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A SIMPLE PROBLEM OF RADIATIVE TRANSFER BY MULTILEVEL ATOMS

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We consider the problem of determining the radiation fields reflected and transmitted by a slab containing multilevel hydrogen atoms and illuminated on one side by a given radiation field. We treat the extreme non-LTE situation in which the populations of the different levels are determined by the radiative processes. We take into account the population and the transfer effects in a self-consistent way by solving the transfer equations in all the lines and continua together with the equations of statistical equilibrium for all levels. We limit ourselves to the idealistic case of rectangular profiles in the lines and continua and to a model of atom with 4 levels and a continuum. Under conditions close to thermodynamic equilibrium we empirically derive a Schuster-like law for the continua with transmitted radiation fields varying as the inverse of the optical thicknesses. Turning to out-of-equilibrium conditions we emphasize the crucial role of the loss probability of the Ly α photons. Owing to the rapid decrease of the excitation/ionization degree in the medium and contrary to the conservative case the optical thicknesses of the subordinate transitions remain now finite even when the population of the fundamental level along the line-of-sight becomes infinite. As a result of this relative transparency the strong emission lines formed by recombination mechanisms can escape from the medium. Although the present problem remains largely academic because of the number of simplifications introduced we suggest some possible applications and developments.

1. *Introduction.* Although very complex transfer problems have been solved successfully the physical understanding of the results rather rests on the comprehension of idealized simpler exercices which can be discussed thoroughly and which display more basic effects. For instance the influence of the scattering in particular cases will often be clarified on the basis of Schuster's [1] model. The study of the radiative transfer problem in a two-level atom for a homogeneous medium (e.g. Avrett & Hummer [2]; Hummer [3]; Ivanov [4]) has thrown light on the non-LTE effects that may appear under other more realistic specific conditions. The model of planetary nebulae by Menzel [5] and his collaborator in the well-known limiting cases A or B still remains the basic scheme for understanding subsequent studies. Milne's

problem constitutes a powerful tool for partly explaining the radiative mechanisms within a stellar atmosphere and the characteristics of the emerging radiation field.

The present work is devoted to a generalization of Schuster's problem that is straightforward in its principle yet much more difficult to solve. Whereas Schuster was concerned by scattering at a single frequency we generalize the problem to a multifrequency radiation field and to multilevel atoms. So we consider a slab containing multilevel atoms – say hydrogen atoms – and allow all the possible corresponding radiative transitions to take place according to the usual quantum probabilities. The slab is illuminated on one side by a given radiation field at all relevant frequencies in the lines and continua. We ask for the radiation fields reflected and transmitted by such a medium. We stick to the rule of the game consisting in coupling the population and the transfer effects in a fully self-consistent way. In other words we simultaneously solve the equations of statistical equilibrium for all levels and the transfer equations at all frequencies: the equations of statistical equilibrium express the dependence of the level populations upon the radiative intensities whereas the transfer equations tell us how the radiation fields react to the level populations via the absorption and emission coefficients.

To our knowledge this archetype problem of multifrequency scattering posed in that manner has never been examined before except by Nikogosyan [6] with analytical means in a three-level atom and for a semi-infinite medium. Of course the problem is now too difficult to be solved analytically and we have to resort to numerical procedures. In spite of that fact we think that the exercice is interesting because it contains the essence of the radiative mechanisms occurring in a medium in which the LTE assumption is not valid and may thus lead to results that are general enough to elucidate more complex situations. On the other hand our schematic model can in no way supersede realistic calculations which include many more factors than the limited number of parameters used here.

The specifications of the model will be given in Sect.2. When losses of photons are ignored we arrive at the pure scattering problem for multilevel atoms, which is examined as a starting point in Sect.3. In Sect.4 we shift to non-conservative cases and show the dramatic role of the losses of $Ly\alpha$ photons. In Sect.5 we examine the influence of some parameters upon the radiation fields. In outlining the main results we conclude in Sect.6 about the relevance of the present model to astrophysical objects.

2. *Description of the model.* As we will later allude to the extended atmospheres of some stars illuminated by the radiation coming from the inner parts we may think of the present model as a schematic representation of a hydrogen ionization zone from

the HII region up to the adjacent HI region and will lead the discussion according to this terminology.

We take a hydrogen atom with 4 levels and a continuum. In principle the electronic temperature should be determined by the equation expressing thermal equilibrium. However in our idealized model the temperature does not really count because the collisions are not explicitely taken into account and the temperature only fixes the coefficients of recombination from the continuum. In practice the temperature of most HII regions as determined observationally or theoretically fall in a relatively narrow range of values around the "standard" 10 000 K figure. Therefore we may safely take a value of that order. Of course this value does not imply that the real electronic temperature in the HI outer region is so high. But in our model the thermal temperature of the HI does not play any role.

The profiles of the absorption and emission coefficients in the 6 lines (Lyα, Lyβ, Lyγ, Hα, Hβ and Paα are taken as rectangular with a total width corresponding to velocities in the range $(-v,+v)$, where v is given either by the thermal value $\sqrt{2kT_e/m_H}$ or by a convenient value characterizing both the thermal and the random macroscopic velocity fields. This assumption is clearly unrealistic as the random walk of the photons through the medium heavily depends upon the line profile and the presence of "Doppler wings" at the edge of the core rules out the use of a constant cross-section for scattering. However beyond the simple Schuster monochromatic problem this modeling assumption allows us to explore the effects of including the transitions between several levels without adding at the same time the redistribution mechanisms within a given line. More realistic profile with Doppler and damping wings will be considered in coming works.

Likewise we treat each continuum at a single frequency or, more exactly, as if it extended over a certain frequency interval with a constant absorption and emission coefficient. That hypothesis is also quite severe-but certainly less severe than in the case of the lines. We define the radiative coefficients of a mean equivalent continuum as follows, with the Lyman continuum taken as an example.

