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VELOCITIES OF MAGNETO-ELASTIC AND MAGNETO-ELECTRON WAVES IN DARK MOLECULAR CLOUDS

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We present analytic and numerical estimates for group velocity of wave motions in two models of cold interstellar medium presumably constituting interior of cores of magnetically supported dark molecular clouds. Namely, in the model of gas-based ferrocolloidal soft matter and in the model of non-compensated electron magnetoplasma. The prediction of these models are given in juxtaposition with data on recent Zeeman measurements of the molecular linewidths detected from dark central regions of star-forming interstellar clouds.

Key words: *waves: magnetic fields-ISM: molecular clouds*

1. *Introduction.* Current study of dynamics of star-forming molecular clouds leads to the conclusion that the intercloud motions are dominated by strong coupling between gas-dust flows and Galactic magnetic fields; the widths of molecular lines provide a direct evidence for such motions. Understandingly that real dynamics of interstellar gas-dust medium in the presence of magnetic fields is a fairly complex topic depending on subtle interplay between different unpredictable factors one of which is chemical composition of interstellar medium (ISM). In such situation a progress in getting insight into the ISM motions is made through idealized models [1] the most familiar of which is the magnetohydrodynamics (MHD). The underlying idea of MHD approach is that all mass of ISM of star-forming molecular clouds possesses properties of highly ionized magnetoplasma. This attitude has been extensively exploited over the years in interpretation of supersonic broadening of molecular lines either in terms of MHD turbulence or wave motions of Alfvén type. It has been found that such an approach provides a fairly reasonable account of data in CO regions of clouds where the temperature and the ionization factor are pretty high [2-5]. However, recent Zeeman measurements of magnetic field in highly obscured cores of molecular clouds have revealed the fact that velocity of intercloud motions are predominately sub-Alfvénic [6]. This discrepancy between MHD estimates and data might mean that the gas-dust composition of the dark cloud cores is quite different from what is implied by model of magnetoplasma.

In this communication we analyze the intercloud motions within the framework of two models alternative to the standard MHD model. First is the ferrohydrodynamical (FHD) model of magneto-elastic waves owing their existence to high degree of magnetic polarizability of interstellar gas-dust matter. Second is the electrodynamical model of spiral magneto-electron waves whose propagation is governed by Faraday's induction with the electric field generated by Hall effect caused by flows of thermal electrons. The predictions of these models regarding the group velocity of intercloud motions are' presented in juxtaposition with data on recent Zeeman measurements of the molecular linewidths detected from cores of star-forming interstellar clouds.

2. *Ferrocolloidal model of ISM.* One of conceivable hypothesis regarding the material composition of ISM constituting central regions of dark molecular clouds, poorly ionized by ultraviolet radiation is that it possesses properties of non-conducting gas-based ferrocolloid consisting of tiny ferromagnetic grains suspended in the dense gas of molecular hydrogen. This kind of superparamagnetic ISM has first been discussed by Jones and Spitzer [7] in the context of the starlight polarization problem. The most striking feature of the Jones-Spitzer ISM is its capability of sustaining long-ranged magnetization in form of linear, chain like, agglomerations of ferrograins extended along the magnetic fields threaded the cloud. Evidence is provided by filamentary structure of some of dark molecular clouds. Understandingly that the laws of continuum mechanics of non-conducting magnetically polarized matter are different from those for non-magnetic perfectly conducting magnetoplasma. In [8,9] the argument have been given that gas-dynamics of Jones-Spitzer ISM can be properly described by equations of magneto-elastodynamics of single-axis magneto-elastic insulators; similar equations are utilized in ferrohydrodynamics of ferrocolloidal magnetic fluid. In this paper we present more detail analytic and numerical analysis of this model. The basic idea underlying the ferrohydrodynamical model is to identify the behavior of two-component gas-dust ferrocolloidal ISM with single-component superparamagnetic soft matter of equivalent density and to describe its continuum mechanics in terms of the bulk density $\rho(\mathbf{r},t)$, the velocity $\mathbf{V}(\mathbf{r},t)$, and the field of magnetization $\mathbf{M}(\mathbf{r},t)$ (magnetic moment per unit volume) which are cosidered in one line as independent dynamical variable of motion. We use the version of FHD model whose governing equations have the form

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\rho\mathbf{V}=0, \tag{1}$$

$$\frac{\partial\mathbf{V}}{\partial t}=\frac{1}{2}\nabla\times[\mathbf{M}\times\mathbf{B}], \tag{2}$$

$$\frac{\partial\mathbf{M}}{\partial t}=[\mathbf{\Omega}\times\mathbf{M}],\quad \mathbf{\Omega}(\mathbf{r},t)=\frac{1}{2}[\nabla\times\mathbf{V}(\mathbf{r},t)]. \tag{3}$$

The equilibrium state of such a fluid is described by linear constitutive equation,

$$\mathbf{M}=\chi\mathbf{B}, \tag{4}$$

where the positive constant χ is the paramagnetic susceptibility of ferrocolloidal.

