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MAGNETIC FIELD OF A NEUTRON STAR WITH  
COLOR SUPERCONDUCTING QUARK MATTER CORED.M.SEDRAKIAN<sup>1</sup>, D.BLASCHKE<sup>2,3</sup>

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The behaviour of the magnetic field of a neutron star with a superconducting quark matter core is investigated in the framework of the Ginzburg-Landau theory. We take into account the simultaneous coupling of the diquark condensate field to the usual magnetic and to the gluomagnetic gauge fields. We solve the Ginzburg-Landau equations by properly taking into account the boundary conditions, in particular, the gluon confinement condition. We found the distribution of the magnetic field in both the quark and hadronic phases of the neutron star and show that the magnetic field penetrates into the quark core in the form of quark vortices due to the presence of Meissner currents.

Key words: *stars: neutron, stars: magnetic fields*

1. *Introduction.* Recently, possible formation of diquark condensates in QCD at finite density has been re-investigated in series of papers following [1,2]. It has been shown that in chiral quark models with non-perturbative 4-point interaction motivated from instantons [3] or non-perturbative gluon propagators [4,5] the anomalous quark pair amplitudes in the color antitriplet channel can be very large: of the order of 100 MeV. Therefore, one expects the diquark condensate to dominate the physics at densities beyond the deconfinement/chiral restoration transition density and below the critical temperature (of the order of 50 MeV). Various phases are possible. The so called two-flavor (2SC) and three-flavor (3SC) phases allow for unpaired quarks of one color. It has been also found [6,7] that there can exist a color-flavor locked (CFL) phase for not too large strange quark masses [8], where color superconductivity is complete in the sense that diquark condensation results in a pairing gap for the quarks of all three flavors and colors.

The high-density phases of QCD at low temperatures are relevant for the explanation of phenomena in rotating massive compact stars which might manifest themselves as pulsars. Physical properties of these objects (once being measured) could constrain our hypotheses about the state of matter at the extremes of densities. In contrast to the situation for the cooling behaviour of compact stars [9,10], where the CFL phase is dramatically different from the 2SC and 3SC phases, we don't expect qualitative changes of the magnetic field structure for these phases. Therefore, below we shall restrict ourselves to the discussion of the simpler two-flavor theory first. Bailin and Love [11] used a

perturbative gluon propagator which yielded a very small pairing gap and they concluded that quark matter is a type I superconductor, which expells the magnetic field of a neutron star within time-scales of  $10^4$  years. If their arguments would hold in general, the observation of life-times for magnetic fields as large as  $10^7$  years [12,13] would exclude the occurrence of an extended superconducting quark matter core in pulsars. These estimates are not valid for the case of diquark condensates characterized by large quark gaps. Besides, in [14] the authors found that within recent non-perturbative approaches for the effective quark interaction that allow for large pairing gaps the quark condensate forms a type II superconductor. Consequently for the magnetic field  $H < H_{c1}$ , there exists a Meissner effect and for  $H_{c2} > H > H_{c1}$ , the magnetic field can penetrate into the quark core in quantized flux tubes. However, they have not considered in that paper the simultaneous coupling of the quark fields to the magnetic and gluomagnetic gauge fields.

Though color and ordinary electromagnetism are broken in a color superconductor, there is a linear combination of the photon and the gluon that remains massless. The authors of [15] have considered the problem of the presence of magnetic fields inside color superconducting quark matter taking into account the possibility of the so called "rotated electromagnetism". They came to the conclusion that there is no Meissner effect and the external static homogeneous magnetic field can penetrate into superconducting quark matter because in their case it obeys the sourceless Maxwell equations. To our opinion this result is obtained when one does not pose correct boundary conditions for the fields. Obviously it is energetically favorable to expell the magnetic field rather than to allow its penetration inside the superconducting matter. Using for the description of the diquark condensate interacting with two gauge fields the same model as in [9,8] and [16], we will show that the presence of the massless excitation in the spectrum does not prevent the Meissner currents to effectively screen the static external magnetic field.