We start from the number of radiative recombinations per unit volume per unit time onto Level 1 written as $N_e^2\beta_1$, where N_e is the electronic density and the coefficient β_1 is regarded as known. We introduce a would-be Einstein coefficient B_{15} defined in such a way that the number of photoionizations $(cm^{-3}s^{-1})$ is $N_1B_{15}J_{15}$, where J_{15} is the mean radiative intensity at the frequency $\nu_{15}\equiv\nu_1$ of the Lyman limit. Neglecting the induced emissions, we would get for the LTE population N_1^* the equilibrium relation

$$N_1^* B_{15} J_{15} = N_e^2 \beta_1 \ , \tag{1}$$

where

$$J_{15} = (2h\nu_1^3/c^2) e^{-h\nu_1/kT} \ , \tag{2}$$

$$(N_e^2/N_1^*) = (2/g_1)\,(2\pi m_e kT/h^2)^{3/2} e^{-h\nu_1/kT} \ . \tag{3}$$

We thus find B_{15} in terms of β_1 as

$$B_{15} = (2h\nu_1^3/c^2)^{-1} (2/g_1)\,(2\pi m_e kT/h^2)^{3/2} \beta_1 \ . \tag{3}$$

We now consider that the continuum is equivalent to a rectangular "line" with equivalent width $\Delta\nu = kT/h$. This value expresses that the energy $h\Delta\nu$ is given by the mean energy kT of the electrons. As in the case of a line, we derive the absorption coefficient (in cm^2).

$$a_1 = (h\nu_1/4\pi)\,(kT/h)^{-1} B_{15} \ . \tag{4}$$

With $T = T_e = 10000$K, $\beta_1 = 15.8 \cdot 10^{-14} cm^3 s^{-1}$, we get $a_1 = 6.1 \cdot 10^{-18} cm^2$, which is practically the value of the cross-section at the Lyman limit: this shows the consistency of our approximations.

The whole problem is solved by the method of addition of layers, which has been generalized so as to handle non-linear problems (Magnan [7]; Gros & Magnan [8]). We recall that this method is derived from Ambartsumian's principle of invariance (see e.g. Mnatsakanian & Pickichian [9]) and consists in building a medium by successive additive steps. The addition formulae that are used at each such step give the radiative properties of an ensemble of two layers when the properties of each individual layer are known. The method proved very efficient and showed no special problem of convergence. It made it possible to deal with very large optical thicknessess (up to 10^{10} in the Lyα transition) with some tens of layers. As we are not interested in the angular dependence of the radiation fields we have used only one angular quadrature point.

Finally each calculated medium is characterized by its optical thickness in the Lyman transitions, which is fixed. Actually, as explained by Magnan [7], the Lyα optical depth is chosen as the independent spatial variable of the problem. Thus the Lyman optical thicknesses of the successive layers are taken as constant during the iterative procedure.

3. *Pure conservative multifrequency scattering.* We first consider the case of *pure* radiative processes in the atom. We neglect all mechanisms capable of converting radiative energy into thermal energy and vice-versa, which amounts to ignoring all inelastic collisional processes. Likewise we neglect every absorption of photons outside the line to which they belong. For a single transition, this case reduces to the pure scattering problem in a line. Here the problem is much more complex as we take into account the interlocking of many different lines: we are now dealing with a multi-line pure scattering problem.

Nearly all classical *academic* problems of radiative transfer are *linear* problems in which the physical properties of the medium do not depend on the actual radiation fields. In particular the possible influence of the radiation upon the optical properties of the medium is not taken into account. If this effect is ignored, the internal and external radiative intensities become proportional to the sources. In the present case the populations of the levels depend upon the radiation fields and hence the opacities are not known in advance. The problem is non-linear and the response is not proportional to the incident sources of energy.

The present highly non-linear case of pure scattering for multi-level atoms does not seem to have ever been considered as such in its essence-although of course such non-linear mechanisms *are included* in any realistic multilevel non-LTE transfer calculation. Yet as recalled above the issue is very simple in its principle and in its formulation and appears as the basic model for studying the coupling between the transfer and population effects. As a noteworthy exception Nikogosyan [6] formulated the scattering problem of a semi-infinite atmosphere composed of 3-level atoms by using Ambartsumian's invariance principle. He succeeded in giving an analytical solution of the equations but since the generalization to more complex atoms and to finite media seems impossible by analytical means other results have not been produced. We extend here Nikogosian's original work but by a numerical approach.

3.1 *Near-equilibrium conditions.* In this subsection we consider a finite layer illuminated by an undiluted Planckian 10-frequency radiation field Q. If the slab were illuminated from both sides, strict thermal equilibrium would be realized. The departure from equilibrium comes only from the asymmetrical boundary conditions. We want to determine the radiation S returning towards the source and the radiation R emerging from the opposite side. In order to present the results in a non-dimensional form, which is easier to handle, we conveniently express the radiation fields in terms of the pseudo-reflection coefficients S_{ij}/Q_{ij} and pseudo-transmission coefficients R_{ij}/Q_{ij} for each transition ij in the lines or in the continua.

Of course since the problem is not linear those "coefficients" depend on the incident field Q at all the frequencies and do not deserve the names of genuine reflection or transmission coefficients.

