АСТРОФИЗИКА

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RELATIVISTIC ROTATION AND PULSAR ELECTRODYNAMICS

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Two illustrative examples are presented as an application of the relativistic rotation transformation and the nonlinear speed-distance relation.

1. Introduction. The hypothesis that the magnetosphere of a pulsar is a plasma in rotation, the rotation velocity approaching the light velocity for regions located close to the light cylinder, permit to describe certain characteristics of the pulsar phenomenon. The phenomenological models [1-4], describing the properties of pulsar emission, as well as the theoretical investigations [5-8] treating of the electrodynamics of pulsar are all based on the hypothesis that the rotation velocity v, at a distance r from the axis of rotation, is given by the well known linear law

$$v = r \cdot \omega \tag{1}$$

Although this law is rendered compatible with special relativity by limiting the rotation to regions interior to the light cylinder $r_L = c/\omega$, it is not satisfactory from the relativistic standpoint for the following reasons:

a) It implies the absolute character of simultaneity at a distance;

b) It does not satisfy the relativistic composition law of velocities;

In the 1950's, Takeno [9] and Trocheris [10] proposed to describe uniform rotation in special relativity by the following transformation

$$\begin{aligned} r'\varphi' &= r\varphi\,\mathrm{ch}\lambda - ct\,\mathrm{sh}\lambda \qquad r' = r \\ ct' &= ct\,\mathrm{ch}\lambda - r\varphi\,\mathrm{sh}\lambda \qquad z' = z \end{aligned} \qquad \lambda = \frac{\omega r}{c} \tag{2}$$

This transformations, which respects the relativity of simultaneity at a distance, the relativistic composition law of velocities and the addition law for angular velocities, leads to the non linear speed - distance relation

$$v = c\,\text{th}\lambda \tag{3}$$

As we see the rotation velocity v is never larger than c and reduces to its galilean value when $\lambda \ll 1$. The light-cylinder is then no more meaningful as it goes to infinity. It therefore appears necessary to investigate how the overall picture of the pulsar phenomenon is modified when one uses the relativistic rotation transformation (2) and the non linear speed - distance relation (3). We briefly present two illustrative examples of such modifications. For further details and applications see ref. [11-13].

2. *The co-rotating source model.* According to this model [1-4] the radiation source is located close to the light cylinder and the main characteristics of the pulsar radiation appear as relativistic effects of a very rapidly rotating source ($v\sim c$). The use of transformation (2) and formula (3) modifies slightly the characteristic parameters of pulsars. Assuming that the energy flux $f'(v')dv'$ in the instantaneous rest frame K' of the radiation source is given by a power law spectrum

$$f'(v') = E\,v'^{-\varepsilon} \qquad (E, \varepsilon = \text{const}) \tag{4}$$

we obtain from (2) the following expression for the observed spectral distribution [11]

$$f(v,\varphi) = E[\gamma(1 - \text{th}\lambda\cdot\cos\varphi)]^{-(3+\varepsilon)}\cdot v^{-\varepsilon}$$
$$\gamma = \text{ch}\lambda. \tag{5a}$$

Equation (5a) permits the determination of the beam width $\Delta\varphi = 2\varphi_{1/2}$ corresponding to the halfpower level, we have

$$\left(\frac{1-\text{th}\lambda\cdot\cos\varphi_{1/2}}{1-\text{th}\lambda}\right)^{3+\varepsilon} = 2.$$

which for $\text{th}\lambda \cong 1$ leads to

$$\Delta\varphi = 2\varphi_{1/2} \cong 2a\gamma^{-1}, \quad a = (2^{\frac{1}{3+\varepsilon}} - 1)^{1/2} \tag{5b}$$

the duration of the pulse is given by the difference between the time during which the radiation pattern of width $\Delta\varphi$ is directed to the earth and the time for radiation to cover the shift of source, we have [2]

$$\Delta P = r\frac{\Delta\varphi}{v} = \frac{2r}{c}\sin\Delta\varphi/2$$

using $\mathrm{th}^{-1}\beta = \lambda$ and $P = 2\pi/\omega$ period of the pulsar relative to the observer we obtain

$$\Delta P = \frac{P}{2\pi}\frac{\mathrm{th}^{-1}\beta}{\beta}(\varphi_{1/2} - \beta\sin\varphi_{1/2}) \tag{5c}$$

which for ultrarelativistic motion, i.e. $\beta \approx 1$, reduces to

$$\Delta P \cong \frac{aP}{2\pi}\gamma^{-3/2}\log 2\gamma .$$

Table 1 illustrates the modifications brought by the non linear laws in the values of the characteristic parameters for the pulsar PSR 0833 [2], period $P = 9\cdot 10^{-2}$s, pulse duration $\Delta P = 2\cdot 10^{-3}$s, index of frequency spectrum $\varepsilon = 1$.

