АСТРОФИЗИКА

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STAR SPOTS STUDIES FOR THREE SHORT PERIOD RS CVn-TYPE BINARIES

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The light curve analysis for three short-period RS CVn-type binaries were performed, using: a) Fourier analysis techniques for the light curve changes in frequency domain. b) Photometric curve fit by means of numerical quadratures to develop theoretical light curves appropriate to RS CVn stars. The two methods were applied to the systems ER Vul, BH Vir, and UV Psc. Improved physical and geometrical parameters are found by taking the average of the results measured by the above two methods.

1. *Introduction*. RS CVn type binaries are a special group of variable stars, with specific characteristics. They typically have highly variable light curves, and differ drastically, when it obtained in two or more consecutive seasons. These binaries have been studied and developed rather gradually by many astronomers [1-8]. RS CVn type stars are a chromospherically-active binaries. A working definition was set down by Hall [5]. He divided the RS CVn systems into five subgroups: regular systems, short period systems, long period systems, flare star systems and V471 Tau type systems.

The Catalog of Chromospherically Active Binary Stars [9] is good reference for the elements study of RS CVn type binaries and listed an important short period subset of these binaries. Hall [5], Oliver [10] studied more than twenty systems in Popper's list [2] and proposed the main properties of these group. These binaries exhibit various peculiarities, such as H and K of CaII emission, indicating the presence of active chromospheres and soft X-ray emission, which suggest the presence of large scale solar-type activity such as star spots on the surface of one or both components. The light curves of these systems show a peculiar semiperiodic structure outside of eclipse. This structure has been referred to as a distortion wave in the light curve. The source of the distortion was due to circumstellar material around the hotter component [11]. Hall concluded that these mechanisms is due to "Starspots" phenomenon, which, in analogy to sunspots are large, cool active regions on the photosphere [12].

Generally, RS CVn show all the characteristic features indicative of chromospheric activity, but while the qualitative interpretations of the observations in terms of the solar stellar connection and magnetic activity cycles are well described [13].

The starspot model was originally proposed by Kron [14] and refined by Hall [4,15,5], Oliver [10], Eaton and Hall [16]. The light curve does not give information on the latitude of the spots, or spot groups directly, and even detailed light curve modeling may not give the latitude uniquely. But information on longitude can be obtained from the light curve. Good observations and studies for the CF Tuc (RS CVn type binary) has been made by Budding & Zeilik [17]. A comparative study of the X-ray for RS CVn versus Algol-type binaries has been made by Singh et al [18]. They compiled a list of 59 RS CVn and 29 Algol-type binaries with well-known orbital parameters and observed with ROSAT PSPC detector. They tried to detect the influence of mass transfer on the X-ray emission properties of these two types of active binaries.

In the present work we have studied three RS CVn binaries by two method of light curve analysis, taking most of the involved phenomena in account, such as limb darkening, gravity darkening, reflection, geometrical distortions, tidal effects and atmospheric eclipses. The fundamental problem is to extract the information of the systems from the light curve. The standard methods are well proven by many authors [19-33].

The three RS CVn type binaries under investigations are:

1.1. *UV Psc*. The eclipsing variable UV Psc (BD + 6°189) was first noticed as a variable at Bamberg in 1957. Hut (1959) has published the first light curve based on 348 plates. It was included in the short period group of RS CVn-type binaries [34,8]. The classification of the primary component of UV Psc was G2V [35,3].

Several observers have observed the system photoelectrically. Sadik [36] stated that the irregularities in the light curves were caused by locally hotter, rather than cooler region on one of the components. Busso et al [37] discussed the presence of cycles of variability in the system on the basis of the analysis of the photometric observations. Sarma et al [38] and Antonopoulou [39] suggested that the hotter component in the system could be an intrinsic variable. A rough migration period was given to be (1.5-2) years for the minimum of the wave-like distortion [40,41]. Akan [42] investigated the amplitude variation of the wave and its migration.