ISM. It is to be emphasized, in the gas-dust ferrofluid an interaction between the magnetic field and the field of magnetization is not a direct as is the case of solid ferromagnetics, but is mediated by verticalflow resulting in precession of the magnetization around that axis. Analytic form for driving force, which is defined as curl of magnetic torque, Eq.(3), shows that this force arises only under deflection of the magnetization from axis of background magnetic field. The most silent property of the Jones-Spitzer ISM is its capability of transmitting perturbation by magneto-elastic wave having many features in common with above discussed Alfvén wave in the perfectly conducting ISM. This comes from the following considerations. By applying the standard procedure of linearization to equations (1)-(3): $\mathbf{V} \to \mathbf{V} + \delta\mathbf{V}(\mathbf{r},t)$, and $\mathbf{M} \to \mathbf{M} + \delta\mathbf{M}(\mathbf{r},t)$, where $\mathbf{V} = 0$ and $\mathbf{M} = \chi\mathbf{B}$, we obtain

$$\nabla \cdot \delta\mathbf{V}(\mathbf{r},t) = 0, \quad \nabla \cdot \delta\mathbf{M}(\mathbf{r},t) = 0, \tag{5}$$

$$\rho\frac{\partial\delta\mathbf{V}(\mathbf{r},t)}{\partial t} = \frac{1}{2\chi}\nabla \times [\delta\mathbf{M}(\mathbf{r},t) \times \mathbf{M}], \tag{6}$$

$$\frac{\partial\delta\mathbf{M}(\mathbf{r},t)}{\partial t} = \frac{1}{2}[[\nabla \times \delta\mathbf{V}(\mathbf{r},t)] \times \mathbf{M}]. \tag{7}$$

This set of equations describes small-amplitude fluctuations of superparamagnetic incompressible fluid around axis of magnetic saturation which are not accompanied by appearance of density of magnetic poles [right of equations (5)]. Substitution into Eqs. (5) the plane-wave form of fluctuating variables

$$\delta\mathbf{V} \propto \exp(i\omega t - i\mathbf{kr}), \quad \delta\mathbf{M} \propto \exp(i\omega t - i\mathbf{kr}), \tag{8}$$

leads to

$$\mathbf{k} \cdot \delta\mathbf{V} = 0, \quad \mathbf{k} \cdot \delta\mathbf{M} = 0. \tag{9}$$

Inserting (8) into (6) yields

$$\omega\rho\delta\mathbf{V} = -\frac{1}{2\chi}(\mathbf{k} \cdot \mathbf{M})\delta\mathbf{M}. \tag{10}$$

After substitution of (8) into (7) we obtain

$$\omega\delta\mathbf{M} = -\frac{1}{2}[(\mathbf{k} \cdot \mathbf{M})\delta\mathbf{V} - \mathbf{k}(\delta\mathbf{V} \cdot \mathbf{M})]. \tag{11}$$

By taking scalar product of last equation with $\mathbf{k} \neq 0$ and considering (9), one finds $(\delta\mathbf{V} \cdot \mathbf{M}) = 0$. Given this, the link between frequency and wave vector in the magneto-elastic wave is defined by the following set of equations

$$\omega\rho\delta\mathbf{V} + \frac{1}{2\chi}(\mathbf{k} \cdot \mathbf{M})\delta\mathbf{M} = 0, \tag{12}$$

$$\omega\delta\mathbf{M} + \frac{1}{2}(\mathbf{k} \cdot \mathbf{M})\delta\mathbf{V} = 0. \tag{13}$$

One sees that perturbation is transmitted most effeciently when $\mathbf{k} \parallel \mathbf{M}$. In this case from (12) and (13) it follows

$$\omega = V_M k, \quad V_M = \frac{MB}{4\rho}. \tag{14}$$

By eliminating $\mathbf{k}\cdot\mathbf{M}$ from equations (12) and (13), one finds that magnetoelastic oscilatory motions satisfy the principle of energy equipartition

$$\frac{\rho\delta\mathbf{V}^2}{2}=\frac{\delta\mathbf{M}^2}{2\chi}, \tag{15}$$

which states that in the magneto-elastic wave the kinetic energy of fluctuating elastic displacements equals to the mean potential energy of fluctuating magnetization.