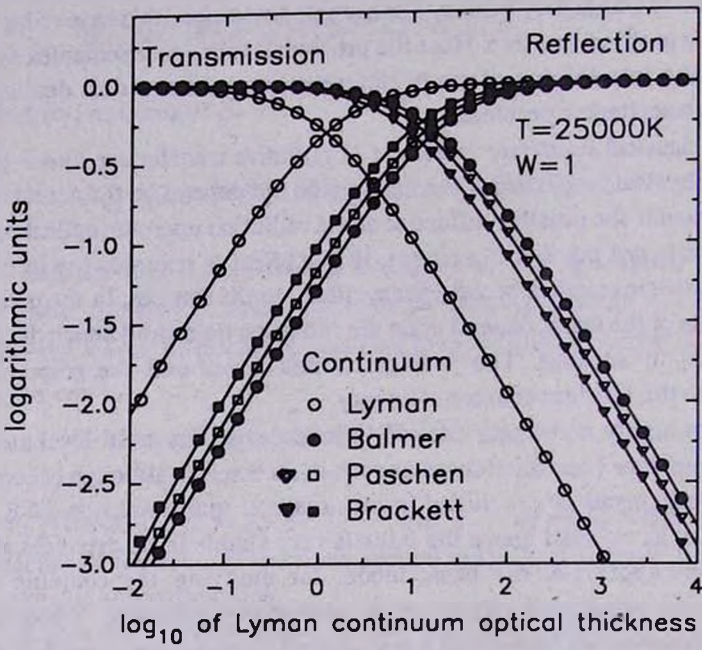

Fig. 1. Transmission and reflection in the continua. The logarithms of the emerging fluxes in the continua (scaled to the incident fluxes) are shown as a function of the optical thickness of the hydrogen layer, which is illuminated by a Planckian radiation field at 25 000 K. The symbols refer to the calculations. The full drawn lines correspond to the Schuster law for pure scattering in a single line.

Figs. 1-3 report the values of the radiation fields emerging from the atmosphere for increasing values of the optical thickness of the hydrogen layer and for a temperature of the source $T=25\,000$ K. (We recall that the optical thickness of each medium in the Lyman transitions is fixed during the iterative numerical procedure.) Since we will use both the Lyα scale and the Lyman continuum scale in the figures it is not unuseful to say the ratio between the Lyα and LyC opacities, which depends on the line-width parameter, amounts here to $7.022\cdot10^3$ for a broadening velocity of 10 km/s roughly corresponding to a temperature of 10 000 K. The intensities in the continua are shown in Fig. 1 and the intensities in the lines are shown in Figs. 2-3. We see that the "reflection coefficients" of all the transitions tend

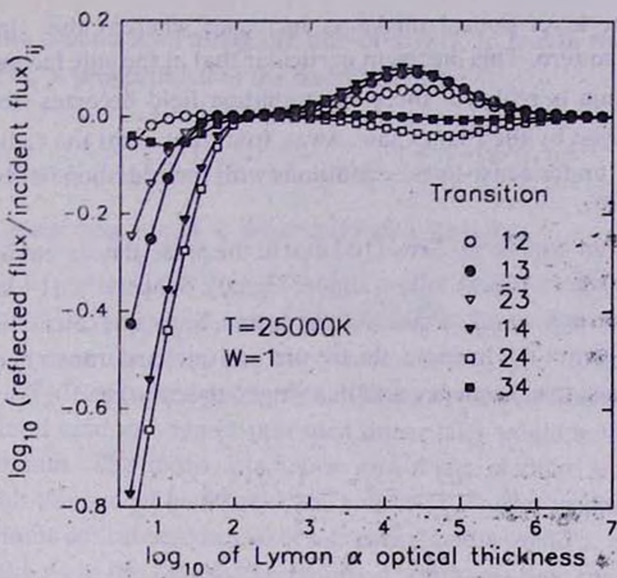

Fig. 2. Reflection in the lines. The curves give the fluxes of the radiation returning towards the illuminating source, which radiates as a black-body at 25 000 K, as a function of the optical thickness of the scattering layer.

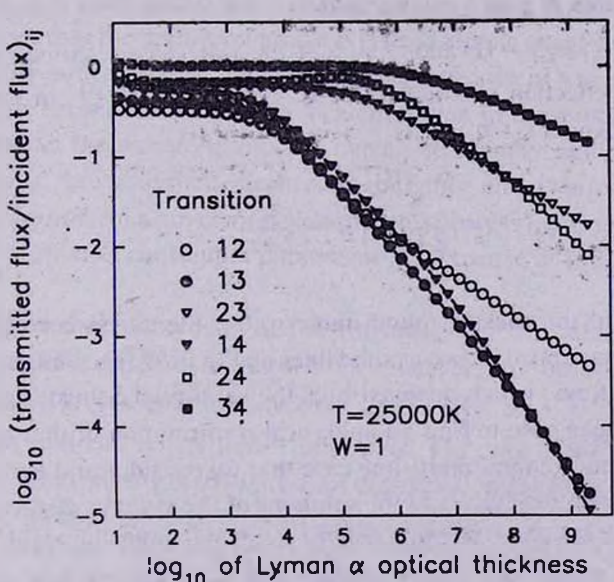

Fig. 3. Transmission in the lines. The curves give the fluxes emerging from the surface of the scattering layer as a function of its optical thickness. The layer is illuminated on the other side by a black-body spectrum at 25 000 K.

to unity when the layer optical thickness increases whereas the "transmission coefficients" tend to zero. This means in particular that at the side facing the source a certain equilibrium is realized: there the radiation field becomes isotropic with $S_{ij} = Q_{ij}$ and is given by the Planck law. Away from that point the radiation fields decrease regularly under quasi-linear conditions with the radiation fields remaining quasi-isotropic.