1.2. *BH Vir* (BD - 0°2769, HD 121909) is an eclipsing system with orbital period less than one day. The system consist of G0V primary star and G2V secondary star [43]. Tessevich [44] gave the light elements as:

$$\text{Min.I} = \text{J.D.}24312421.393 + 0^d.81679\ E.$$

The eclipses regions of the light curve were observed photoelectrically by Huruhata and Nakamura [45] and Szczepenwska [46]. Kitamura el al [47] obtained a complete light curves using *B* and *V* filters. Koch [48] reported observations in *UBV* filters and two narrow band filters. A large light variations outside eclipses was detected, besides, it was considered the occurance of intrinsic variation in the system. The system observed also by Sadik [49], and Hoffmann [50]. They associated the system with the short period group of RS CVn type.

1.3. *ER Vul* (HD 200391, BD + 27°3952, SAO 089396) is a short period system ($P=0^d.69809510$). The system was discovered spectroscopically by Northcott and Bakos [51]. The light curve changes and CaII H and K emission were studied by Hall [5]. The irregular light curve variations were identified by Al-Naimiy [52,53]. The time of minima have been monitored by Akan et al [54].

Photometric observations by Zeilik et al [55] indicated variations on the scale of one week and observations of color with orbital plane. Starspot analysis by Zeilik and Budding [56] indicates that two spots are needed to adequately describe the light variations and observations of Akan et al [54] indicate a photometric wave migration rate on the order of 8 months.

2. *Methods of Analysis*. The analysis of eclipsing binaries is not easy task due to the complexity of the phenomena involved. Many authors developed the attempt to solve the light curve in different procedures. In present paper we used two techniques of analysis:

2.1. *Fouries analysis of the light changes in frequency domain.* This techniques for the derivation of the geometrical elements depends on the equations of the moments A_{2m} [20,57] in the form

$$A_{2m} = a_{2m} - x_j - b_{2m} \tag{1}$$

where

$$x_j = -m! \sum_{j=1}^{n} \frac{\left(\frac{1}{2}j+1\right)}{\left(\frac{1}{2}j+m+1\right)} k_j, \tag{2}$$

a_{2m} is the area subtended by the observed light curve in $\left(l - \sin^{2m}\theta\right)$ coordinates and given by:

$$a_{2m} = \int_0^{\pi/2} \left[l\left(\frac{\pi}{2}\right) - l(\theta) \right] d\left(\sin^{2m}\theta\right), \tag{3}$$

$l\left(\frac{\pi}{2}\right)$ denotes the maximum brightness of the system at the time of quadratures, a_{2m} are calculated from photometric observations for both minima with different colors.

The constants k_j in equation (2) evaluated by appropriate modulation of the uneclipsed part of the light curve by the use of

$$k_j = \int_{-a}^{+a} \left[l\left(\frac{\pi}{2}\right) - l(\theta) \right] p_j^{(a,n)}(\cos\theta)\, d(\cos\theta), \tag{4}$$

j denotes the degree of the respective harmonic factoring k_j, n is the total number of such harmonics included in simultaneous solutions, and $a=\cos\theta_1$; θ_1 is the angle of first contact which was estimated for ER Vul, UV Psc and BV

Vir to be around 30°. For the range $30° \le \theta \le 150°$, $a = \sqrt{3}/2$ and $n = 4$, the modulation Legendre polynomials $P_j^{(a,n)}(\cos\theta)$ are given in a paper of Kopal [58]. After calculating k_i the weighted sum x_i of equation (2) were formed. The photometric perturbations b_{2m} have been ignored.