The obtained dispersion equation and equation of energy equipartition bears strong resemblance to those for transverse Alfvén wave in the single-component MHD model. However, the very existence of hydromagnetic wave is attributed to the perfect conductivity of cosmic dusty plasma; whereas the magneto-elastic wave owe its existence to magnetic polarizability of non-concluding ferrocolloidal gas-dust medium. As was stressed, the transverse MHD wave in interstellar magnetoplasma is accompanied by coupled fluctuations in the density of magnetic flux and velocity of hydrodynamic flow. The equations (12) and (13) show that in the FHD wave, the field of magnetization and velocity of hydrodynamic flow undergo coupled oscillations [10]. The transverse FHD wave in gas-based ferrocolloidal soft matter can be visualized by vibrations of flexible filaments of magnetization frozen in hydrodynamic flow about axis of background magnetic field; this filaments can be thought of as magnetic chains composed of tiny single domain grains coupled by dipole-dipole forces. It is noteworthly that in contrast to Bloch waves of magnetization in ferromagnetic dielectric solids which is characterized by quadratic in k dispersion relation, $\omega\sim k^2$, the wave transport of magnetization in gas-based ferrocolloidal interstellar medium is described by dispersion free law of propagation, $\omega\sim k$. Kinematically, the magneto-elastic wave have many features in common with the magneto-torsion wave in uniformly magnetized nematic liquid crystals [10]. It is instructive to consider the model of a spherical cloud with uniform magnetization inside. In such a cloud the internal magnetic field is described by the equation [11]: $\mathbf{B}+2\mathbf{H}=0$ and $\mathbf{B}-\mathbf{H}=4\pi\mathbf{M}$, from which it follows $\mathbf{M}=(3/8\pi)\mathbf{B}$. This latter equation can be taken as a constitutive equation for superparamagnetic medium, since the value $\chi=3/8\pi\approx 0.1$ is typical of magnetic fluids. From this free any adjustable constants and a highly idealized model it follows that at equal B and ρ, the speed of magneto-elastic wave is sub-Alfvénic - $V_M\approx 0.6V_A$. In Fig.1 we plot group velocity V_M of magneto-elastic wave as function of magnetic flux density and particle density; the data have been taken from [6]. One sees that prediction of the model are in a line with this data what can be interpreted as indirect evidence of ferrocolloidal composition of intercloud ISM.

3. *Electron interstellar magnetoplasma*. One more quite plausible wave process owing its existence to the strong coupling intercloud flows with Galactic magnetic field can be helicoidal magneto-electron waves [12]. This

process can take place in the clouds in which large scale motions are dominated by electrons whereas mobility of ions and neutrals is heavily suppressed. The cold ISM of central regions of dark molecular clouds is one of the