It has already been noticed by Gros [10] that in the present near-equilibrium case the continuum emergent fluxes follow almost exactly Schuster's [11] law for pure scattering, which we now recall to ease the discussion. Since our calculations use one Gaussian division point (μ_1) in angle, the inward and outward transfer equations for pure scattering at a single frequency and in a single direction read

$$\mu_1 \frac{dI^+}{d\tau} = I^+ - \frac{1}{2}(I^+ + I^-)$$
$$-\mu_1 \frac{dI^-}{d\tau} = I^- - \frac{1}{2}(I^+ + I^-) \qquad (5)$$

with obvious usual notations and $\mu_1 = 1/\sqrt{3}$. By solving Eq. (5) we immediately get the radiation fields R and S emerging from a layer with total optical thickness τ_0 illuminated by $I^+_{(\tau=\tau_0)} = 0$ and $I^-_{(\tau=0)} = Q$ and derive the corresponding transmission and reflection coefficients $t = R/Q$ and $r = R/Q$ in terms of the effective optical thickness $\tau_1 = \tau_0/\mu_1$ as

$$t = \frac{2}{2+\tau_1}\ , \quad r = \frac{\tau_1}{2+\tau_1}\ . \qquad (6)$$

By taking the optical thicknesses found numerically, the curves corresponding to those functions (6) have been drawn as solid lines on Fig. 1. We see that the computed points fall on those lines, which demonstrates the validity of Schuster's law in our case. We have not been able to find an analytical confirmation of that law-derived here empirically-in the general multi-line case that we consider and leave to others the task of studying the structure and the solutions of the systems of equations from a mathematical point-of-view. We only briefly recall the following result 'ated to the two-level case, which may partly explain our more general law: for a given illumination, the number of atoms along the line-of-sight in the upper level is proportional to the optical thickness of the layer (or equivalently to the number of atoms in the ground level along the line-of-sight). In fact in the two-level case the ratio N_2/N_1 is proportional to the mean intensity

$$J \equiv \frac{1}{2}\left(I^{+}+I^{-}\right)=Q\,\frac{1+\tau_1-(\tau/\mu_1)}{2+\tau_1}\ , \tag{7}$$

which is a linear function of τ. We immediately find that

$$\int_0^{\tau_0} J\,d\tau = Q\tau_0/2\ . \tag{8}$$

Therefore $\int N_2 ds$ is indeed proportional to the optical thickness of the medium. In the multi-level case we suggest that such linear laws would hold approximately for each continuum. The mean intensities within the medium would then decrease linearly with the optical distance to the source at the corresponding frequency and the subordinate optical thicknesses of successive media would be proportional to their optical thickness in the Lyman continuum. At the same time the emerging intensity of each continuum would vary in inverse proportion to the corresponding optical thickness (Schuster's law). For all those reasons we may say that in this near-equilibrium configuration quasi-linear conditions seem to be satisfied.

Figures 2-3 show that the behavior of the lines is more complex, which demonstrates that the multilevel problem is not a simple quasi-linear problem either. We especially notice that the reflection coefficients present a maximum a little larger than 1 in the resonance lines and a local minimum in the subordinate lines. Those extrema reflect the transformation of Lyman continuum photons into Lyman line photons by a recombination mechanism but the effect is not very pronounced because in the present near-equilibrium case this process tends to be counterbalanced by the production of continuum photons at the expense of line photons via reverse cycles.

In Fig. 3, which is concerned with the transmission of the radiation, the comparison of the slopes of the different curves at large optical depths indicates that the net effect of the diffusive processes is to transform Lyβ, Lyγ and Hβ photons —namely the photons that can be splitted— into Lyα, Hα and Paα photon –which are indivisible. This splitting is efficient since the optical thicknesses we consider are very large, up to 10^9 in Lyα, which corresponds to about $4 \cdot 10^7$ in Hβ.

3.2. *Out-of-equilibrium conditions and recombination spectrum.* We assume that the medium has the same characteristics as before but now turn to a situation departing from equilibrium by letting the hydrogen layer be located at some distance from the central source. This "geometrical" departure from equilibrium is also equivalent to a departure from a Planckian law for the illumination. If d designates the distance between the envelope and the central star with radius R, the flux per unit surface per unit time per unit frequency interval received on the layer at frequency ν is $\pi(R/d)^2 B_\nu(T)$ (the intensity leaving the star is assumed to be independent of direction). In a two-stream approximation with 2π steradians in each direction, this corresponds to an equivalent intensity $WB_\nu(T)$ in the incident direction, where $W = (R/d)^2$, and thus to a mean intensity $(1/2)WB_\nu(T)$ in 4π steradians. Note that this value differs by a factor 2 (for large distances) from the correct value of the mean radiation field, namely $(1/2)\ [1-\sqrt{1-(R/d)^2}\,]B_\nu(T)$ or $(1/4)\ (R/d)^2 B_\nu(T)$ when $d>>R$, but it is here physically more meaningful to work with the right value of the incident flux. In what follows we will call $W = (R/d)^2$ the dilution parameter in order to distinguish it from the usual dilution factor $(1/4)\ (R/d)^2$. Our boundary condition at the illuminated surface is then $Q_\nu = WB_\nu(T)$. As the dilution parameter decreases the radiation field increasingly departs from equilibrium conditions and the solution to the multi-line pure diffusion problem becomes more difficult to predict. We illustrate the results in the case of a central black-body source spectrum at 25 000 K and a dilution parameter of 10^{-4}.

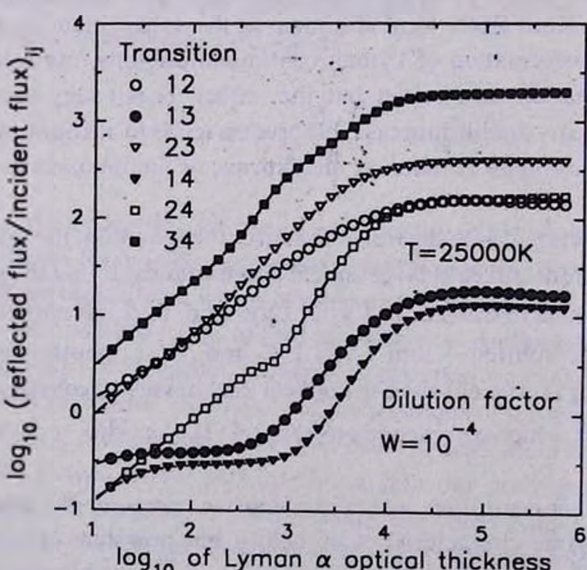

Fig. 4. Reflected line fluxes. The temperature of the illuminating central black-body is 25 000 K.