In [16] two of us have derived the Ginzburg-Landau equations of motion for the diquark condensate placed in static magnetic and gluomagnetic fields,

$$ad_p + \beta(d_p d_p^*)d_p + \gamma \left( i \nabla - \frac{e}{3} \bar{A} + \frac{g}{\sqrt{3}} \bar{G}_8 \right)^2 d_p = 0, \quad (1)$$

where  $d_p$  is the order parameter,  $a = tdn/dE$ ,  $\beta = (dn/dE)7\zeta(3)(\pi T_c)^{-2}/8$ ,  $\gamma = p_F^2 \beta / (6\mu^2)$ ,  $dn/dE = p_F \mu / \pi^2$ ,  $t = (T - T_c) / T_c$ ,  $T_c$  being the critical temperature,  $p_F$  - the quark Fermi momentum, and for the gauge fields

$$\lambda_q^2 \text{rotrot} \bar{A} + \sin^2 \alpha \bar{A} = i \frac{\sin \alpha (d_p \nabla d_p^* - d_p^* \nabla d_p)}{2qd^2} + \sin \alpha \cos \alpha \bar{G}_8, \quad (2)$$

$$\lambda_q^2 \text{rotrot} \bar{G}_8 + \cos^2 \alpha \bar{G}_8 = -i \frac{\cos \alpha (d_p \nabla d_p^* - d_p^* \nabla d_p)}{2qd^2} + \sin \alpha \cos \alpha \bar{A}. \quad (3)$$

These equations introduce a "new" charge of the diquark pair  $q = \sqrt{\eta^2 e^2 + g^2} P_8$ ,  $P_8 = 1/\sqrt{3}$ , and for the diquark condensate with paired blue-green and green-blue  $ud$  quarks one has  $\eta = 1/\sqrt{3}$ . The penetration depth of the magnetic and gluomagnetic fields  $\lambda_q$  and the mixing angle  $\alpha$  are given by

$$\lambda_q^{-1} = qd \sqrt{2\gamma}, \quad \cos\alpha = \frac{g}{\sqrt{\eta^2 e^2 + g^2}}. \quad (4)$$

At neutron star densities gluons are strongly coupled ( $g^2/4\pi \sim 1$ ) whereas photons are weakly coupled ( $e^2/4\pi = 1/137$ ), so that  $\alpha \approx \eta e/g$  is small. For  $g^2/4\pi = 1$  we get  $\alpha \approx 1/20$ . The new charge  $q$  is by an order of magnitude larger than  $e/\sqrt{3}$ . Please notice also that since red quarks are normal in the 2SC and 3SC phases, there exist the corresponding normal currents  $j_\mu^r(A) = -\Pi_{\mu\nu}^{r\prime} A^\nu$  and  $j_\mu^r(G_8) = -\Pi_{\mu\nu}^{r\prime} G_8^\nu$  which however do not contribute in the static limit under consideration to the above Ginzburg - Landau equations, cf. [17]. Thus, the qualitative behavior of the static magnetic field for all three 2SC, 3SC, and CFL phases is the same. Recently the influence of a constant uniform chromomagnetic field on the formation of color superconductivity and the role of the Meissner effect for tightly bound states have been considered in the papers [18,19].

We will solve the Ginzburg-Landau equations (1), (2), (3) by properly taking into account the boundary conditions, in particular, the gluon confinement condition. We will find the distribution of the magnetic field in both the quark and hadronic phases of the neutron star and will show that the magnetic field penetrates into the quark core in the form of quark vortices due to the presence of Meissner currents.

We assume a sharp boundary between the quark and hadron matter since the diffusion boundary layer is thin, of the order of the size of the confinement radius ( $\sim 0.2 + 0.4$  fm), and we suppose that the coherence length  $l_\xi = \sqrt{\gamma/(-2\alpha)}$  is not less than this value and the magnetic and gluomagnetic field penetration depth  $\lambda_q$  is somewhat larger than the confinement radius. Also we assume that the size of the quark region is much larger than all mentioned lengths.