The determination of the zeroth moment $A_0(m = o)$ depends on the light of the system $(\lambda = l(\theta))$ at the moment of conjunctions of the respective minimum such that:

$$A_0 = 1 - \lambda = L_1\,\alpha_0(a, c_0), \tag{5}$$

where $\alpha_0(a, c_0)$ is the maximum obscuration of the star undergoing eclipse of luminosity L_1, depending on the ratios:

$$a = r_1/(r_1 + r_2) \tag{6}$$

$$c_0 = \cos i/(r_1 + r_2). \tag{7}$$

Therefore, the following ratios can be formed [59]

$$g_{2m}(a, c_0) = (A_{2m})^2/(A_{2m-2}A_{2m+2}). \tag{8}$$

Therefore, for $m = 0$, 1, 2, 3 we form the combination of

$$g_2(a, c_0) = A_2^2/A_0A_4 \quad \text{and} \quad g_4(a, c_0) = A_4^2/A_2A_6.$$

The empirical values of g_{2m} were evaluated from A_{2m}, which evaluated from the observed light curves. The unknown constants a and c_0, can be solved numerically [59]. Then the elements $r_{1,2}$ and i can be determined from any ratios of A_{2m+2}/A_{2m}. Finally, the fractional luminosity $L_{1,2}$ can be evaluated from the individual moment A_{2m} where $L_2 = 1 - L_1$.

2.2. ***Light curve fitting techniques.*** The synthetic light-curves was the solution of the Roche Models basic equation. We consider that the shape of the two components follows the equipotential surface of Roche Model given by the equation [19].

$$\begin{aligned} \xi &= \frac{1}{r} + q\left[\left(1 - 2x + r^2\right)^{-0.5} - x\right] + \left(\frac{1+q}{2}\right)\left(x^2 + y^2\right) \\ &= \frac{1}{r} + q\left(\frac{1}{r'} - x\right) + \left(\frac{1+q}{2}\right)\left(x^2 + y^2\right) \end{aligned} \tag{9}$$

Where x, y, z are rotating cartesian coordinates, q mass ratio, r, r' are position vectors relative to the primary and secondary centers, respectively, ξ is the normalized potential, given by

$$\xi = \frac{|\Omega| R}{Gm} - \frac{1}{2}\frac{m^2}{m(m + m')}.$$

Ω is the gravitational potential at an arbitrary point of (x, y, z). By using spherical polar coordinates:

$$x = r\cos\Phi\sin\Psi = r\lambda$$

$$y = r\sin\Phi\sin\Psi = r\mu$$

$$z = r\cos\Psi = r\nu$$

Equation (9) can be written in more familiar form such as:

$$\xi = \frac{1}{r} + q\left[\left(1-2\lambda r+r^2\right)^{-0.5} - \lambda r\right] + \left(\frac{q+1}{2}\right) r^2\left(1-\upsilon^2\right)$$
$$= \frac{1}{r} + q\left(\frac{1}{r'} - \lambda r\right) + \left(\frac{q+1}{2}\right) r^2\left(1-\upsilon^2\right) \qquad (10)$$

The solution of equation (10) has been given by Kopal [19], Al-Naimiy [60,30] and Sabat [33]. We used a linear orthogonal transformation to move from the fixed cartesian coordinates x, y, z, to rotating coordinates X, Y, Z and vice versa, utilizing the transformation matrices defined by Kopal [19] and Sabat [33]. We have assumed a linear limb-darkening law [29] and also assumed a linear approximation of the gravity-darkening law:

$$\left(H/H_0\right)_\lambda = 1 + \tau_0\left(g/g_0 - 1\right),$$

where τ_0 is the gravity-darkening index, given by

$$\tau_0 = \frac{1}{4}\frac{hc/\lambda k T_e}{1-e^{-\frac{hc}{\lambda k T_e}}}.$$

According to Al-Naimiy techniques, for developing theoretical light curve, we used a mesh of cells over the system, and calculated the amount of theoretical