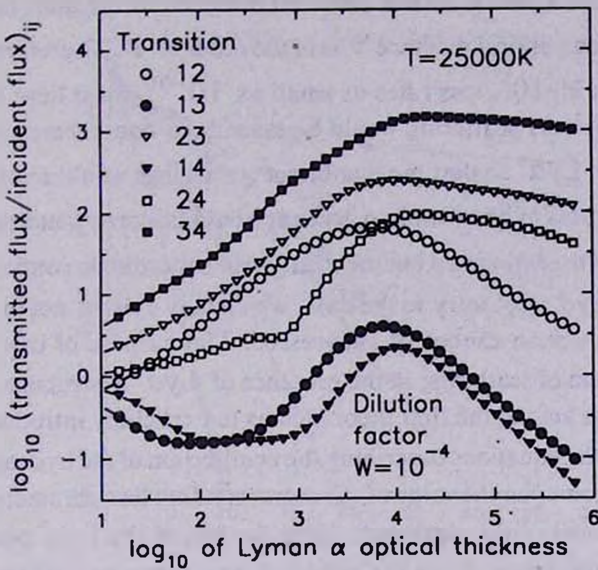

Fig. 5. Transmitted line fluxes. The temperature of the illuminating central black-body is 25 000 K.

The behavior of the continua (not shown here) does not differ significantly from the previous case as the deviations from Schuster's law for pure scattering are small. This indicates that the direct and reverse cycles still partially balance each other. Nevertheless Rosseland's theorem, according to which quanta of higher energy are transformed into quanta of lower energy under nebular conditions, now begins to manifest itself. Figures 4-5 show the high increase of the outgoing fluxes in the lines with respect to the incident ones. Since the total radiative energy is strictly conserved, the extra energy fed into the lines is obviously borrowed from the continua. The $Ly\beta$ and $Ly\gamma$ lines are the least strong as they can be degraded into photons of higher series and $Ly\alpha$ photons.

4. *Non-conservative scattering.* Although the pure scattering problem in a multilevel atom is interesting as an illustrative example of radiative effects in dilute media, the case is hardly realized in practice since photons are lost from the radiation pool of hydrogen line transitions by absorptions and collisions. The loss of the $Ly\alpha$ photons is particularly important to include in that respect because those photons cannot be destroyed by degradation into other lines, as is the case for the other Lyman and subordinate lines. Moreover since $Ly\alpha$ photons suffer a large number of scatterings before escaping from the medium, the loss mechanisms become

effective in the long run, even if their probability of occurrence *per scattering* is very small. In the present artificial case of coherent diffusion in the lines the number of scatterings to cover an optical distance τ is of the order of τ^2. Therefore for a typical Lyα optical depth of 10^5, loss rates as small as 10^{-10} must here be taken into account. (In practice the scattering would be essentially non-coherent up to optical thickness $\approx 10^4$ in Lyα so that the number of scatterings would scale as τ rather than τ^2 and loss rates to be taken into account would be correspondingly larger, of order τ^{-1}). Dust absorption and photoionization in subordinate continua constitute possible sinks of Lyα. Contrary to the dust, which may exist or not in practice, the latter cause of absorption cannot be suppressed. The purpose of this section is to examine the problem of scattering in the presence of Lyα destruction.

Sobolev [11] was among the first theoreticians to explicitely introduce the loss of Lyα photons into the equations describing the equilibrium of the hydrogen atom and to base his discussion upon the value of the corresponding loss parameter. Our study confirms the rightness of this approach. Here the loss of the Lyα photons will be measured by a parameter β in the following way: along an infinitesimal optical path $d\tau$ the probability for a Lyα photon to interact with the medium is written $(1+\beta)d\tau$ where the fraction $\beta d\tau$ represents the probability that the photon will be removed from the Lyα photon pool (pure absorption) and the fraction $d\tau$ represents the probability of being absorbed by an hydrogen atom in Transition 1-2.

The loss parameter β should be estimated in each specific case. To see its influence we will regard it as a free parameter in Subsect. 4.1 but will specify it in Subsect. 4.2 by considering the destruction of Lyα photons in the Balmer continuum.

4.1 *Role of the loss probability* β. The introduction of a loss factor in Lyα alters the picture dramatically in comparison to the conservative case. The emergent flux in the Lyman continuum is plotted in Fig. 6 against the optical thickness of the medium for several values of the loss parameter. The curves show that the emergent flux now decreases *exponentially* with the optical thickness whereas in the pure scattering case ($\beta=0$) it decreased as a *power law*, namely as the inverse of the total optical distance. This means that cycles creating Lyman continuum photons cannot any longer balance the cycles destroying them. There fore continuum photons are eventually lost. Another important consequence of introducing losses in Lyα concerns the degree of excitation of the upper levels and correlatively the optical