Table 1

Parameter	linear law	nonlinear law
β	0.79	0.83
$\Delta\varphi$	0.64 rad	0.56 rad
r	$3.4\cdot 10^8$ cm	$5.1\cdot 10^8$ cm
$f(\nu,0)/f_0$, $f_0 = E\nu^{-\varepsilon}$	73	116

We see that

1) the source is located at a distance close to the light cylinder $r_L = 4.3\cdot 10^8$cm for the linear law and larger than r for the non linear law;
2) is less than unity in both cases;
3) the beamwidth is smaller for the non linear law while the increase in the energy flux is more important.

3. The quasi-static condition. In pulsar electrodynamics one often uses the so-called quasi-static condition [5-7]

$$\frac{\partial}{\partial t} = -\omega\frac{\partial}{\partial\varphi}, \tag{6}$$

where the time t and the azimuthal angle φ refer to the laboratory frame K. Condition (6) may be stated in a coordinate independent manner with the help of the Lie derivative. When applied to a 4-vector A^l it takes the form:

$$L(X)A_l = X^k \partial_k A_l + A_k \partial_l X^k = 0, \quad \partial_k = \frac{\partial}{\partial r^k} \tag{7}$$

where $X_l^l = (1,0,\omega/c,0)$ in coordinates $(x^0 = ct, r, \varphi, z)$. We have $X^l X_l = -1 + \omega^2 r^2/c^2$; hence X^l is timelike inside the light cylinder and spacelike outside. This leads Ardavan [7] to claim that under condition (6) the wave equation, for the 4 potential A, changes type (elliptic $\to$ hyperbolic) when crosses the light cylinder. From transformation (2) and the non linear law (3) the quasi static requirement can be generalized in an invariant form to:

$$L(u)A_l = 0, \quad u^l = (\mathrm{ch}\lambda, 0, \mathrm{sh}\lambda/r, 0). \tag{8}$$

In the case of the 4 potential A_l equation (8) gives

$$L(u)A_l = (\mathrm{ch}\lambda\partial_0 + \mathrm{sh}\lambda/r\,\partial_\varphi)A_l = 0 \tag{9}$$

The generalized quasi static condition (9) clearly gives the classical condition (6) when $\lambda \ll 1$.

Now, in the Lorentz gauge, the 4 potential A_l satisfies the wave equation

$$\Box A_l = g^{hk}\nabla_h \nabla_k A_l = -J_l \tag{10}$$

where J_l is the 4 current. It reads for instance in the case of A_0, when $\partial_{00} A_0 = \frac{1}{r^2}\mathrm{th}^2\lambda \frac{\partial^2 A_0}{\partial\varphi^2}$ is substituted from equation (9)

$$r^{-1}\frac{\partial}{\partial r}(rA_0) + r^{-2}(1-\mathrm{th}^2\lambda)\frac{\partial^2 A_0}{\partial\varphi^2} + \frac{\partial^2 A_0}{\partial z^2} = -J_0. \tag{11}$$

Unlike with condition (6), the coefficient of $\partial^2 A_0/\partial\varphi^2$ has its sign constant with the generalized condition (9) and the wave equation does not change type.

4. Conclusion. The use of transformation (2) and of the non linear speed-distance relation (3) by sending the light cylinder at infinity modifies the characteristic parameters of pulsars in the co-rotating source model circumvents the change of type of the 4 potential equation when a quasi-static type constraint is assumed. In brief, the light cylinder concept is a byproduct improper use of the linear speed-distance relation at relativistic velocities.

РЕЛЯТИВИСТСКОЕ ВРАЩЕНИЕ И ЭЛЕКТРОДИНАМИКА ПУЛЬСАРОВ

Р.А.КРИКОРЯН

В качестве применения преобразования релятивистского вращения и нелинейного соотношения скорость - расстояние приведены два иллюстративных примера.

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