# АСТРОФИЗИКА

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## DIRECT AND INVERSE PROBLEMS OF RADIATIVE TRANSFER


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The operator approach to the linear problems of Radiative Transfer ( RT ) and corresponding apparatus of analytical semigroups (see [1,2]) are very flexible and general means for solution of Direct and Inverse problems of RT, which are of a great importance in astrophysics . Here we present some results, which are mainly taken from [3].

The integral differential equation of stationary RT in a homogeneous slab $\Pi(\tau_o)$ of thickness $\tau_o \le +\infty$ admits of the following operator representation

$$\pm\frac{dJ^{\pm}}{d\tau} = -AJ^{\pm} + L^{+}J^{\pm} + L^{-}J^{\mp} \qquad (1)$$

where the vector functions $J^{+}$ and $J^{-}$ describe the radiation intensities at a depth $\tau$ in directions of increasing and decreasing $\tau$ respectively. $J^{\pm}$ may depend on the direction $\omega$, frequency $\nu$ and etc. A and $J^{\pm}$ are operators: A describes absorption of radiation by infinite thin slab and $J^{\pm}$ describe redistribution by $\omega$ , $\nu$ of emitted radiation.

Let $R(\tau_o)$ and $T(\tau_o)$ be the reflection and transmission operators for a slab $\Pi(\tau_o)$.

*Problem* 1. By given of $R(\tau_o)$ and $T(\tau_o)$ to find $R_m = R(m\tau_o)$ and $T_m = T(m\tau_o)$, $m \ge 2$.

This problem can be solved by multiple application of well known formulas of addition of layers.

Here we describe a more effective approach to this problem.

Let $W = W(\tau_o)$ is the Canonic solution (CS) of the equation

$$W = (R + TW)(T + RW). \qquad (2)$$

The CS is the limit of natural iteration $W_n$, with $W_0 = 0$. We have $W \ge 0$ $||W|| \le$

1 and

$$\rho=R+TW,\ X=T+RW \qquad (3)$$

where $\rho=R(\infty)$, and $X(\tau)=\exp(-G\tau)$ is the semigroup of operators with generator

$$G=-A+L^{+}+L^{-}\rho,\ \ X(0)=I.$$

We have

$$X(m\tau_o)=X_m(\tau_o) \qquad (4)$$

$R_m$ and $T_m$ may be determined by formulae

$$R_m=(\rho-X_m)W_m(I-W_m^2)^{-1},\ T_m=(I-\rho)X_m(I-W_m^2)^{-1} \qquad (5)$$

where $W_m=\rho X(m\tau_o)$.

*Problem* 2. (Division slab in half). By given of $R(\tau_o)$, $T(\tau_o)$ to find $R\left(\frac{\tau_0}{2}\right), T\left(\frac{\tau_0}{2}\right)$.

*Problem* 3. By given of $R(\tau_o)$, $T(\tau_o)$ to find local properties of the medium.

*Solution of the problem* 2. We determine W, ρ and X from (2), (3), then we extract the root from $X(\tau_o)$: $X(\tau_o/2)=[X(\tau_o)]^{1/2}$. Operators $R\left(\frac{\tau_0}{2}\right), T\left(\frac{\tau_0}{2}\right)$ are determined from (5) at $m=1/2$.

*Solution of the problem* 3. Applicating $n$ times repeated operation of division in half the operators $R(2^{-n}\tau_o)$ and $T(2^{-n}\tau_o)$ are created, which for enough large $n$, describe local properties of the medium. The operators A and $L^{\pm}$ participating in (1) may be found by means of them.

The operator $\rho=R_\infty$ is the CS of Ambartsumian's equation

$$A\rho+\rho A=L^{-}+\rho L^{+}+L^{+}\rho+\rho L^{-}\rho. \qquad (6)$$

At $L^{+}=L^{-}=L$ we obtain

$$L=(I+\rho)^{-1}(A\rho+\rho A)(I+\rho)^{-1} \qquad (7)$$

The formula (7) solves inverse problem 4.

*Problem* 4. By given ρ to determine $L(L=L_{+}=L^{-})$.

From Eq. (7) and well known connection between A and L may be determined A and L by iteration way.

In work [4] a matrix representation of $L^*$ in case of coherent anisotropy scattering were found.

Above mentioned results may be applied to the problems of distance probe for atmosphere of earth and other planets.

***Прямые и обратные задачи переноса излучения.*** На основании операторного подхода к линейным задачам переноса (ЛЗП) и соответствующего аппарата аналитических полугрупп предлагаются некоторые методы решения ЛЗП и задач по восстановлению локальных оптических свойств плоского слоя по наблюдамым характеристикам всей среды.

## REFERENCES

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