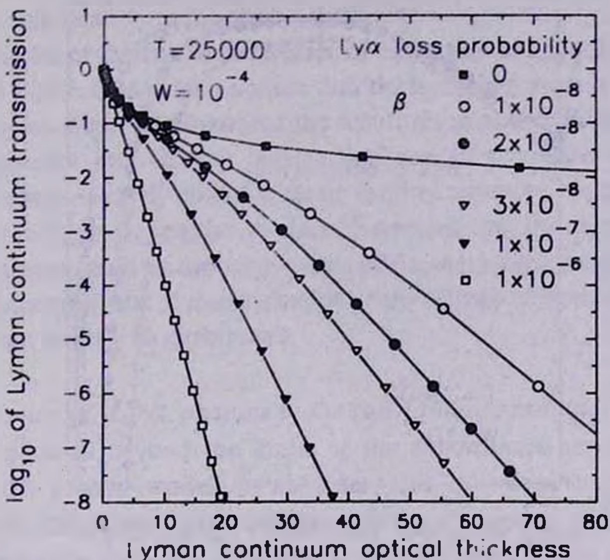

Fig. 6. Transmission of the Lyman continuum as a function of the optical thickness of the medium for different values of the loss parameter.

thicknesses of the subordinate lines and continua. In the conservative case, we have seen that the intensities and the degrees of excitation within the medium varied slowly and linearly with the optical depth. In such a case the total optical thicknesses of the medium in the subordinate transitions increased with the optical thickness in the Lyman transitions without apparent limits. In fact the number of atoms in a given excited level along the line–of–sight (i.e. the so-called column density) was proportional to the corresponding number of atoms in the ground level. On the contrary, once photons are destroyed within the medium, the degrees of excitation become exponentially decreasing functions of the Lyman optical depth. As a result for increasingly deeper points away from the source in a given medium, the optical distances in the subordinate transitions *reach a maximum and then stay constant*. Correlatively, for different media of growing optical thicknesses, the column density of the atoms in the subordinate levels tends to a finite value when the column density of the atoms in the ground level tends to infinity. In physical terms the passage from the HII ionized region to the HI unionized one is so sharp that it is only in the narrow HI* transition zone that atoms may be found in the upper levels in significant amounts. Beyond that point in the unionized region the degree of

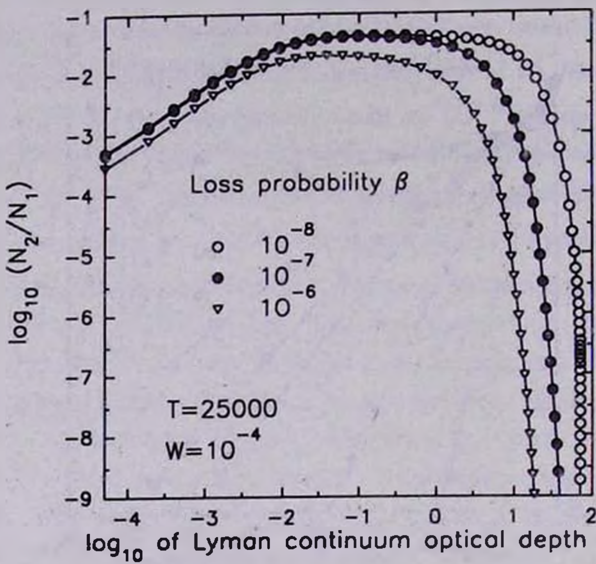

Fig. 7. Degree of excitation N_2/N_1 versus optical distance to the illuminating source for various values of the Lyα loss probability β.

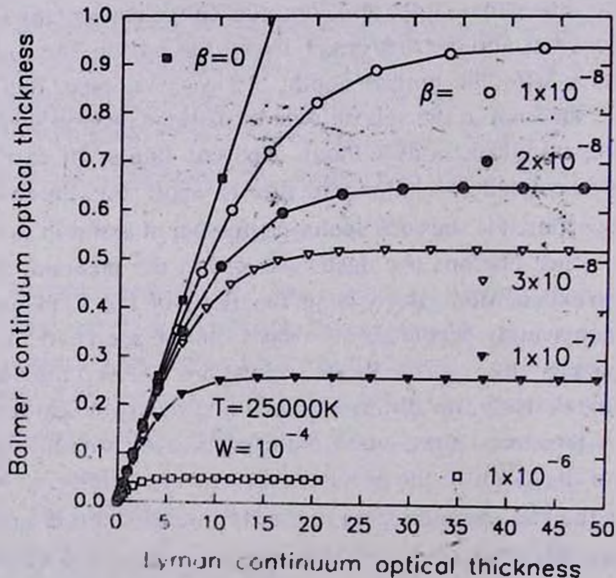

Fig. 8. Balmer optical thickness of the medium as a function of its Lyman optical thickness for various values of the Lyα loss probability β.

excitation drops so rapidly that the populations of the upper levels become too small to contribute significantly to the optical thickness.

Some examples of the run of the degree of excitation of Level 2 within a medium are shown on Fig. 7. One indeed notices that the hydrogen atom is excited only over a restricted zone. Figure 8 illustrates the result given above: for increasing Lyman optical thicknesses the Balmer optical thickness of successive media first grows and then reaches an upper bound without varying anymore beyond. Owing to our hypothesis of rectangular profiles, it must be stressed that the reported values of the optical thicknesses could be unrepresentative of actual realistic values. Nevertheless the result concerning the *finite* character of the values of the subordinate optical thicknesses can hardily be questioned.