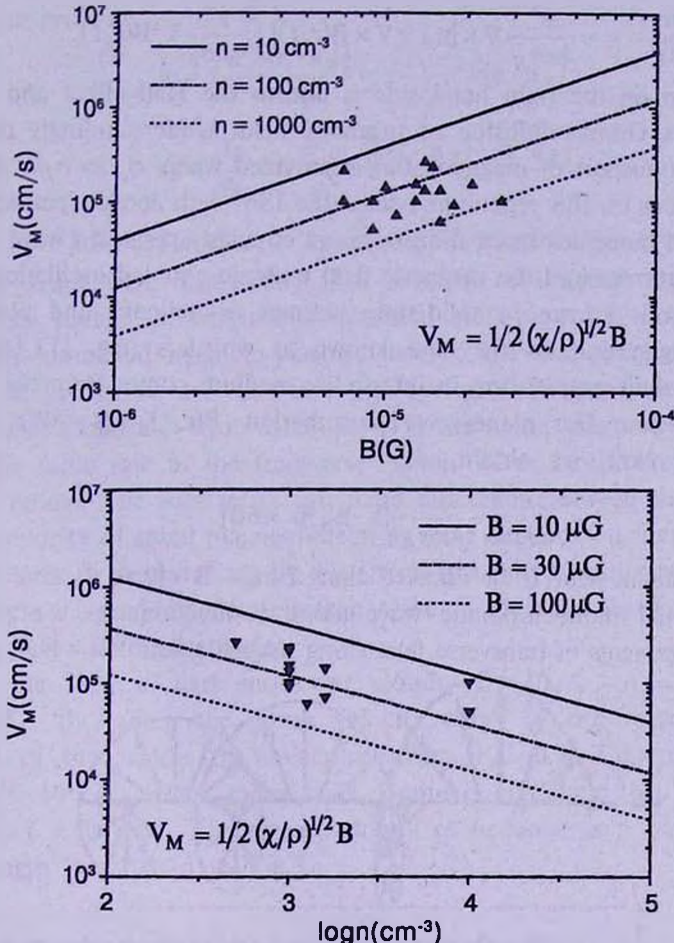

Fig.1. Velocity of the magnetization wave as function of magnetic field (upper panel) and particle density (lower panel); symbols - data from [6].

plausible sites of such a medium. The analysis of motions in such an ISM can be performed on the equation for Faraday's induction

$$\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} = -c\nabla\times\mathbf{E}(\mathbf{r},t), \tag{16}$$

supplemented by constitutive equation

$$\mathbf{E}(\mathbf{r},t) = \frac{\mathbf{j}(\mathbf{r},t)}{\sigma_c} + \frac{[\mathbf{n}_B\times\mathbf{j}(\mathbf{r},t)]}{\sigma_H}, \quad \mathbf{n}_B = \frac{\mathbf{B}}{B}, \tag{17}$$

$$\sigma_c = \frac{n_e e^2}{m_e \nu_c}, \quad \sigma_H = \frac{e n_e c}{B}, \quad \mathbf{j}(\mathbf{r},t) = \frac{c}{4\pi}\nabla\times\mathbf{B}(\mathbf{r},t), \tag{18}$$

where the Ohmic conductivity σ_c is given by Drude formula, and σ_H stands for the Hall conductivity; $\mathbf{j}(\mathbf{r},t)$ is the density of Ampére's curent. Inserting (17) into (16) we obtain

$$\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} = -\frac{c^2}{4\pi\sigma_H}\nabla\times[\mathbf{n}_B\times\nabla\times\mathbf{B}(\mathbf{r},t)] + \frac{c^2}{4\pi\sigma_c}\nabla^2\mathbf{B}(\mathbf{r},t). \tag{19}$$

The first term on the right hand side is due to the Hall effect and second term describes Ohmic diffusion of magnetic field. Understandingly that dissipative-free transport of magnetic flux is provided when $\sigma_c >> \sigma_H$. In what follows we focus on this regime and show the ISM with above electrodynamic properties can transmit perturbation by spiral circularly polarized wave in wich the electron current and the magnetic field undergo coupled oscillations. This sort of waves is known in solid-state plasmas as helicons and physics of planetary magnetospheres they are known as whistlers (e.g. [13,14]). The possibility of their propagation in interstellar medium comes from the following consideration. For plane-wave perturbation $\mathbf{B}(\mathbf{r},t)\to\mathbf{B}+\delta\mathbf{B}(\mathbf{r},t)$ with $\delta\mathbf{B}=\mathbf{b}(t)\exp(-i\mathbf{k}\mathbf{r})$, we obtain

$$\frac{\partial\delta\mathbf{B}}{\partial t} = -\frac{c^2}{4\pi\sigma_H}(\mathbf{k}\cdot\mathbf{n}_B)[\mathbf{k}\times\delta\mathbf{B}]. \tag{20}$$

Let the permanent field $\mathbf{B}$ be directed along z-axis: $\mathbf{B}=[0,0,B]$, and consider one-dimensional monochromatic wave along z direction, $\mathbf{k}=k\,\mathbf{e}_z$. In this case the components of transverse fluctuating magnetic field $\delta\mathbf{B}=[\delta B_x,\delta B_y,0]$

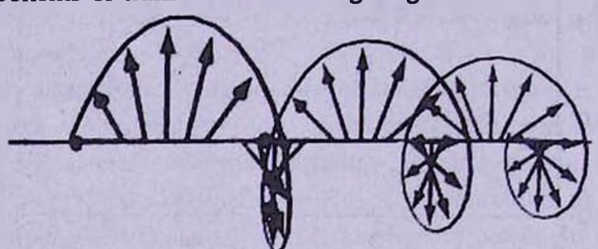

Fig.2. Hodograph of the magnetic field vector in the helicoidal magneto-electron wave.