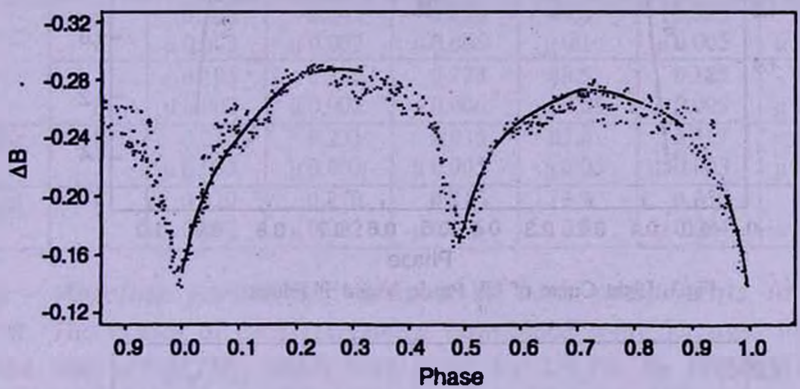

Fig.1. Blue Light Curve of ER Vul.

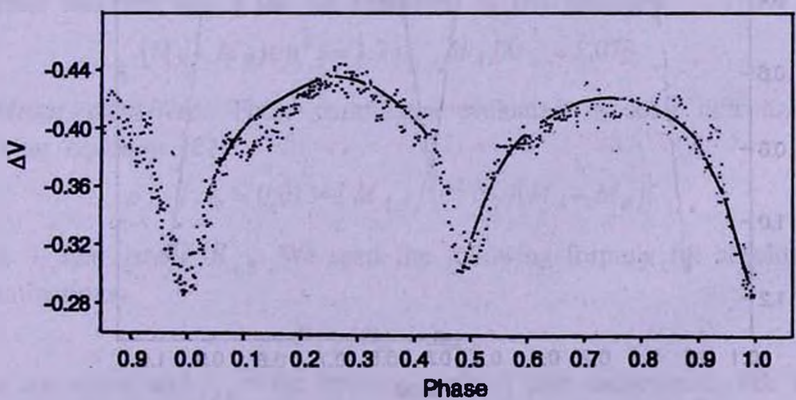

Fig.2. Yellow Light Curve of ER Vul.

light from each cell which is given by the following final equation [60].

$$\Delta l_d = \frac{Q}{4}\frac{L_{un}\left[H_o\left(1+\tau_o\left(\frac{g-g_0}{g_0}\right)\right)(1-u+\cos\gamma_d)+J^{*}(x,y)\right]}{\pi r_o^2(1-u/3)}(\Delta x\cdot\Delta y)_d, \quad (11)$$

where $J^{*}(x, y)$ is the increase in radiative flux due to reflection and heating.

Thus, the total light of the eclipsing binary system at a given phase angle is the total sum of light from each cell of the mesh, and the synthetic light-curve as whole results when the total theoretical light is calculated at sufficiently small intervals of the phase-angle, over a complete cycle of the system.

The theoretical light curve for each of RS CVn system has been calculated,

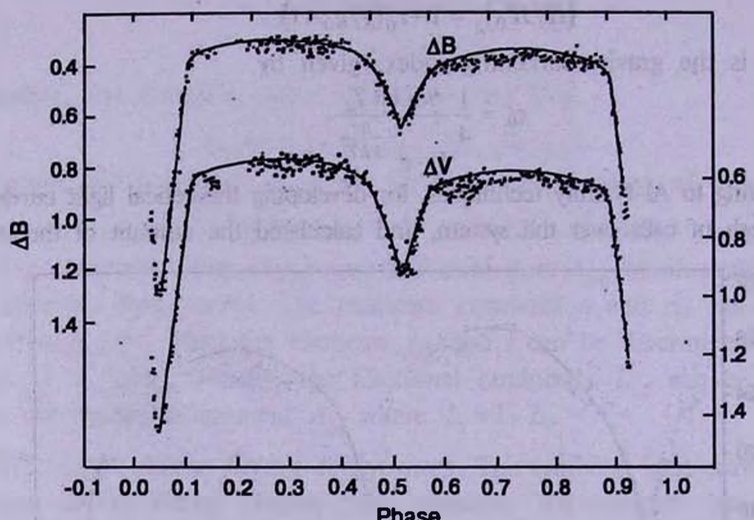

Fig.3. Light Curve of UV Psc in V and B Filters.