4.2. *Absorption of Ly α photons in the subordinate continua*. The absorption of Lyman line photons beyond the limits of the subordinate series is a loss factor inherent to the atomic model we consider and therefore cannot be excluded a priori. We will particularize the discussion on the absorption of Lyα in the Balmer continuum, which is the main point for the reasons given above, but the absorption of the photons from other Lyman and subordinate lines will also be taken into account in the calculations. The non-linearity of the problem results from the fact that the

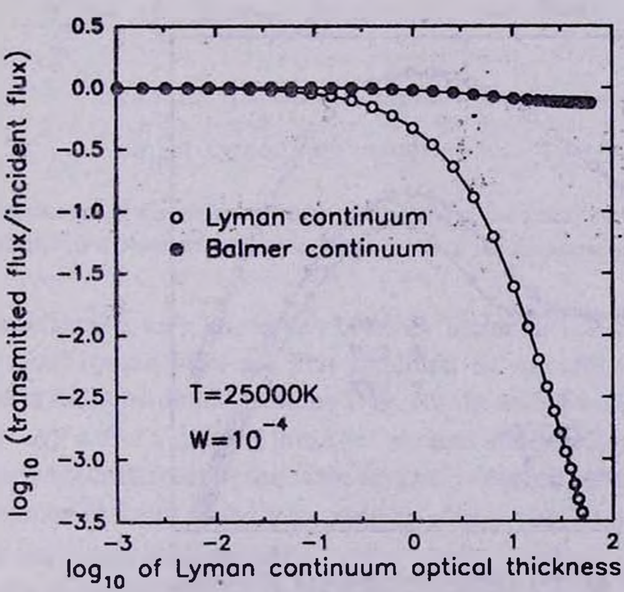

Fig. 9. Emergent Lyman and Balmer continua as a function of the optical thickness of the medium. The destruction of the Lyα photons in the subordinate continua is taken into account.

opacity due to the second level is determined by the degree of excitation, which is in turn determined by the losses and thus by the opacity itself. In the present case this interesting feedback loop is expected to act as a *regulator* of the degree of excitation. If the degree of excitation of the second level increases, the Balmer continuum will become optically thicker so that the loss probability of Lyα photons will also increase. But in return this will tend to decrease the degree of excitation and thus to counterbalance the initial perturbation. Conversely a decrease of the population of Level 2 will entail a lowering of the opacity and a resulting tendency to increase the radiation field and the population. According to this self-regulating mechanism, the degree of excitation of Level 2 should reach an equilibrium value such that the competing population effects balance each other.

Our calculations show that under the same illumination conditions as before (T=25 000 K, W=10^{-4}) the optical thickness of the Balmer continuum saturates at the value $\tau_{2c} \approx 0.5$. The maximum value of N_2/N_1 within the medium was found equal to 0.0383, which corresponds to an excitation temperature of about 25 500 K, close to the temperature of the ionizing core. (The fact that this excitation temperature is a little larger than the temperature of the source is not a priori excluded in our model, which ignores the tranfer between mechanical and radiative energies.

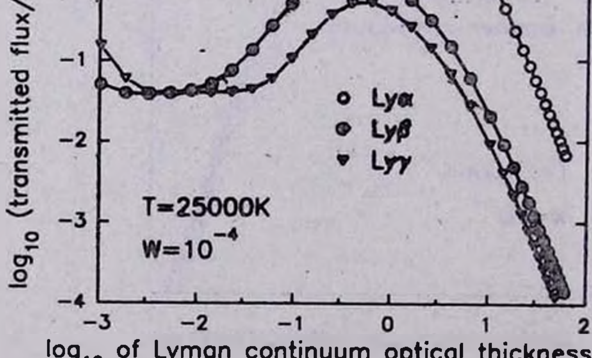

Fig. 10. Transmission of the Lyman lines as a function of the optical thickness of the medium. The destruction of the Lyα photons (and photons from the other lines) is taken into account.

The near equality between the maximum excitation temperature and the black-body temperature of the source seems a general law of our model as it was verified for other values of the parameters.) As shown by Fig. 9, the optical thickness of the Balmer continuum is too low to significantly affect the corresponding continuum intensity, which stays nearly constant in that specific case. On the contrary the Lyman continuum is very sensitive to the Lyα loss by the Balmer continuum absorption and decreases very rapidly beyond $\tau_{1c} \approx 1$.

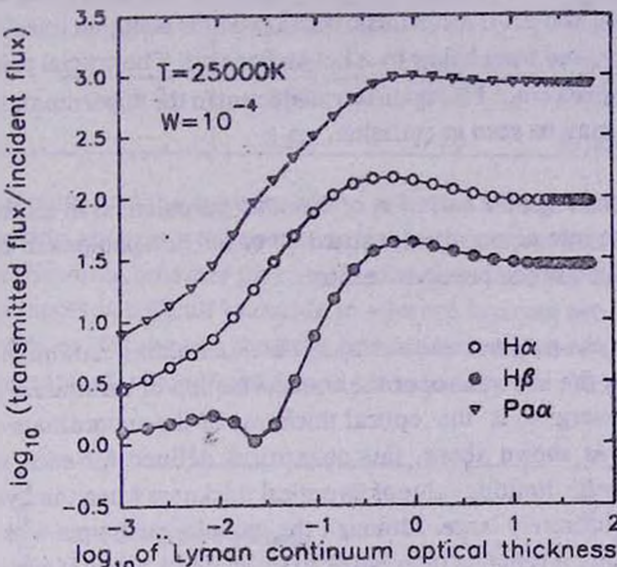

Fig. 11. Transmission of the subordinate lines as a function of the optical thickness of the medium. The destruction of the Ly α photons (and photons from other lines) is taken into account.

The lines show a very interesting behavior under such circumstances. Broadly speaking the Lyman lines are first amplified by recombination until they are destroyed in the subordinate continua (Fig. 10). In addition the upper Lyman lines Lyβ and Lyγ suffer a splitting into Lyα photons and accompanying photons from the Balmer and/or Paschen series. This effect is evidenced by the fact that the factor of transmission of Lyβ and Lyγ remains always smaller than the Lyα factor by nearly two orders of magnitude.

The lines of the subordinate series behave very differently (Fig. 11). As a broad rule their intensities increase by recombination with the optical thickness of the medium until all the available energy of the Lyman continuum has been transformed into quanta of lower energy. As was the case above for the higher members of the Lyman series, $H\beta$ suffers a splitting into quanta of lower energies (namely $Pa\alpha$ and $H\alpha$) and this may explain why its intensity undergoes a momentary decline. The crucial new point with non-conservative scattering is that, once formed, the subordinate lines are able to escape from the medium without being absorbed any more and may appear as strong emission features.