depend only upon z and t. It is easy to see that the equation (20) amounts to the equation for precession $\delta\dot{\mathbf{B}}=[\mathbf{\Omega}\times\delta\mathbf{B}]$ of the vector $\delta\mathbf{B}$ about z-axis with the angular frequency $\mathbf{\Omega}=-(cB)/(4\pi n_e e)k^2\,\mathbf{e}_z$. Fig.2 pictures the spiral motions of magnetic field density in helicoidal magneto-electron wave. In cartesian coordinates Eq. (20) reads

$$\delta\dot{B}_x = +\frac{c^2k(\mathbf{k}\cdot\mathbf{n}_B)}{4\pi\sigma_H}\delta B_y,\quad \delta\dot{B}_y = -\frac{c^2k(\mathbf{k}\cdot\mathbf{n}_B)}{4\pi\sigma_H}\delta B_x. \tag{21}$$

The wave motions in which two components of the vector field are coupled by a rotation, are customarily described in terms of the right-hand δB_+ and the left-hand δB_- circularly polarized wave's fields $\delta B_+=\delta B_x+i\delta B_y=b(z)\exp(-i\omega t)$ and $\delta B_-=\delta B_x-i\delta B_y=b(z)\exp(-i\omega t)$. After simple algebra we obtain

$$\frac{\partial\delta B_\pm}{\partial t} = \mp i\frac{c^2k(\mathbf{k}\cdot\mathbf{n}_B)}{4\pi\sigma_H}\delta B_\pm \tag{22}$$

From (22) it follows that the frequency and group velocity of magneto-electron wave are given by

$$\omega = \pm\frac{c^2k(\mathbf{k}\cdot\mathbf{n}_B)}{4\pi\sigma_H} = \pm\frac{\omega_c}{\omega_p^2}c^2k(\mathbf{k}\cdot\mathbf{n}_B), \tag{23}$$

$$\mathbf{V}_h = \frac{\partial\omega}{\partial\mathbf{k}} = \pm\frac{c^2\,\omega_c}{\omega_p^2}\frac{\left[\mathbf{k}(\mathbf{k}\cdot\mathbf{n}_B)+k^2\,\mathbf{n}_B\right]}{k}, \tag{24}$$

where $\omega_c = [eB/m_ec]$ and $\omega_p = \left[4\pi e^2n_e/m_e\right]^{1/2}$ stand for the cyclotron and the plasma frequencies, respectively. The helicons or whistlers represent a low-frequency branch of magneto-mechanical exitation in a non-compensated electron-dominated magneto-plasma, that is, the frequency of electron oscillations in this wave is less than the cyclotron frequency $\omega < \omega_c$ [13]. In the electron magneto-hydrodynamics [15] the spiral magneto-electron waves play the same role as the transverse Alfvén waves in the single-component MHD model. The substantial kinematic difference between them is that the group velocity of spiral magneto-electron wave depends on frequency whereas Alfvén wave is characterized by dispersion-free law of propagation: kinematic character of spiral magneto-electron wave is identical to above mentioned Bloch wave in ferromagnetics.

In the ISM of dark molecular clouds, $B \approx 10\mu G$, $n_e \approx 10^{-3}\text{cm}^{-3}$ and $T \approx 10\text{K}$. By taking the group velocity equal to the velocity dispersion typical of the widths of molecular lines, $V_h \sim 0.3 - 5.0$ km/s, one finds $\lambda \sim 10^{12} - 10^{13}$ cm. This space scale is much less than the linear sizes of clouds, $L \sim 10^{17}$ cm. The group velocity of helicons as a function of their wavelength is pictured in Fig.3.