*2. Solution of Ginzburg-Landau equations.* Let us rewrite equations (2) and (3) for a homogeneous superconducting matter region being a type II superconductor in the following form

$$\lambda_q^2 \text{rotrot } \bar{A} + \sin^2 \alpha \bar{A} = \bar{f} \sin \alpha + \sin \alpha \cos \alpha \bar{G}_8, \quad (5)$$

$$\lambda_q^2 \text{rotrot } \bar{G}_8 + \cos^2 \alpha \bar{G}_8 = -\bar{f} \cos \alpha + \sin \alpha \cos \alpha \bar{A}, \quad (6)$$

where

$$\bar{f} = \lambda_q^2 4\pi i q \gamma [\bar{d} \bar{\nabla} \bar{d}^* - \bar{d}^* \bar{\nabla} \bar{d}] \quad (7)$$

and obeys the equation

$$\text{rotrot } \vec{f} = 0. \quad (8)$$

If we introduce

$$\vec{A}' = \vec{A} - \frac{\vec{f}}{2\sin\alpha}, \quad \vec{G}'_8 = \vec{G}_8 + \frac{\vec{f}}{2\cos\alpha}, \quad (9)$$

then Eqs. (5) and (6) can be rewritten in the form:

$$\frac{\lambda_q^2}{\sin\alpha} \text{rotrot } \vec{A}' = \cos\alpha \vec{G}'_8 - \sin\alpha \vec{A}', \quad (10)$$

$$-\frac{\lambda_q^2}{\cos\alpha} \text{rotrot } \vec{G}'_8 = \cos\alpha \vec{G}'_8 - \sin\alpha \vec{A}'. \quad (11)$$

We can define  $\vec{G}'_8$  from (10) as follows

$$\vec{G}'_8 = \frac{\lambda_q^2 \text{rotrot } \vec{A}' + \sin^2\alpha \vec{A}'}{\sin\alpha \cos\alpha}. \quad (12)$$

We derive from Eqs. (11) and (12) the relation

$$\text{rotrot } \vec{G}'_8 = -\cot\alpha \text{rotrot } \vec{A}'. \quad (13)$$

We can consider instead of the system of coupled equations (10) and (11) the equivalent system (12) and (13). We substitute  $\vec{G}'_8$  from (12) into (13) and obtain the following equation

$$\text{rotrot} \left( \lambda_q^2 \text{rotrot } \vec{A}' + \vec{A}' \right) = 0. \quad (14)$$

We introduce the new function  $\vec{M}'$  as

$$\vec{M}' = \text{rotrot } \vec{A}', \quad (15)$$

we obtain

$$\lambda_q^2 \text{rotrot } \vec{M}' + \vec{M}' = 0. \quad (16)$$

So we can determine the function  $\vec{A}'$  by simultaneous solution of the equations (15) and (16). Then we can find the electromagnetic potential  $\vec{A}$  and the gluonic potential  $\vec{G}_8$  from equations (12) and (9).

In order to determine the distribution of electromagnetic and gluonic potentials inside the superconducting quark matter core in response to an external magnetic field we shall require on the quark-hadronic matter boundary both the continuity of the magnetic field and the vanishing of the gluon potential ( $\vec{G}_8 = 0$ ) due to gluon confinement. As we shall see below, these conditions are sufficient for a unique determination of the magnetic and gluomagnetic fields inside the quark matter region.

We shall assume that a neutron star with radius  $R$  possesses a spherical core of radius  $a$  consisting of color superconducting quark matter surrounded by a spherical shell of hadronic matter with thickness  $R - a$ . The functions  $\vec{M}'$ ,  $\vec{A}$  and  $\vec{G}_8$  in spherical coordinates  $(r, \vartheta, \varphi)$  have only  $\varphi$ -components  $M'_\varphi(r, \vartheta)$ ,  $\vec{A}_\varphi(r, \vartheta)$  and  $\vec{G}_{8\varphi}(r, \vartheta)$ . For the solution of the equation (16) we make the ansatz  $M'_\varphi(r, \vartheta) = M_\varphi(r) \sin\vartheta$ . Then equation (16) can be written as

$$\frac{d^2 M_\varphi(r)}{dr^2} + \frac{2}{r} \frac{dM_\varphi(r)}{dr} - \left( \frac{2}{r^2} + \frac{1}{\lambda_q^2} \right) M_\varphi(r) = 0. \quad (17)$$

