sigma} k^\sigma &= -\eta_{24} = g_{22}u^2(L)u_2(L), \\ k^\rho s_{\rho\sigma} m^\sigma &= s\sqrt{-g}\, u^1(L) \end{aligned} \tag{38}$$

in Eqs.(35) and (36) yields

$$\begin{aligned} u^1(L) &= -\frac{g_{22}\Omega(s)/c}{\sqrt{-g}}\frac{\kappa}{1+\kappa^2}\left(1 + u^2(L)u_2(L)\right), \\ u^2(L) &= -\frac{\Omega(s)}{c}\frac{\kappa}{1+\kappa^2}\frac{1+u_2(L)u^2(L)}{u_4(L)} , \end{aligned} \tag{39}$$

where the components $u^\sigma(L)$ satisfy the normalization condition:

$$u^4(L)u_4(L) + u^2(L)u_2(L) + u^1(L)u_1(L) = -1 . \tag{40}$$

Taking into account the relation

$$\Omega(s) = \frac{c}{e} \frac{j_2}{g_{22} n(s) u^4(s)}, \tag{41}$$

which is easily derived from definition (5) and $u^2(s) = \Omega(s)/cu^4(s)$, Eqs.(39) and (40) determine the expressions of the radial and azimuthal components of the vortex velocity as a function of the only non-zero component of the 4-supercurrent, $j_2(s)$.

5. *The relaxation equation.* As it is well known a variation of the applied magnetic field induces a change in the structure of the vortices; this change corresponds to a relaxation process. We now derive the corresponding equation.

The substitution $(\nu = 3, \mu = 4)$ in Eq.(11) gives

$$\eta^{43\rho\sigma} \partial_\rho M_\sigma = s \frac{u^4(L)}{\sqrt{g_{11}}}. \tag{42}$$

Because of axial symmetry, all quantities entering in equation (42) are independent of the azimutal coordinate x^2. From definition (12) of $\eta^{\mu\nu\rho\sigma}$, equation (42) may be written in the following form:

$$\frac{\partial M_2}{\partial x^1} = -s \frac{\sqrt{-g}}{\sqrt{g_{11}}} u^4(L). \tag{43}$$

Writing explicitly the equation of conservation of vortex number as given by (15), we have

$$\frac{\partial}{\partial x^4}\left(s \frac{\sqrt{-g}}{\sqrt{g_{11}}} u^4(L) \right) = -\frac{\partial}{\partial x^1}\left(s \frac{\sqrt{-g}}{\sqrt{g_{11}}} u^1(L) \right). \tag{44}$$

Differentiating Eq.(43) with respect to x^4 and using Eq.(44), we obtain the following partial differential equation:

$$\frac{\partial}{\partial x^1}\left[\frac{\partial M_2}{\partial x^4} - s \frac{\sqrt{-g}}{\sqrt{g_{11}}} u^1(L) \right] = 0. \tag{45}$$

Integration of Eq.(45) with respect to x^1, with the time independent initial conditions: $u^1(L) = A_2 = j_2 = 0$ at the origin $x^1 = 0$, gives:

$$\frac{\partial M_2}{\partial x^4} = s \frac{\sqrt{-g}}{\sqrt{g_{11}}} u^1(L). \tag{46}$$

By virtue of Eqs.(4), (39) and (41) the quantities M_2 and $u^1(L)$ may be expressed in terms of the supercurrent component j_2, accordingly Eq.(46) may be exhibited in the form:

$$\frac{\partial j_2}{\partial T} + \frac{j_2}{\tau} = -\frac{e^2 n(s)}{mc^2} \frac{\partial A_2}{\partial t}, \tag{47}$$

where the relaxation time τ is defined by

$$\frac{1}{\tau}=\frac{es}{2mc}\frac{\kappa}{1+\kappa^2}\frac{1+u^2(L)u_2(L)}{\sqrt{-g_{44}}\sqrt{g_{11}}\,u^4(s)}. \qquad (48)$$

In Eq.(47) $dT=\sqrt{-g_{44}}\,dx^4/c$ represents the proper time registered by a standard clock at the rest with respect to the normal. The quantity s entering in the definition (48) of the relaxation time is defined by the following formula

$$s=-\frac{\sqrt{g_{11}}}{\sqrt{-g}}\frac{1}{u^4(L)}\frac{\partial M_2}{\partial x^1}, \qquad (49)$$

which may be derived from Eq.(11). As wee see the quantity s depends on j_2, consequently the relaxation Eq. (47) is a nonlinear equation with respect to j_2. However in the particular case, when the condition

$$j_2 \ll \frac{1}{4\pi\lambda^2}A_2 \qquad (50)$$

is satisfied, this equation can be linearized. Condition (50) states that the relaxation current is much smaller than the Meissner current. When the nonequilibrium vortex structure tends to equilibrium the relaxation current j_2 tends to zero, accordingly the last stage of the relaxation process may be regarded as linear. Linearity of the process is conserved when the change in the applied magnetic field ΔH is smaller compared to the magnetic field H. In the case of equilibrium the quantity s defined by Eq.(49) reads:

$$s_0=-\frac{\sqrt{g_{11}}}{\sqrt{-g_{44}}\,u^4(L)}\frac{1}{\sqrt{\gamma}}\frac{\partial A_2}{\partial x^1} \qquad (51)$$

where we have used relation $\sqrt{-g_{44}}\sqrt{\gamma}=\sqrt{-g}$, which connects the determinant g to the determinant γ of the spatial quadratic form $ds_1^2=\gamma_{ij}\,dx^i dx^j$. Following [8] we define the component $B^3\equiv B$ of the magnetic induction 3-vector by

$$B^3=-\frac{1}{\sqrt{\gamma}}\frac{\partial A_2}{\partial x^1}. \qquad (52)$$

Substitution of (52) in (51) yields for s_0 the following expression:

$$s_0=\frac{\sqrt{g_{11}}\,B}{\sqrt{-g_{44}}\,u^4(L)}. \qquad (53)$$

Taking into account the condition $u^i(L)u_i(L)=-1$ (resp. $u^i(s)u_i(s)=-1$), the final expression for the relaxation time is given by:

$$\frac{1}{\tau_0}=\frac{eB}{mc}\frac{\kappa}{1+\kappa^2}\frac{1+u^2(L)u_2(L)}{\left[\left(1+u^i(L)u_i(L)\right)\left(1+u^i(s)u_i(s)\right)\right]^{1/2}}. \qquad (54)$$

In the limiting case of small velocities ($u^i(L)u_i(L)\ll 1$ and $u^i(s)u_i(s)\ll 1$) the non relativistic expression of the τ_0 reads:

$$\frac{1}{\tau_0}=\frac{eB}{mc}\frac{\kappa}{1+\kappa^2}. \tag{55}$$

As it can be seen from Eq.(55), the inequality $(\tau_0/\tau_L)>>1$, where $\tau_L=mc/eB_3$ is the Larmor characteristic time, is satisfied for both small and large friction coefficients. This result is a consequence of the smallness of the vortex mobility.

6. *Conclusion.* The system of equations that we have derived describes the motion of quantum vortices regarded as a 2-dimensional polarized liquid. Accordingly we have introduced the hydrodynamical concepts of density, velocity and polarization of a fluid particle. Having specified the forces acting on a given fluid particle, from the equilibrium condition we have obtained the equations satisfied by the velocity field of the fluid. The solution of the equations of motion determines the dependence of the velocity field on the friction coefficient, the density of vortices and the superconducting current. From the closed system of Eqs.(11), (15), (35) and (36) we derive the relaxation equation, when a variable magnetic field is applied. From these equations one may also derive the dependence of magnetic induction on the density of vortices in the stationary case. The investigations of the above problems require the specification of the superconducting medium under consideration.

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КОВАРИАНТНАЯ ФОРМУЛИРОВКА ДИНАМИЧЕСКИХ УРАВНЕНИЙ КВАНТОВЫХ ВИХРЕЙ В СВЕРХПРОВОДНИКАХ ВТОРОГО РОДА

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Получена замкнутая система ковариантных уравнений, описывающая движение двумерной поляризованной жидкости квантовых вихрей. Найдены ковариантные выражения для сил, действующих на вихрь, из условия равновесия которых выведено уравнение, определяющее поле скоростей рассматриваемой жидкости. Показано, что поле скоростей зависит от коэффициента трения, плотности вихрей и сверхпроводящих токов.

Полученная замкнутая система уравнений позволит изучить процесс релаксации вихрей при приложении переменного магнитного поля.

Ключевые слова: *сверхпроводник: динамические уравнения*

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Appendix

Derivation of London's equations for type II superconductors

Following London let us consider free particles of charge q ($q<0$) and mass m moving without friction and subject to the action of the electromagnetic field. These particles are assumed to form a non viscous charged liquid with velocity fields $u^{\alpha}\left(x^{\beta}\right)$ satisfying the equations:

$$u^{\mu}\left[\partial_{[\mu} u_{\nu]}+\frac{q}{mc^2}F_{\mu\nu}-\frac{q}{mc^2}s_{\mu\nu}\right]=0. \qquad \text{(A.I)}$$

These equations are obtained by considering the motion of a "fictitious" charged particle (q, m) subject to the Lorentz force and the reaction of the Magnus force, i.e.

$$\frac{\nabla u^{\mu}}{ds}=\frac{q}{mc^2}F^{\nu\mu}u_{\mu}-\frac{q}{mc^2}s^{\nu\mu}u_{\mu}. \qquad \text{(A.II)}$$

Using the fact that $u^{\mu}\nabla_{\nu}u_{\mu}=0$, Eq.(A.II) may be exhibited in the form (A.I).

According to London the transition from the non viscous liquid to the superconductor is performed by replacing (A.I) by the following six equations:

$$\frac{mc^2}{q}\nabla_{[\mu} u_{\nu]}+F_{\mu\nu}=s_{\mu\nu}. \qquad \text{(A.III)}$$

Taking into account definition (4) Eq.(A.III) give Eq.(10).