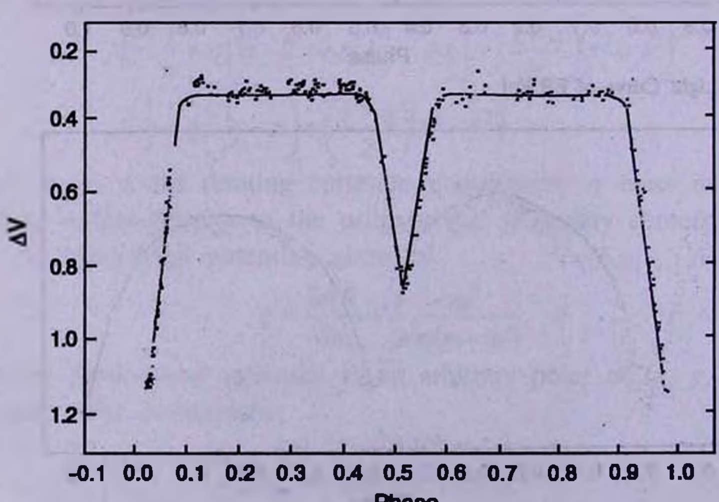

Fig.4. Light Curve of BH Vir.

then we obtained the best fit with the observational light curve at the final physical and geometrical elements. The observational data for ER Vul taken from Al-Naimiy [52,53], while for BV Vir and UV Psc are taken from Sadik [49]. Figures 1, 2, 3 and 4 show the fitting of theoretical light curve with the observed one.

3. *Computation of the parameters.* i - *Limb darkening coefficients (u).* According to the spectral types of each component in the three system, we evaluated $u_{1,2}$ for both components theoretically, using tables based on comprehensive range of model atmospheres of Carbon and Gingerich [61], Al-Naimiy [29].

ii - *The elements, $r_{1,2}$, i and $L_{1,2}$.* The derivation of the elements carried out by using the two techniques: Fourier analysis of the light changes and the synthetic procedure for light curve fitting, then the average value for the elements taken for those derived by the two above techniques. Table 1 contain the average values of the geometrical and physical elements.

Table 1

THE ELEMENTS OF THE THREE SYSTEMS

System	Filter	r_2	r_1	k	i	L_2	L_1
UV Psc	*V*	0.203 µ 0.003	0.245 µ 0.002	0.823 µ 0.006	88.8 µ 0.1	0.225 µ 0.005	0.773 µ 0.004
	B	0.195 µ 0.002	0.250 µ 0.002	0.773 µ 0.006	88.5 µ 0.05	0.185 µ 0.005	0.814 µ 0.007
BH Vir	*V*	0.215 µ 0.003	0.231 µ 0.003	0.935 µ 0.005	87.0 µ 0.05	0.317 µ 0.003	0.680 µ 0.003
ER Vul	*V*	0.220 µ 0.002	0.270 µ 0.002	0.815 µ 0.005	73.2 µ 0.05	0.350 µ 0.004	0.650 µ 0.004

iii - *Absolute parameters. Masses of both components in each system.* The masses of each component determined from the mass function and the ratio $q = M_2/M_1$, which have given for UV Psc by Popper [3], for BH Vir by Abt [43] and for ER Vul by Northcott & Bakos [57]. For example, the mass function and q for ER Vul given as the following

$$(M_A + M_B)\sin^3 i = 1.73, \quad M_A/M_B = 1.078.$$

Mean densities. These parameters evaluated in solar unit using the following equation [62].

$$\rho_{A,B}/\rho_{\odot} = 0.01344\, M_{A,B}/\{P^2\, r_{1,2}^3/(M_A + M_B)\}.$$

iv - *The radii $R_{A,B}$.* We used the following formula for absolute radii determinations:

$$R_{A,B}/R_{\odot} = a r_{A,B},$$

where $a = a_1 + a_2$ and $r_{A,B}$ = the fractional radii of both components. For ER Vul:

$$a_1 \sin i = 1.326 \times 10^6 \text{ km},$$

$$a_2 \sin i = 1.433 \times 10^6 \text{ km}.$$

Therefore $a = 2.759/\sin i \times 10^6 \text{ km} = (29.5 + 0.02) \times 10^6 \text{ km} = 4.25\, R_\odot$.

