АСТРОФИЗИКА

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KONISHI - KANEKO MAP AND FEIGENBAUM UNIVERSALITY

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The study of chaotic properties of N - body gravitating systems has become one of essential problems of stellar dynamics [1]. An interesting possibility of study of N - body dynamics is provided by the iterated maps [2, 3]. From this point of view the demonstrated by Inagaki [2] agreement of Konishi - Kaneko map with the thermodynamic formalism and the results of numerical simulations, is of particular importance.

The Konishi - Kaneco map is defined as follows [4,5]:

$$p_i^{n+1} = p_i^n + k\sum_{j=1}^{N}\sin 2\pi\left(x_j^n - x_i^n\right), \qquad (1)$$

$$x_i^{n+1} = x_i^n + p_i^{n+1};(mod\,1). \qquad (2)$$

This system describes 1 - dimentional N - body system with a pontential of interaction which is free of singularity and is attractive if $k>0$.

To study the possibility of existing of period - doubling bifurcations, first, we have to check the necessary condition, i.e. the negativity of the Schwartzian derivation:

$$Sf = f'''/f'' - 3/2\left(f''/f'\right)^2 < 0. \qquad (3)$$

This condition is fulfilled for Eqs. (1) and (2) since $f'''/f'<0$ for any value of k.

The numerical calculation were performed as follows. The point is to observe via computer analysis the period - doubling befurcations and to calculate the scaling defined by the Feigenbaum number δ for Konishi - Kaneko map. To obtain the bifurcation scale δ we have to find out the values of period - doubling bifurcation points which must satisfy the conditions

$$\sum_j^N \left|x_j^{n+1} - x_j^n\right| < \varepsilon, \left(2^1 = 2\right); \sum_j^N \left|x_j^{n+2} - x_j^n\right| < \varepsilon, \sum_j^N \left|x_j^{n+3} - x_j^{n+1}\right| < \varepsilon, \left(2^2 = 4\right);$$

for each k_n, n=1,2,..., respectively. The ε is the accuracy of the obtained values of k_n.

Our numerical calculation of k_1 e. g. for N=10 $\varepsilon=10^{-5}$, and 10^{-4} for k_2 and k_3. These calculations were enough to find out the Feigenbaum universal number δ=8.72... (cf. [3]). The result of calculations for N=10 are given in Table 1.

Table 1

k_1	k_2	k_3	δ
0.015000...	0.015196...	0.01521822...	8.82...
0.015000...	0.015194..:	0.01521622...	8.78...
0.015000...	0.01519368...	0.01521590...	8.71584...
0.015000...	0.01519368...	0.01521578...	8.76359...
0.015000...	0.01519368...	0.01521593...	8.70490...
0.015000...	0.01519368...	0.01521591...	8.71219...
0.015000...	0.01519368...	0.01521589...	8.71950...
0.015000...	0.01519368...	0.01521587...	8.72682...
0.015000...	0.01519368...	0.015215885...	8.72315...
0.015000...	0.01519368...	0.015215889...	8.71950...
0.015000...	0.01519368...	0.015215887...	8.72315...

The results for other values of N are absolutely identical with those for N=10, though with different accuracy ε. We have noticed a clear decrease in the accuracy with the increase of the number of particles.

Using the obtained values of k_n and the formula

$$k_\infty = \left(\delta\, k_{n+1} - k_n\right) / (\delta - 1), \qquad (4)$$

we also estimate the k_∞, from which the chaotic behavior of the system is established and map (sequence) never repeats itself:

$$k_\infty = 0.1307....$$

At $k>k_\infty$ the system should have positive Lyapunov numbers. This fact supports the results in [5] on the estimation of Lyapunov numbers.

Thus, we showed the existence of period - doubling bifurcations and numerically calculated the Feigenbaum number for the Konishi - Kaneko map. The period - doubling points correspond to the phase transitions of second order and can enable the study of such systems via the methods of thermodynamical formalism.

Система Кониши-Канеко и Фейгенбаум универсальность. Компьютерный анализ системы Кониши-Канеко показывает существование бифуркаций удвоения (периода), что обусловлено соответствующим условием для производной Шварца. Значение контрольного параметра k_∞, соответствующего переходу в хаотическую фазу оценено наряду с универсальными постоянными Фейгенбаума. Подобный факт может быть решающим в порядке исследования хаоса в звездной динамике.

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