The solution of equation (17) is

$$M_\varphi(r) = \frac{1}{r^2} \left[ c_1' \left( 1 - \frac{r}{\lambda_q} \right) e^{r/\lambda_q} + c_2' \left( 1 + \frac{r}{\lambda_q} \right) e^{-r/\lambda_q} \right]. \quad (18)$$

The condition that  $M_\varphi(r)$  tends to zero at the center of the quark core gives  $c_1' = -c_2'$ , so that

$$M_\varphi(r) = \frac{c_1}{r^2} \left[ \sin h \frac{r}{\lambda_q} - \frac{r}{\lambda_q} \cos h \frac{r}{\lambda_q} \right]. \quad (19)$$

Substituting the solution (19) into equation (15) for  $\bar{A}'$  we obtain the following solution

$$A_\varphi'(r, \vartheta) = M_\varphi'(r, \vartheta) + c_0' r \sin \vartheta. \quad (20)$$

Taking into account (20) and using equation (9) for the electromagnetic potential we obtain

$$A_\varphi(r, \vartheta) = M_\varphi'(r, \vartheta) + c_0' r \sin \vartheta + \frac{f_\varphi(r, \vartheta)}{2 \sin \alpha}. \quad (21)$$

The unknown function  $f_\varphi(r, \vartheta)$  we will find from the solution of equation (8) which gives

$$f_\varphi(r, \vartheta) = c_0' r \sin \vartheta. \quad (22)$$

Finally we get for the electromagnetic potential

$$A_\varphi(r, \vartheta) = \left[ M_\varphi(r) + c_0' r + \frac{c_0' r}{2 \sin \alpha} \right] \sin \vartheta. \quad (23)$$

Using equations (12) and (9), we find the gluonic potential  $G_{8\varphi}$  in the form

$$G_{8\varphi}(r, \vartheta) = \left[ -\cot \alpha M_\varphi(r) + \tan \alpha c_0' r - \frac{c_0' r}{2 \sin \alpha} \right] \sin \vartheta. \quad (24)$$

The constant  $c_0'$  we can define using the gluon confinement condition on the surface of the quark matter core  $G_{8\varphi}(a, \vartheta) = 0$ . This condition will define  $c_0'$  as

$$c_0' = \cot^2 \alpha \frac{M_\varphi(a)}{a} + \frac{c_0}{2 \sin \alpha}, \quad (25)$$

to be substituted into equations (23) and (24) so that we obtain for the final expressions for the electromagnetic and gluonic potentials

$$A_\varphi(r, \vartheta) = \left[ M_\varphi(r) + \cot^2 \alpha \frac{r}{a} M_\varphi(a) + \frac{c_0' r}{\sin \alpha} \right] \sin \vartheta, \quad (26)$$

$$G_{8\varphi}(r) = - \left[ M_\varphi(r) - \frac{r}{a} M_\varphi(a) \right] \cot \alpha \sin \vartheta. \quad (27)$$

In ending this section we mention that the electromagnetic potential in the hadronic matter part of the neutron star can be found from the solution (18) by replacing the penetration depth for quark matter  $\lambda_q$  with that for hadronic matter  $\lambda_p$ .

3. *The magnetic field components for the neutron star.* The components of the magnetic fields in quark and hadronic matter can be found from those of the vector potentials using the formula  $\vec{B} = \text{rot } \vec{A}$ . In spherical coordinates we have

$$B_r = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\phi(r, \theta)), \quad (28)$$

$$B_\theta = -\frac{1}{r} \frac{\partial}{\partial r} (r A_\phi(r, \theta)). \quad (29)$$

For the case of quark matter we have to insert into these formulae the expression (refm1) for  $A_\phi(r, \theta)$ . Then finally we get (for  $r \leq a$ )

$$B_r^q = \left[ \frac{2M_\phi(r)}{r} + 2\cot^2 \alpha \frac{M_\phi(a)}{a} + \frac{2c_0}{\sin \alpha} \right] \cos \theta, \quad (30)$$

$$B_\theta^q = -\left[ \frac{1}{r} \frac{d}{dr} (rM_\phi(r)) + 2\cot^2 \alpha \frac{M_\phi(a)}{a} + \frac{2c_0}{\sin \alpha} \right] \sin \theta, \quad (31)$$

where  $M_\phi(r)$  is defined by equation (19).