For UV Psc and BV Vir, we used

$$a/R_\odot = \{74.55 (M_1/M_\odot)(1+q) P^2\}^{1/3}$$

v - ***The luminosity and bolometric mangnitudes.*** These parameters evaluated for each components by using the well known relation (Stephan-Boltzmann Law).

$$L_{A,B}/L_\odot = (R_{A,B}/R_\odot)^2 (T_{A,B}/T_\odot)^4, \text{ where } T_\odot = 5770\text{K}.$$

The bolometric magnitudes M_{bol} can be evaluated using the following equation [63]:

$$M_{bol_{A,B}} = 42.36 - 10\log T_{A,B} - 5\log(R_{A,B}/R_\odot).$$

The absolute parameters of the two components for each system are listed in Table 2.

Table 2

ABSOLUTE PHYSICAL ELEMENTS

	Stars		
Elements	UV Psc	BH Vir	ER Vul
$A/R_\odot$	4.9	4.42	4.31
$R_1/R_\odot$	1.23	1.05	1.16
$R_2/R_\odot$	0.95	1.02	0.96
$L_1/L_\odot$	1.45	1.38	1.58
$L_2/L_\odot$	0.39	0.88	0.91
$\rho_1/\rho_\odot$	0.66	0.82	0.73
$\rho_1/\rho_\odot$	1.13	1.1	1.2
M_{bol1}	4.34	4.36	4.25
M_{bol2}	5.75	4.64	4.85
$M_1/M_\odot$	1.2	0.86	1.15
$M_2/M_\odot$	0.9	0.88	1.06
T_1	5740	6150	6000
T_2	4750	5850	5750

vi - ***Effective temperature.*** It is not a straightforward matter to directly deduce T_{eff} of both components independently. However, we may consider the planckian approximation for the flux ratio F_A/F_B and using the following expression in terms of the reciprocal effective temperature [64].

$$F_A/F_B = \exp[a(\theta_B - \theta_A)][1 - \exp(-a\theta_B) + \exp(-a\theta_A) + \exp(-a\theta_A)^2],$$

$$a = 2.857 \times 10^{-4}/\lambda_{eff}^{(cm)} = 6.57 \quad \text{in blue},$$

$$= 5.15 \quad \text{in yellow}, \ \theta_{A,B} = 5040/T_{eff_{A,B}}.$$

λ_{eff} has been taken from Ibanoglu [65] for ER Vul, while for UV Psc from

Popper [3] and for BH Vir from Abt [56]. For F_A/F_B, we used the following formulae:

$$F_A/F_B = k^2 L_A/L_B, \text{ where } k = r_B/r_A.$$

Then we find the value of $(\theta_B-\theta_A)$. If we assume the temperature of G0V component from Cox and Gouli Table [66] as 6000 K, this leads to an effective temperature of the secondary component for each system (see Table 2).

4. *The final results.* Tables 1 and 2 give the mean values of physical and geometrical elements for the three system, using the two mentioned procedures.

A, $R_{A,B}$, $L_{A,B}$, $\rho_{A,B}$, $M_{bol_{A,B}}$ stand for the separation between the two components, absolute radius, absolute luminosity, density, and the bolometric magnitude, respectively, and $M_{A,B}$, $T_{1,2}$ represent the mass of two component, and temperature, respectively.

