АСТРОФИЗИКА

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THE BEHAVIOR OF KS-ENTROPY OF N-BODY SYSTEMS AND RICCI CURVATURE

A.A.MELKONYAN
Yerevan Physics Institute, Yerevan, Armenia

1. The study of various statistical properties of N-body systems can be crucial for the understanding of many astrophysical problems - from the dynamics of the Solar system up to the dynamics of clusters of galaxies [1].

Below will consider two methods of the study of chaotic properties of N-body gravitating systems:

1. Lyapunov exponents and KS-entropy;
2. Ricci curvature.

In the first case the aim is to reveal whether the behavior of the system with given boundary conditions is chaotic or not. In the second case we investigate the "comparative" behavior of two close systems and will estimate their relative instability with respect to each other.

2. Lyapunov numbers show the exponential rate of divergence of close trajectories, and are defined as:

$$\lambda(x,t) = \lim_{t\to\infty} 1/t \ln \frac{d(x_0,t)}{d(x_0,0)},$$

with a spectrum: $\lambda_1 < ... < \lambda_M$, where M is the dimension of the phase space. The existence of the limits and of the order is established by Oseledets theorem.

KS-entropy is the sum of positive Lyapunov exponents and defined as:

$$KS = \sum_{\lambda_i>0} \lambda_i .$$

To demonstrate the complicated behavior of the chaotic properties of the systems on control parameters even for relatively simple systems, we will consider the following 1D Hamiltonian system:

$$H = \sum_{i=1}^{n} p_i^2 / 2m + \sum_{i=1}^{n} \left(Ax_i^4 - Bx_i^2 + B^2 / 4A \right) + C/2 \sum_{i=1}^{n} \left(x_{i+1} - x_i \right)^2,$$

where A,B,C>0 are positive constants.

Lyapunov numbers and KS-entropy of this system have been calculated using the algorithm by Bennetin et al [2]. Some results of calculations are represented in Fig.1 and Fig.2. It's interesting, that A and B parameters are symmetrical.

Such behavior has been overlooked in some previous studies of this system [3].

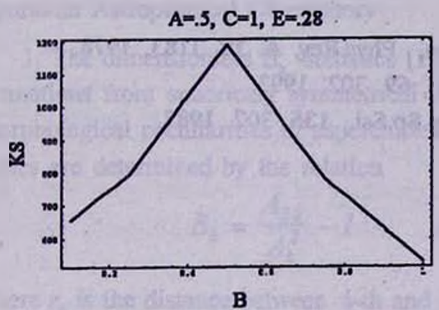

Fig. 1.

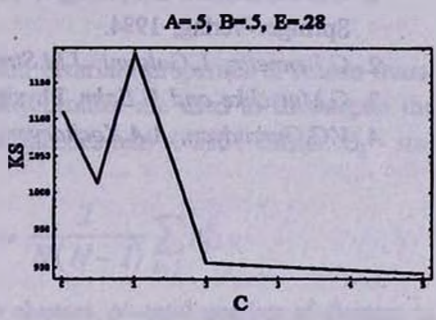

Fig. 2.

3. Ricci curvature is a geometrical method for study relative chaotic behavior of close geodesics, introduced in [4].

Ricci curvature is defined as:

$$r_u(s) = \frac{Riemu^i u^j}{|u|^2}$$

Consider some interval S, and denote r, as:

$$r = 1/3N inf\left[r_u(s)\right].$$

Consider two systems with value of Ricci curvature r_1 and r_2 accordingly within this interval $[0,s_1]$. We will conclude, that first system is less stable with respect second system within this interval, if $r_1 < r_2$ and $r_1 < 0$. Numerical experiments have been performed for various configurations of N-body gravitating systems with fixed values of control parameters (energy, momentum, etc.).

Поведение KS-энтропии систем N-тел и кривизна Ричи. **На отдельных примерах рассмотрены два метода изучения хаотических свойств многомерных нелинейных динамических систем. Представлены результаты расчетов KS-энтропии на примере 1D системы N-тел для различных параметров.**

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