The magnetic field in hadronic matter phase we can find from the solution (18) by taking into account that proton vortices in this phase generate a homogeneous mean magnetic field with amplitude  $B$  and direction parallel to the axis of rotation of the star [20]. For the components of the magnetic field  $\vec{B}^p$  in the hadronic phase (for  $a \leq r \leq R$ ) we get the following expressions

$$B_r^p = \left[ \frac{2A_\phi(r)}{r} + B \right] \cos \theta, \quad (32)$$

$$B_\theta^p = -\left[ \frac{1}{r} \frac{d}{dr} (rA_\phi(r)) + B \right] \sin \theta, \quad (33)$$

where

$$A_\phi(r) = \frac{c_2}{r^2} \left( 1 - \frac{r}{\lambda_p} \right) e^{r/\lambda_p} + \frac{c_3}{r^2} \left( 1 + \frac{r}{\lambda_p} \right) e^{-r/\lambda_p}. \quad (34)$$

As we have mentioned above  $\lambda_p$  is the penetration depth in hadronic matter.

The external magnetic field  $\vec{B}^e$  in the region outside of the star ( $r \geq R$ ) has to be dipolar and their components are given by the following expressions

$$B_r^e = \frac{2\mathcal{M}}{r^3} \cos \theta, \quad B_\theta^e = \frac{\mathcal{M}}{r^3} \sin \theta, \quad (35)$$

where  $\mathcal{M}$  is the full magnetic moment of the star. The unknown constants  $c_0, c_1, c_2, c_3$  and  $\mathcal{M}$  in equations (28)-(35) have to be defined from the continuity conditions of the magnetic field components at  $r=a$  and  $r=R$  and from the condition

$$B^q V_1 + B V_2 = 8\pi/3\mathcal{M}, \quad (36)$$

where  $B^q$  is the  $z$  component of the magnetic field in the quark matter region with the volume  $V_1$ ,  $V_2$  is the volume of the hadronic matter region. Here we suppose that the magnetic field in both regions is mainly constant and parallel

to the axis of rotation  $z$ . As we will see later this supposition is fulfilled with high accuracy because  $a$ ,  $R$  and  $R-a$  are much greater than  $\lambda_p$  and  $\lambda_q$ .

The continuity of the magnetic field components at  $r=a$  and  $r=R$  gives us the following equations

$$\frac{2M_\varphi(a)}{a} + 2\cot^2 \frac{M_\varphi(a)}{a} + \frac{2c_0}{\sin\alpha} = \frac{2A_\varphi(a)}{a} + B, \quad (37)$$

$$\frac{1}{r} \frac{d}{dr} (rM_\varphi(r)) \Big|_{r=a} + 2\cot^2 \frac{M_\varphi(a)}{a} + \frac{2c_0}{\sin\alpha} = \frac{1}{r} \frac{d}{dr} (rA_\varphi(r)) \Big|_{r=a} + B, \quad (38)$$

$$\frac{2A_\varphi(R)}{R} + B = \frac{2\mathcal{M}}{R^3}, \quad (39)$$

$$\frac{1}{r} \frac{d}{dr} (rA_\varphi(r)) \Big|_{r=R} + B = -\frac{\mathcal{M}}{R^3}. \quad (40)$$

Substituting the functions  $M_\varphi(r)$  and  $A_\varphi(r)$  from equations (19) and (21) into the system of equations (37) - (40) we find the following system of equations

$$0 = c_1 \left[ \left( 1 + \frac{a^2}{3\lambda_q^2} \right) \sin h \frac{a}{\lambda_q} - \frac{a}{\lambda_q} \cos h \frac{a}{\lambda_q} \right] - c_2 \left[ 1 - \frac{a}{\lambda_p} + \frac{a^2}{3\lambda_p^2} \right] e^{\frac{a}{\lambda_p}} - c_3 \left[ 1 + \frac{a}{\lambda_p} + \frac{a^2}{3\lambda_p^2} \right] e^{-\frac{a}{\lambda_p}}, \quad (41)$$

