АСТРОФИЗИКА

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FORMATION OF SPECTRAL LINES IN STELLAR ATMOSPHERES. LINEAR AND NONLINEAR THEORY

N.B.YENGIBARIAN, A.Kh.KHACHATRIAN

Byurakan Astrophysical Observatory

The present paper represents a short review of some directions of Radiative Transfer Theory in spectral lines (RTSL) concerning investigations of authors in this field.

1. *Nonlinear transfer theory of coherent or completly non coherent scattering*. It is well known, that due to an interaction of the strong radiation field with medium the local optical properties of the medium depend on the state of radiation field. At RTSL strong radiation field reduces to the high excitation of atoms and as a result of it the absorption capacity of the medium decreases (in a series of the ground state) and the effects of stimulated absorption begin to play role (negative absorption). Then the Transfer equation becomes nonlinear. Nonlinear Transfer Problems (NLTP) are of a great importance not only in astrophysics, but also in the theory of quantum optical generators.

Up to the 60 - s. the analytical theory of NLTP was absent. Although in that period the physics of local act of interaction of the strong radiation field was well known.

In 1963 the creation of the nonlinear theory of RTSL was undertaken by academician V.Ambartsumian and his followers.The investigations have been carried out by two basic directions.

a) Application of the principle of invariance (PI) of Ambartsumian to NLTP.

b) Development and creation of the method of self-consistent optical depths (SCOD).

The basic results due to the method of SCOD were obtained. The idea of this method which was suggested by V.Ambartsumian in 1964, was based on the possibility of exact linearization of NLTP in homogeneous plane layer by means of transition from limiting optical depth to real optical depth (which depends on radiation field) (see [1]).

The method of SCOD was realized by N.B.Yengibarian [2] where for the first time NLTP in three dimensional medium was exactly solved. The electron collision of the second kind was taken into account. It was also assumed, that in spectral line coherent scattering took place. This assumption means that the profile of absorption coefficient is directangle.

One of the most basic questions connected with application of SCOD (within linearization of the transfer equation) is determination of the real optical thickness of medium in spectral lines.

In [2] the following formula connected with limiting (y_0) and real (τ_0) optical thickness in case of two level atoms was found

$$y_0 = \tau_0 + \gamma \cdot \int_0^{\tau_0} S(\tau, \tau_0)\, d\tau \tag{1}$$

where S- is the source function for linear transfer problem in slab of thickness τ_0 and γ=const.

In [2] NLTP in the medium, consisting of three level atoms, when the transition $1\leftrightarrow 2$ are forbidden, was also solved.

In the work [3] by Therebij the method and results of the [2] were propagated for the case, when in spectral lines takes place complete redistribution in frequencies with an assumption coincidence of the profiles of absorption, stimulated and spontaneous emissions.

A number of monochromatic and polychromatic scattering problems representing essential astrophysical interest, were solved in the papers of Yengibarian, Vardanian, Therebij, Abramov, Dikhne, Napartovich and others (see [3-6]) by using methods of [2]; NLTP in presence of scattering and absorbing atoms, internal energy sources, taking into account electron collisions of the first and second kinds and so on among them were also solved.

The paper [7] of Khachatrian is devoted to the mathematical substantiation of the SCOD's method. In Yengibarian's work [8] nonlinear Miln's problem has been studied by using the method of [2]. In particular it was shown, that in the case of coherent scattering even for $\lambda < 1$, the solution of the problem lineary decreases with the geometrical depth.

The application of the results of the above mentioned works to the concrete astrophysical problems is not enough advanced yet.

In the 60-s some NLTP in a one dimensional medium and nonlinear nonstationary transfer problems were solved by means of application of PI and direct methods (see [9-11]). Some NLTP connected with an application in the theory of quantum optical generators have been studied too.

2. *Transfer in spectral lines with partial redistribution in frequencies (PRF).* The redistribution of radiation by frequencies at an elementary act of scattering (noncoherent scattering) plays an important role. It is described by redistribution function (RF)

$$r(x,x') = g(x,x')\alpha(x') \tag{2}$$

where $g(x,x')$ is the probability that an incident photon with a dimensionless frequency x' before scattering will acquire a frequency in the range from x to $x+dx$; and $\alpha(x)$ is the profile of absorption.

In general r depends also on the angle of scattering γ

$$r = r(x,x',\gamma).$$

The basic physical processes reducing to the noncoherent scattering are the following: natural width of spectral lines, Doppler broadening due to thermal or other kind of motions moving of atoms, pressure effects and etc.

In the middle of the 60-th the expressions of the function r corresponding to the basic mechanism of redistribution already were well known. Although up to the 70-th Transfer problems in spectral lines with simplest assumption on complete redistribution by frequency (CRF), as a rule, were considered (see [12]). Then $r = A\,\alpha(x)\alpha(x')$. In those cases, when the Doppler effect plays an essential role in process of redistribution, the assumption CRF become inadequated.

In 1971 in [13] the creation of analytical theory of RTSL at real physical laws of noncoherent scattering was begun byYengibarian.Here we shall use the therm "partial redistribution by frequencies" (PRF) adopted in astrophysical literature.Further a number of papers of Yengibarian, Nikoghossian, Gevorkian, Harutyunian, Pikichian, Khachatrian and others appeared, devoted to the development of PRF theory (see [14-20]).

