

WAVE MOTIONS IN MAGNETIC INTERSTELLAR MEDIUM

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This communication considers continuum approach modelling large-scale dynamics of non-conducting interstellar medium capable of sustaining long-ranged filamentary agglomeration of tiny superparamagnetic grains suspended in dense molecular cloud. The filamentary ordering of permanently magnetized grains, oriented in the direction of regular galactic field threading the cloud, is thought of as an effect of soft magnetic solidification of nonconducting gas-dust substance imparting to interstellar material the mechanical features of single-axis magnetoelastic insulators. With this physical picture in mind, we set up macroscopic equations furnishing opportunity to study dissipative-free motions of superparamagnetic gas-dust non-ionized matter in terms of continuum mechanics magnetoelastic materials. Particular attention is given to oscillatory behavior in the regime of strong magnetization-flow coupling. The most remarkable inference of this model is that nonconducting magnetically polarized interstellar medium can transmit perturbations by transverse waves of magnetization which can be regarded as a counterpart of Alfvénic waves generic to cosmic dusty plasma.

1. Introduction. The magnetohydrodynamics (MHD) serves as one of the powerful tools in studying a vast diversity of astrophysical phenomena ranging from Alfvén waves in interstellar medium (ISM) to dynamo processes supporting long-term stability of stellar and planetary magnetic fields. The standard argument for applicability of MHD model is that it has to do with highly ionized fluid placed in a uniform magnetic field. The hydromagnetic waves are considered to be a common property of both the main sequence stars [1] and compact stellar objects like neutron stars [2,3]. Supersonic motions of ISM observed in dark molecular clouds, presumably ionized by ultraviolet photons, are customarily associated with hydromagnetic Alfvénic wave motions [4]. In the meantime, the continuum treatment of large-scale motions of interstellar matter with highly pronounced properties of magnetic polarizability is developed less. The existence of gas-dust ISM with strong, superparamagnetic, properties of dusty grains aligned by galactic field has been hypothesized long ago by Jones and Spitzer [5] whose arguments provide considerable guidance in current investigations on the starlight polarization problem [6]. The purpose of this paper is to explore one of conceivable theoretical approaches to macroscopic dynamics of superparamagnetic gas-dust ISM.

2. *Governing equations.* In the model under consideration the gas-dust interstellar matter is pictured by gas-based ferrocolloidal substance consisting of tiny ferromagnetic or superparamagnetic solid grains suspended in dense gaseous cloud of molecular hydrogen. It is assumed that both gaseous and dust components of ISM are poorly ionized, and internal temperature is so low that equilibrium and dynamical properties are dominated by long-ranged magnetic ordering of dusty particles along the regular galactic field threaded the cloud. As was suggested in [5], superparamagnetic grains can form flexible filamentary structures or magnetic chains, presumably by means of dipole-dipole interaction between magnetic moments of grains. The detailed theory of chain-like structures in ferrocolloidal soft matter made of permanently magnetized grains suspended in magnetopassive fluid has been developed in [7] to explain superparamagnetism of magnetic liquids. From standpoint of the condensed matter physics, the filamentary agglomeration of superparamagnetic dusty particles, oriented in the direction of regular galactic field, can be thought of as an effect of soft magnetic solidification imparting to interstellar gas-dust nonconducting medium mechanical properties of single-axis magnetoelastic insulators [8]. One can expect, therefore, that large-scale dynamics of uniformly magnetized gas-dust interstellar matter should manifest mechanical behavior having some features in common with that for magnetoelastic continuum. This attitude constitutes a basis of our theoretical treatment of macroscopic dynamics of superparamagnetic gas-dust ISM in terms of continuum mechanics of magnetoelastic materials, and explains term magnetoelastodynamics coined in further discussion.