$$D = c_1 \left( \sin h \frac{a}{\lambda_q} - \frac{a}{\lambda_q} \cos h \frac{a}{\lambda_q} \right) - \left[ c_2 \left( 1 - \frac{a}{\lambda_p} \right) e^{\frac{a}{\lambda_p}} + c_3 \left( 1 + \frac{a}{\lambda_p} \right) e^{-\frac{a}{\lambda_p}} \right] \sin^2 \alpha, \quad (42)$$

$$\mathcal{M} = c_2 \left( 1 - \frac{R}{\lambda_p} + \frac{R^2}{3\lambda_p^2} \right) e^{\frac{R}{\lambda_p}} + c_3 \left( 1 + \frac{R}{\lambda_p} + \frac{R^2}{3\lambda_p^2} \right) e^{-\frac{R}{\lambda_p}}, \quad (43)$$

$$\mathcal{M} = c_2 \left( 1 - \frac{R}{\lambda_p} \right) e^{\frac{R}{\lambda_p}} + c_3 \left( 1 + \frac{R}{\lambda_p} \right) e^{-\frac{R}{\lambda_p}} + \frac{BR^3}{2}, \quad (44)$$

where

$$D = \frac{Ba^3}{2} \sin^2 \alpha - c_0 a^3 \sin \alpha. \quad (45)$$

The solution of this system of equations using the fact that  $a$ ,  $R$  and  $R-a$  are much larger than  $\lambda_q$  and  $\lambda_p$  we obtain the following expressions for  $c_1$ ,  $c_2$ ,  $c_3$  and  $\mathcal{M}$

$$c_1 = -\frac{\lambda_q^2}{a\lambda_p} \frac{D}{\sin^2 \alpha + \frac{\lambda_q}{\lambda_p} \sin h \frac{a}{\lambda_q}}, \quad (46)$$

$$c_2 = \frac{\lambda_p}{2a} \frac{De^{\frac{R}{\lambda_p}}}{\sin^2 \alpha + \frac{\lambda_q}{\lambda_p} \sin h \frac{R-a}{\lambda_p}}, \quad (47)$$

$$c_3 = -\frac{\lambda_p}{2a} \frac{De^{\frac{R}{\lambda_p}}}{\sin^2\alpha + \frac{\lambda_q}{\lambda_p} \sinh \frac{R-a}{\lambda_p}} \cdot 1, \quad (48)$$

$$\mathcal{M} = \frac{BR^3}{2} - \frac{R}{a} \frac{D}{\sin^2\alpha + \frac{\lambda_q}{\lambda_p} \sinh \frac{R-a}{\lambda_p}} \cdot 1. \quad (49)$$

In order to obtain a final result we have to determine the constant  $c_0$  assuming  $B$  is constant. The latter is given since in the hadronic phase entrainment currents generate proton vortices which provide this field [20].

Before we go over to the determination of  $c_0$  let us consider the behaviour of the magnetic field at distances  $r$  much larger than  $\lambda_p$  and  $\lambda_q$ . Also we take into account that  $\lambda_q \ll \lambda_p \ll a$ ,  $\lambda_q \ll \lambda_p \ll R$  and  $\lambda_q \ll \lambda_p \ll R-a$ . Using the expressions (46) - (49) for the constants, which we have obtained within this approximation, the components of the magnetic field in the different regions of the neutron star take the following form.

For  $r \leq a$

$$B_r^q = \left[ \frac{2D}{ar^2} \frac{e^{\frac{a-r}{\lambda_q}}}{\sin^2\alpha + \frac{\lambda_q}{\lambda_p}} \frac{\lambda_q}{\lambda_p} + 2\cot^2\alpha \frac{M_\varphi(a)}{a} + \frac{2c_0}{\sin\alpha} \right] \cos\theta, \quad (50)$$