In these works a number of problems of spectral lines formation in stellar atmosphere and other cosmic objects have been effectively solved. In [21-25,12] a rich arsenal of classical and new methods of the transfer theory and the theory of integral equations have been applicated; PI method, Probability method of Sobolev [12], Yengibarian's nonlinear factorization equation method [21,22], matrix method [19,20], the special method which reduced transfer problem in the plane layer of finite thickness to the corresponding transfer problem in half space [23,24], modification of

Chandrasekhar's discrete ordinate method [25], methods of analytical semigroups [23] and so on among them.

In [14] and in the next works the following bilinear expansion of RF plays a principle role

$$r(x,x') = \sum_{k=1}^{\infty} A_k \, \alpha_k(x)\alpha_k(x'). \tag{3}$$

It was shown the importancy of the expansion (3) in the case,when the $\{\alpha_k(x)\}$ are orthogonal with the weighting factor $1/\alpha$

$$\int_{-\infty}^{\infty} \frac{\alpha_k(x)\alpha_m(x)\,dx}{\alpha(x)} = \delta_{km} \,. \tag{4}$$

The works of Hummer and Avrett and other astrophysists (see [26-32]) for studing RF and creation of the expansion of type (3) a very important role have played.

In the case, when r depends on γ, the following decomposition is used

$$r(x,x',\gamma) = \sum_{l} P_l(\cos\gamma)\sum_{k} A_{lk}\, \alpha_{lk}(x)\alpha_{lk}(x') \tag{5}$$

where P_l - are Leghandre polynomials.

During the numerical solution of transfer problems with PRF in many cases various approximation methods were parallelly used.Comparison of the results shows neglected divergence of results. Matrix method is simpler from the point of view of calculation.

For the last years in astrophysical literature a number of investigations for the problems RTSL with PRF appeared,where different methods of discretization by frequencies were used (see [26-32]).

One of the most interesting case of the theory of RTSL with PRF is the problem, when

$$r(x,x') = a\alpha(x)\alpha(x') + b\alpha(x)\delta(x-x'). \tag{6}$$

This problem in work [33] of authors of the present paper, in context with application to the problems of the γ-quantum scattering on Mossbauer nuclei was solved.

3. *Nonlinear problems RTSL at PRF.* Consideration of the real laws of redistribution by frequencies is the actual problems of nonlinear theory of RTSL. In 70-s in astrophysical literature a number of s.c. works concerning above mentioned problems appeared, some results of which are published in monograph by D.Mihalas [29].

In these works "Full linearization method" (FLM) of Milkey and Mihalas takes the main place. FLM based on special iteration processes.Although (by admition of authors) this iteration processes are very laborious.

In the works [34,35] an analytical approach to the solution of NLTP with PRF is developed.This approach is based on the method of SCOD and the methods of linear theory of noncoherent scattering with PRF.

An assumption is made on the coincidence of the absorption coefficient profile $\alpha(x)$ with that of stimulated coefficient profile $\psi^*(x)$. It allows us exact linearize NLTP and reduce it to the analytical solution of the problem,by means of Ambartsumian's generalizing functions or matrix analogy.

Functions $\alpha(x)$, $\psi^*(x)$ and $\psi(x)$ (profile of spontaneous radiation) theoretically are different,connected with it, that on different levels, particularly in the excited states, the deviation of velocity distribution from Maxwell velocity distribution is appreciable, that is conditioned by the selective character of atom excitation by quanta of this or that frequency (see [36-38]). The numerical results show (see [37]) , that the relative deviation $\frac{\psi^* - \alpha}{\alpha}$ is of the order of 5 per cent in the center,which is well agreed with the results of [38]. That's why the assumption of coincidence α and ψ^* is a good first approximation to the solution of the problem. In [39] the second approximation, taking into account differences between α and ψ^* is given.

4. *Radiative transfer in the random nonhomogeneous medium (RNHM).* For the first time in astrophysics the transfer problem in RNHM with random local optical properties was considered and solved by Yengibarian and Nikoghossian [40]. Solution of such problems contains valuable information about local optical properties of the medium and presents essential astrophysical interest.

The problem of diffuse reflection from random inhomogeneous plane layer is solved by means of application of PI. It was assumed that albedo of scattering $\lambda = \lambda(\tau)$ is a random function of optcal depth, which is described by Gauss-Markov random process.

Solution and studying of the transfer problems (for different models) in RNHM were continued by Yengibarian, Vardanian and others (see [41]).

In the 70-s in astrophysical literature a number investigations devoted to the radiative transfer in RNHM appeared.

ОБРАЗОВАНИЕ СПЕКТРАЛЬНЫХ ЛИНИЙ В ЗВЕЗДНЫХ АТМОСФЕРАХ. ЛИНЕЙНАЯ И НЕЛИНЕЙНАЯ ТЕОРИИ

Н.Б.ЕНГИБАРЯН, А.Х.ХАЧАТРЯН

Статья представлят собой краткий обзор некоторых направлений теории переноса излучения в спектральных линиях, развитых авторами.

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