One of the basic suggestion of the magnetoelastodynamical treatment is to identify the behavior of real two-component gas-dust medium with that for single-component superparamagnetic continuum of equivalent density which obeys the standard continuity equation

$$\frac{d\rho}{dt} + \rho \nabla \cdot v = 0, \quad (1)$$

where d/dt stands for convective derivative. The second suggestion is to consider the field of magnetization $m(r, t)$ (magnetic moment per unit volume) as independent dynamical variable of motion on equal footing with the bulk density $\rho(r, t)$ and the velocity of elastic displacement $v(r, t)$. The distinguishing feature of magnetic insulators is that the rate of changes in the velocity of elastodynamic flow magnetized by the uniform field B is controlled by driving force originating from body-torque density [8]

$$\rho \frac{dv}{dt} = \frac{1}{2} \nabla \times [m \times B], \quad B = \frac{M}{\chi}. \quad (2)$$

It is clearly seen that appearance of this force is associated with deviation of the magnetization vector from direction of magnetic anisotropy. Notice that superparamagnetic state of matter is described by the linear constitutive equation $M = \chi B$, that is, the same as for paramagnetics [9]. The evolution of this latter is governed by equation of precession

$$\frac{dm}{dt} = [\omega \times m], \quad \omega = \frac{1}{2} \nabla \chi v. \quad (3)$$

Here the angular velocity ω is essentially local characteristics of elastodynamical flow in which the direction of the magnetization vector changes, but the magnitude does not. The above closed system of equations describes dissipative-free motions of superparamagnetic ISM in the state of magnetic saturation.

3. *Transverse waves of magnetization.* Let us show that gas-dust matter governed by these equations can transmit transverse wave of magnetization. By applying standard procedure of linearization: $v \rightarrow v_0 + \delta v(r, t)$ and $m \rightarrow m_0 + \delta m(r, t)$, where $v_0 = 0$ and $m_0 = M$, one finds that magnetoelastodynamical equations describing linear fluctuations of incompressible superparamagnetic ISM take the form

$$\nabla \cdot \delta v(r, t) = 0, \quad \nabla \cdot \delta m(r, t) = 0, \quad (4)$$

$$\rho \frac{\partial \delta v(r, t)}{\partial t} = \frac{1}{2\chi} \nabla \chi [\delta m(r, t) \times M], \quad (5)$$

$$\frac{\partial \delta m(r, t)}{\partial t} = \frac{1}{2} [[\nabla \chi \delta v(r, t)] \times M]. \quad (6)$$

In (5) we have used constitutive equation $B = M/\chi$. The right of equations (4) expresses the fact that perturbation is not accompanied by appearance of density of magnetic poles. It is worth noting that from mathematical point of view the above linearized equations of magnetoelastodynamics are very similar to that for nematodynamics of ferronematic liquid crystals whose low-frequency oscillatory behavior has been studied in [10]. In what follows we consider perturbation in the plane-wave form

$$\delta v = v' \exp(i\omega t - ik \cdot r), \quad \delta m = m' \exp(i\omega t - ik \cdot r), \quad (7)$$

where v' and m' are some small constant vectors, k stands for the wave vector, and ω is the frequency of magnetoelastic oscillations. The result of above plane-wave transformation of equations (4)-(6) can be represented as follows

$$\omega \rho \delta v + \frac{1}{2\chi} (k \cdot M) \delta m = 0, \quad \omega \delta m + \frac{1}{2} (k \cdot M) \delta v = 0. \quad (8)$$

By eliminating $(k \cdot M)$ from equations (8), one finds that magnetoelastic oscillatory motions satisfy the principle of energy equipartition

$$\frac{\rho \delta v^2}{2} = \frac{\delta m^2}{2\chi}, \quad (9)$$

which states that in the magnetoelastic wave the kinetic energy of elastic displacements of ISM equals mean potential energy of fluctuating magnetization. From (8) it follows

$$\omega^2 = \frac{(k \cdot M)^2}{4\chi\rho} = V_M^2 k^2 \cos^2 \theta, \quad V_M^2 = \frac{MB}{4\rho} = \frac{M^2}{4\chi\rho} = \frac{\chi B^2}{4\rho}. \quad (10)$$

This dispersion relationship describes transverse wave of magnetization traveling in incompressible uniformly magnetized continuum. By θ we denoted

angle between wave vector k and the direction of equilibrium magnetic anisotropy M . The wave is transmitted most efficiently when $k \parallel M$. In this latter case, the vectors of magnetization and velocity of elastic displacements undergo coupled oscillations in the plane perpendicular to M . The quadratic form of resultant dispersion relationship shows both M and $-M$ directions of propagation are energetically equivalent. The transverse elastomagnetic wave can be visualized by long wavelength vibrations of elastic filaments of magnetizations, frozen-in elastodynamic flow, around axis of equilibrium permanent magnetization. In spirit of MHD treatment of frozen-in lines of magnetic force in hydrodynamic flow, in our case one can say that filamentary field of magnetization turns out frozen in elastodynamic flow.