5. *Discussion and Conclusion.* 5.1. *The parameters.* Tables 1 and 2 show the geometrical and physical elements of the three short period group of RS CVn binaries. We can conclude from the fractional radii values (Table 1) that the primary eclipse for the three system is a transit eclipses.

5.2. *Wave amplitude.* The amplitude is an important parameter that characterizes the light curve. The light variation in RS CVn stars arises from the rotational modulation of the observed flux by the presence of star spots on active components. The amplitude of light variation depends only on the longitude asymmetry in the distribution of spots, and hence a variation in the amplitude implies significant changes in the longitudinal distribution of spots. We have estimated the wave amplitudes in V and B filters and the phases of the maximum and minimum brightnesses for each system (Table 3). It can

Table 3

THE AMPLITUDES AND THE ESTIMATED PHASES OF THE MAX AND MIN BRIGHTNESS AT OUTSIDE ECLIPSE

Star	$\theta_{min}(V)$	$\theta_{max}(B)$	Amp.(V)	Amp.(B)
UV Psc	0.48	0.84	0.07	0.09
BH Vir	0.62	0.21	0.04	-
ER Vul	0.03	0.81	0.06	0.05

be seen from this Table that the amplitude is small for BH Vir and a clear trend of the wave amplitude dependence with wavelength is shown in UV Psc. While for ER Vul shows a smaller trend because of the smallness light changes.

5.3. *Position of each component in H-R diagram.* We managed to determine the position of the three eclipsing binaries on H - R diagram from the absolute physical elements (Table 2). We found that both components of each system lies within the main - sequence band, and the primary star is more evolved than the secondary.

5.4. *Type of eclipse.* The type of eclipse can be defined from the numerical values of the constants a and c_0 as a function of $r_{1,2}$ and i (Equ. 7). The eclipse is total if $c_0 < 1 - 2a$ and partial if $1 > c_0 > 2a - 1$. By applying these formulas to the three binaries, we found that the eclipse partial for BH Vir and ER Vul and total for UV Psc.

5.5. *First contact angle.* The first contact angle (θ_1) can be derived from the following: equation: $\theta_1 = \sin^{-1}\sqrt{(r_1 + r_2)^2 \csc^2 i - \cot^2 i}$

The results for the systems was: for UV Psc = 26°.8, for BH Vir = 27°.7 and for ER Vul = 26°.0. That mean θ_1 vary from $26^\circ \rightarrow 28^\circ$ which is less than that of contact binaries, this concludes that all systems are detached and the two components for each system are well inside the Roche lobes.

5.6. *RS CVn are a Sun like stars.* It is clear from Table 2 that the absolute physical parameters of the primary and secondary star $R_{A,B}$, $M_{A,B}$ and $T_{A,B}$ of the systems are nearly equal to the physical parameters of the Sun, this means that the physical properties of the short period group of RS CVn are in analogy with the Sun.

5.7. *The eccentricity and apsidal motion.* We have not found any significant effect which could be associated either with eccentricity (e) or apsidal motion (ω), therefore, e and ω have been ignored in our solutions.

5.8. *Suggestions.* Finally, we suggest that these groups of stars needs continuous observational programs for the investigation of minimum changes, wave migrations, star spots and the calculation of the period of their phenomena.

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ИССЛЕДОВАНИЕ ЗВЕЗДНЫХ ПЯТЕН ДЛЯ ТРЕХ КОРОТКОПЕРИОДИЧЕСКИХ ДВОЙНЫХ ТИПА RS CVn

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Проведен анализ кривых блеска для трех короткопериодических двойных типа RS CVn с помощью двух методов: а) Фурье-анализ для изменений кривых блеска в области частот и b) подгонка фотометрических кривых с помощью численных квадратур для получения теоретических кривых

блеска для звезд типа RS CVn. Оба метода применены для систем ER Vul, BH Vir и UV Psc. Найдены улучшенные физические и геометрические параметры путем усреднения результатов, полученных обоими методами.

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