$$B_\theta^q = - \left[ \frac{D}{ar\lambda_p} \frac{e^{\frac{a-r}{\lambda_q}}}{\sin^2\alpha + \frac{\lambda_q}{\lambda_p}} + 2\cot^2\alpha \frac{M_\varphi(a)}{a} + \frac{c_0}{\sin\alpha} \right] \sin\theta, \quad (51)$$

for  $a \leq r \leq R$

$$B_r^p = \left[ -\frac{2D}{ar^2} \frac{e^{\frac{r-a}{\lambda_p}}}{\sin^2\alpha + \frac{\lambda_q}{\lambda_p}} \frac{\lambda_q}{\lambda_p} + B \right] \cos\theta, \quad (52)$$

$$B_\theta^p = - \left[ \frac{D}{ar\lambda_p} \frac{e^{\frac{r-a}{\lambda_p}}}{\sin^2\alpha + \frac{\lambda_q}{\lambda_p}} + B \right] \sin\theta, \quad (53)$$

and for  $r \geq R$

$$B_r^e = \frac{BR^3}{r^3} \cos\theta, \quad B_\theta^e = \frac{BR^3}{2r^3} \sin\theta. \quad (54)$$

As can be seen from the obtained solutions, the magnetic field in both quark and hadronic phases depends on  $r$  only close to the phase boundary at  $r=a$ . In more detail, the part of the solution for the field which depends

on  $r$  tends to zero in the quark core at distances  $a-r \gg \lambda_q$  and in the hadronic shell at distances  $r-a \gg \lambda_p$ . But since  $\lambda_p$  and  $\lambda_q$  are much smaller than  $a$  and  $R-a$  except a small layer at the surface of the quark core with the depth of the order  $\lambda_p + \lambda_q$ , the magnetic field in both phases is constant and directed parallel to the rotation axis of the star, see solution (50) - (53). We conclude that in the main part of the volume of the quark and hadron phases the magnetic field is constant and has the direction  $z$ . In this approximation the condition (36) is satisfied. Therefore inserting in (36) the relation  $\mathcal{M} = BR^3/2$ , see equation (54), we have

$$B^q = 2\cot^2\alpha \frac{M_\varphi(a)}{a} + \frac{2c_0}{\sin\alpha} = B. \quad (55)$$

Solving equations (45) and (55) we finally obtain

$$c_0 = \frac{B \sin\alpha}{2}, \quad D = 0. \quad (56)$$

Thus in this approximation the magnetic field  $\vec{B}$  enters from the hadronic phase into the quark phase in the form of quark vortices [16]. The transition zone is of the order  $\lambda_p + \lambda_q$  which entails that quantity  $D$  is small of the order  $(\lambda_p + \lambda_q)/a$  so that the condition  $D=0$  is well fulfilled.

4. *Conclusion.* We have investigated the behaviour of the magnetic field of a neutron star with superconducting quark matter core in the framework of the Ginzburg-Landau theory whereby the simultaneous coupling of the diquark condensate field to the usual magnetic and to the gluomagnetic gauge fields has been taken into account (rotated electromagnetism). In solving the Ginzburg-Landau equations for this problem, we have respected the boundary conditions properly, in particular, the gluon confinement condition. We have found the distribution of the magnetic field in both the quark and hadronic phases of the neutron star and have shown that the magnetic field penetrates into the quark core in the form of quark vortices due to the presence of Meissner currents.

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# МАГНИТНОЕ ПОЛЕ НЕЙТРОННОЙ ЗВЕЗДЫ С ЯДРОМ, СОСТОЯЩИМ ИЗ СВЕРХПРОВОДЯЩЕЙ КВАРКОВОЙ МАТЕРИИ

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Методом теории Гинзбурга-Ландау изучено поведение магнитного поля в нейтронной звезде с ядром, состоящим из сверхпроводящей кварковой материи. Учтено взаимодействие конденсата дикварков с магнитным и глюомагнитным калибровочными полями. Решены уравнения Гинзбурга-Ландау с корректным учетом граничных условий, в том числе с учетом условия конфаймента глюонов. Найдено распределение магнитного поля в кварковой и адронной фазах нейтронной звезды и показано, что магнитное поле проникает в кварковое ядро в виде кварковых вихревых нитей благодаря наличию экранирующих мейснеровских токов.

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