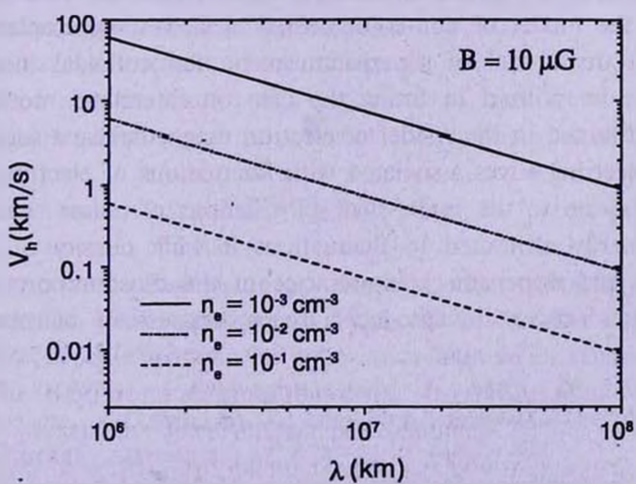

Fig.3. The group velocity V_h of helicoidal magneto-electron wave as a function of wavelength λ.

If one takes into account the effect of the Ohmic decay of the magnetic flux density one can show that the dispersion relation takes the form

$$\omega = \frac{\omega_c}{\omega_p^2} c^2 k^2 \left(1 + i \frac{\sigma_H}{\sigma_c}\right). \tag{25}$$

In a typical dark molecular cloud, $\omega_c \approx 10^2\ s^{-1}$ and $\nu_c \approx 10^{-3} - 10^{-1}\ s^{-1}$ so that the criterion of dissipation-free propagation of helicons $\Gamma = \sigma_H/\sigma_c = \nu_c/\omega_c \ll 1$ is well justified, and its validity remains quite robust to the changes in ν_c up to $\nu_c = 100\ s^{-1}$. These estimates suggest that the helicons can freely propagate in the dark molecular clouds and we conjuncture that they can contribute to the broadening of molecular line [12].

4. *Summary.* While the central role of magnetic fields in interstellar gas dynamics was recognized many years ago, major uncertainties regarding the character of the motions in interstellar clouds followed from inadequate knowledge of the material composition of the intercloud medium. One of the hypothesis is the presence in molecular clouds of a sizable fraction of charged particles (primarily electrons and ions) whose collective flows are strongly coupled with the intercloud magnetic field. On the assumptions that the magnetic field causes both electrons and ions to move with equal velocities and then friction causes the neutral molecules to follow the ions with same velocity, the model of single-component magnetohydrodynamics (MHD) has been extensively exploited in interpreting supersonic broadening of molecular lines in terms of hydromagnetic waves of the Alfvén type. However in inner region of dark molecular clouds where the ionizing ultraviolet radiation is heavily suppressed the applicability of MHD approach becomes questionable. In this paper we investigate two another models of intercloud gas dynamics which are interesting in their own rights because they can have more wide range of applicability. Namely, the ferrohydrodynamical model of gas-based ferrocolloidal soft matter and the model of non-compensated electron magnetoplasma. We have shown that the model of superparamagnetic ferrocolloidal medium of Jones-Spitzer can be utilized in fitting the data on intercloud motions.

The results obtained in the model of electron magnetoplasma suggest that spiral magneto-electron waves associated with fluctuations of electron current density can contribute to the interstellar scincillations of pulsar signals. The latter are customarily attributed to fluctuations in bulk density of electron which determine the dispersion measure, one of the most important characteristics of pulsar activity. In case $\mathbf{n}_B \parallel \mathbf{k}$, the group velocity of these waves is given by

$$V_h = \frac{2c\sqrt{\omega_c}}{\omega_p}\sqrt{\omega} \approx 4.46 \times 10^9 \sqrt{\frac{B}{n_e}}\sqrt{\omega}\ \text{cm/s}. \tag{26}$$

This points to the possibility for determination of helicity measure of ISM depending on ratio B/n_e the quantity analogous to dispersion measure which depends on n_e and rotation measure depending on B; understandably that this issue demands special consideration.

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СКОРОСТЬ МАГНИТО-ЭЛАСТИЧНЫХ И МАГНИТО-ЭЛЕКТРОННЫХ ВОЛН В ТЕМНЫХ МОЛЕКУЛЯРНЫХ ОБЛАКАХ

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Выполнены аналитические и численные оценки групповой скорости предположительно неплохо моделирующих ядра магнито-удерживаемых темных молекулярных облаков для двух случаев холодного межзвездного вещества, а именно для модели газовой ферроколоидной пластичной материи и для модели нескомпенсированной магнитоплазмы электронов. Использование предложенных моделей обосновывается сравнением с результатами недавних измерений молекулярной зеемановской ширины линий в спектрах, детектируемых от темных центральных областей звездообразующих межзвездных облаков.

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