To show that presented model can provide proper account of the recent data on supersonic, but sub-Alfvénic motions of interstellar medium in the cores of dark molecular clouds [4], we consider long-wavelength, non-radial oscillations of a spherical uniformly magnetized cloud. From electrodynamics of continuous media [11] it is known that in a homogeneous spherical mass of paramagnetic matter with constant magnetization M inside, the internal magnetic field is uniform and is expressed by the equations $B + 2H = 0$ and $B - H = 4\pi M$, from which it follows

$$M = \frac{3}{8\pi} B. \quad (11)$$

With above reservations in mind, it would be not inconsistent to consider (11) as a constitutive equation of superparamagnetic continuum with $\chi = 3/8\pi \approx 0.1$. The advantage of this model is that it allows one to avoid uncertainty in the magnitude of χ . First we notice that from analytic estimate for v_M , equation (10), it follows that, at equal B and ρ , the considered wave motions are sub-Alfvénic: $v_M \approx 0.6v_A$, where $v_A = B/(4\pi\rho)^{1/2}$ is the speed of Alfvén's wave. Taking the bulk density $\rho = n\mu_{H_2} \approx 10^{-21}$ g/cm³ (where $n = 10^3$ cm⁻³ and μ_{H_2} is the mass of the hydrogen molecule [4]) and the magnetic field $B = 10\mu G$, one finds that the speed of wave of magnetization $v_M \approx 0.3$ km/s, that is, exceeds the isothermal sound speed $c_s = (k_B T / \mu_{H_2})^{1/2} \approx 0.2$ km/s at the average intercloud temperature $T \approx 10$ K [4]. So, under conditions typical of inner region of dark star-forming molecular clouds, the waves of magnetization are supersonic, but sub-Alfvénic. The fact that above predictions are in line with available data [4] suggests that supersonic motions observed toward core of dark molecular clouds, poorly ionized by ultraviolet photons, may be due to considered here wave motions.

4. *Summary.* It is shown that equations of continuum mechanics of magnetoelastic materials can provide consistent theoretical treatment of large-scale motions of magnetically ordered interstellar gas-dust medium possessing properties of gas-based ferrocolloidal substance whose existence has been justified long ago by Jones and Spitzer in the context of starlight polarization

problem. We have found that superparamagnetic interstellar medium can transmit perturbations by transverse waves of magnetization. These waves uniquely define dissipative-free oscillatory behavior of superparamagnetic nonconducting ISM and can be considered as a counterpart of hydromagnetic Alfvén's waves in cosmic dusty plasma. The obtained numerical estimates for the speed of waves of magnetization in the ISM with parameters typical of cores of dark molecular clouds allows one to suggest that supersonic and sub-Alfvénic linewidths inferred from recent measurements of magnetic field toward core position in dark interstellar molecular clouds may be due to considered here transverse waves of magnetization.

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ВОЛНОВОЕ ДВИЖЕНИЕ В ЗАМАГНИЧЕННОЙ МЕЖЗВЕЗДНОЙ СРЕДЕ

С.И.БАСТРУКОВ, ДЖ.ЯНГ

В сообщении используется континуальный подход, позволяющий моделировать крупномасштабную динамику непроводящей межзвездной среды, в которой допускается спекание взвешенных в плотных молекулярных облаках крошечных суперпарамагнитных зерен в волокнистые образования. Волокнистое упорядочение непрерывно намагничиваемых зерен, ориентированных в направлении обычных галактических полей пронизывающих облако, предложено представлять как эффект магнитного затвердевания непроводящих частичек газообразного вещества, наделяющих межзвездную среду механической способностью одноосного магнитоэластического обособления. Такой физической картине сопоставляются макроскопические уравнения, которые позволяют изучать бездиссипативное движение суперпарамагнитных частичек газа неионизованной материи на языке механики сплошных сред для магнитоэластических материалов. Особое внимание уделено осцилляторному поведению в режиме сильной намагниченности. Наиболее

интересным следствием такой модели является то, что по непроводящей магнитополяризованной межзвездной среде может распространяться возмущение в виде поперечной волны намагничивания, которую можно рассматривать как аналог альфеновской волны, генерируемой космической пылевидной плазмой.

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