АСТРОФИЗИКА

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FACTORIZATION METHODS IN RADIATIVE TRANSFER THEORY

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A wide class of linear and nonlinear problems in Radiative Transfer (RT) theory lead to the integral equation

$$S(\tau) = g(\tau) + \lambda\int_0^\infty K(|\tau - t|)\, S(t)\, dt, \; or \; (\mathrm{I} - \lambda \mathrm{K})\, S = g, \tag{1}$$

where $0<\lambda\leq 1$ is the albedo of scattering .

Let V be the solution of nonlinear factorization equation (NFE)

$$V(\tau) = \lambda K(\tau) + \int_0^\infty V(\tau) V(\tau + t)\, dt;\; t \geq 0 \tag{2}$$

and let $V_\pm$ be Volterra type operators:

$$(\mathrm{V}_+ f)(\tau) = \int_0^\tau V(\tau - t)\, f(t)\, dt;\; (\mathrm{V}_- f)(\tau) = \int_\tau^\infty V(t - \tau)\, f(t)\, dt.$$

Then there holds the factorization (see [1,2])

$$\mathrm{I}\text{-}\lambda\mathrm{K}=(\mathrm{I}\text{-}\mathrm{V}_-)(\mathrm{I}\text{-}\mathrm{V}_+). \tag{3}$$

In applications to RT the kernel K is usually superposition of exponentials

$$K(\tau) = \int_a^b e^{-|\tau| p}\, G(p)\, dp;\; G \geq 0;\; \int_a^b \frac{G(p)}{p}\, dp = 1 \tag{4}$$

In this case (2) is reduced to Ambartsumian's equation (AE)

$$\varphi(s) = 1 + \varphi(s)\int_a^b \frac{\varphi(p)\,G(p)\,dp}{s+p};\; V(\tau) = \int_a^b e^{-\tau s}\varphi(s)\,G(s)\,ds;$$

$$\int_a^b \frac{\varphi(s)}{s} G(s)\,ds = 1 - \sqrt{1-\lambda}. \qquad (5)$$

Equation (1), including Miln's problem (when g=0, λ=1, S>0) is easily solved by using (3).

In conservative case λ=1, if $\int_0^\infty g(t)\,dt < +\infty$, equation (1) possesses of a solution $S \geq 0$ (see [2,3]).

The results, mentioned above, are generalized to systems of the form (1) (see [4]), which are also widely applied in astrophysics.

In [5] on the basis of these results it is proved that Miln's problem in inhomogeneous medium:

$$S(\tau) = \lambda(\tau)\int_0^\infty K(|\tau - t|)\,S(t)\,dt, S > 0,\; 0 \leq \lambda(\tau) \leq 1,\; \tau \in R^+ \;, \qquad (6)$$

under conditions $\int_0^\infty t(1-\lambda(t))\,dt < +\infty$ has a solution $S > 0$.

Some methods of solution of the equation (1) by means of repeated factorization are obtained by B.N.Yengibarian recently.The following representation

$$I-\lambda K=(I-U_-)(I-T)(I-U_+),$$

where $U_\pm$ are Volterra operators, and T is the operator of the form K with a simpler structure, lies on the base of these methods.

(a) The split of albedo: Then $I-U_\pm$ are Volterrian factors for $I-\lambda_1 K$, where $0 \leq \lambda_1 \leq \lambda$

(b) Iterative method: Then $U_\pm = K_\pm$ are Volterrian components of K: $K=K_+ + K_-$.The factorization of this type can be repeated several times.

(c) Factorizational interpretation of Chandrasekhar method of discrete ordinates

(MDO). Let the kernel K, in accordance with the MDO, be reduced to a finite sum

$$K(\tau) \approx T(\tau) = \sum_{m=1}^{n} a_m e^{-s_m|\tau|}$$

Then the operator I-λK may be represented in the form

$$I - \lambda K = (I - U_1^-)\ldots(I - U_n^-)(I - U_n^+)\ldots(I - U_1^+)$$

where $U_m^{\pm}$ are "simplest" Volterrian operators with kernels

$$b_m e^{-s_m \tau},\ b_m > 0.$$

Факторизационные методы в теории переноса излучения. Приводятся факторизационные методы авторов, применяемые при решении и изучении интегральных уравнений переноса в однородном и неоднородном полупространстве. К числу этих ФМ относятся метод нелинейных уравнений факторизации и методы многократной факторизации.

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