АСТРОФИЗИКА

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NEW APPROACHES TO SOME CLASSIC METHODS OF RADIATIVE TRANSFER

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1. *About the Method of Discrete Ordinate (MDO).* Let us consider the well known integral Equation of Radiative Transfer (RT).

$$S(\tau) = S_0(\tau) + \frac{\lambda}{2}\int_0^{\tau_0} K(|\tau - t|)\, S(t)\, dt \qquad (1)$$

where

$$K(\tau) = E_1(\tau) = \int_1^{\infty} e^{-\tau s}\frac{ds}{s}$$

Application of MDO leads to the following reduction

$$K(\tau) \approx T(\tau) = \sum_{m=1}^{n} a_m e^{-|\tau| s_m} \qquad (2)$$

where $\frac{1}{s_m}$ are the positive roots of Legandre polynomial $P_{2n}(\eta)$ and $a_m > 0$.

Such choice of nodes of discretization (2) does not correspond to the essence of the problem.

Let S be the solution of (1) when K is replaced by T. For deviation $\|S-\tilde{S}\|$ we have the estimate

$$\|S-\tilde{S}\| \le (1-\lambda)^{-1}(1-\lambda-\delta)^{-1}\delta\|S_0\|$$

where

$$\delta = \delta(s_1, s_2, \ldots s_n, a_1, a_2, \ldots a_n) = 2\int_0^\infty |K(\tau) - T(\tau)|\, d\tau.$$

In [4] the problem of minimization of δ for fixed n, when K is arbitrary superposition of exponentials

$$K(\tau) = \int_a^b e^{-|\tau|s}\, G(s)\, ds, \quad G \geq 0$$

was solved.

2. *About the Method of Spheric Harmonics (MSH).* Consider the problem of anisotropic scattering. MSH is based on the reduction

$$g(\mu) \approx \tilde{g}(\mu) = \sum_{m=0}^{n} c_m P_m(\mu)$$

where $g(\mu)$ is the indicatrix of scattering, P_m are Legandre polinomials, and c_m are the corresponding Fourier coefficients. It was shown that the choice of this coefficients, based on requirement of minimization of quantity

$$\delta = \delta(c_0, c_1, \ldots c_n) = \int_{-1}^{1} \left[g(\mu) - \sum_{m=0}^{n} c_m P_m(\mu) \right]^{-1} d\mu$$

is more effective. The problem of minimization of δ for fixed n can be solved by numerical methods.

3. *About Principle of Invariance (PI).* Consider the general linear Transfer Equation in homogeneous half-space Π(0,+∞) (see [5]).

$$\pm \frac{d\mathrm{J}^\pm}{d\tau} = -\mathrm{A}\mathrm{J}^\pm + \mathrm{L}^+\mathrm{J}^\pm + \mathrm{L}^-\mathrm{J}^\pm \qquad (3)$$

where the vectors J^+ and J^- are the desired radiation intensities at the optical depth τ in directions of increasing and decreasing τ respectively.

The operators A and $\mathrm{J}^\pm$ describe the absorption and redistribution of radiation by infinite thin slab.

Ambartsumian's equation corresponding to (3) has the form

$$A\rho + \rho A = L^- + \rho L^+ + L^+\rho + \rho L^-\rho \tag{4}$$

where ρ is the operator of reflection from $\Pi(0,+\infty)$. It was shown, that the Eq. (4) has always unique physical solution (PS), which is the limit of iterations ρ_n , determined by

$$A\rho_{n+1} + \rho_{n+1}A = L^- + \rho_n L^+ + L^+\rho_n + \rho_n L^-\rho_n,\ \rho_0 = 0.$$

PS is the minimal positive solution of (4).

Новые подходы к некоторым классическим методам теории переноса излучения. Предлагаются новые, математически обоснованные, подходы к Принципу инвариантности Амбарцумяна, методу дискретных ординат Чандрасекара и методу Сферических гармоник.

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