More realistic transfer models will undoubtedly change the "numbers" but it is difficult to imagine that they could suppress the recombination mechanism and the subsequent line escape. We therefore confirm Sobolev's view according to which the present nebular-like non-LTE mechanism could produce emission lines in extended atmospheres illuminated from below by a hot stellar core. The crucial point is that in our model the unionized outer HI region is transparent in the subordinate lines, which explains why they may be seen in emission.

5. *Effects resulting from the variation of the other parameters*. In all the following calculations we take into account the destruction of the line photons in the adjacent continua, as described in the previous section.

5.1. *Influence of the quality and quantity of the illuminating radiation.* One of the factors determining the importance of the transformation of continuum quanta into quanta of lower energies is the optical thickness of the subordinate continua in the envelope. As was shown above, this quantity is defined for each subordinate continuum as the *finite* limiting value of its optical thickness when the Lyman optical thickness becomes infinitely large. Although the quantity computed with our model may prove unrealistic, it remains interesting to examine its trend of variation under the influence of the physical parameters.

Table 1 shows how the optical thicknesses of the Balmer and Paschen continua, designated respectively by τ (BaC) and τ (PaC), depend on the temperature T of the illuminating black-body spectrum and the dilution parameter W. For the lowest values of T ($\approx$25 000 K) and W ($\approx 10^{-4}$) the subordinate continua are marginally thick. For stronger excitation conditions the optical thicknesses reach values of order 10—15.

As they become optically thick, the subordinate continua undergo a diminution of intensity. Figure 12 gives an example of such a behavior for the Balmer continuum, which may be reduced by a factor 10 whereas it was practically unattenuated under conditions of lower excitation (compare to Fig. 9). The situation is similar for the lines, whose transmission factor decreases when both the degree of excitation and the optical thickness increase. Figure 13 refers for instance to the case of $H\alpha$.

Table 1

OPTICAL THICKNESSES τ (BAC) AND τ (PAC) OF THE BALMER AND PASCHEN CONTINUA IN OUR IDEALISTIC MODEL

	τ (BAC) / τ (PAC)	Temperature 25 000	30 000	40 000	50 000
Dilution parameter	10^{-4}	0.50	0.80	1.70	2.80
		0.30	0.70	1.50	2.60
	10^{-3}	1.40	2.40	4.80	7.40
		1.40	2.60	5.40	8.10
	10^{-2}	2.90	4.60	8.10	11.30
		3.60	6.20	11.80	16.70

Both the quality and the quantity of the radiation change with the temperature. Broadly speaking, we expect the excitation/ionization rate to increase along with the temperature. However, whether the outgoing fluxes in the lines increase or not under such circumstances is difficult to decide in advance because two factors compete in opposite directions. On the one hand the intensification of the energy input tends to increase the rate of recombination and consequently the rate of line production.

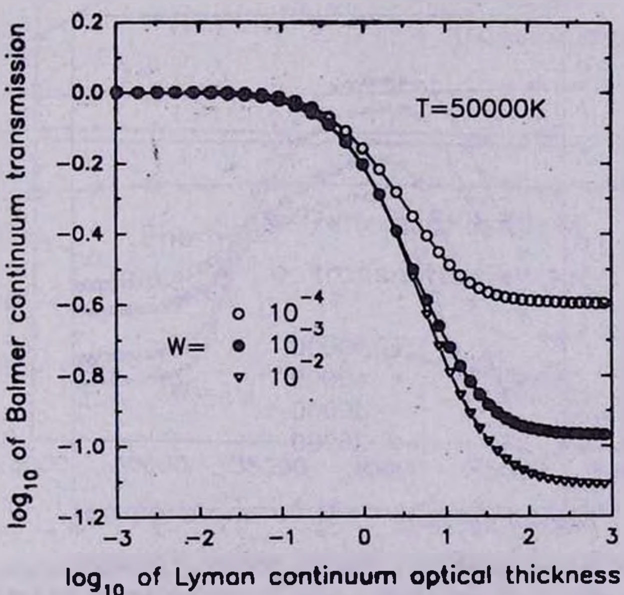

Fig. 12. The absorption of the Balmer continuum is more pronounced when the degree of excitation is higher.

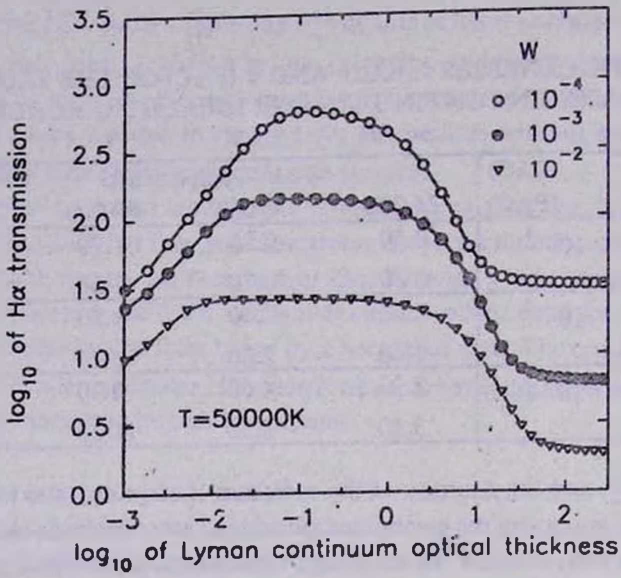

Fig. 13. The transmission in the lines is inhibited by the growth of the optical thickness resulting from an intensification of excitation.

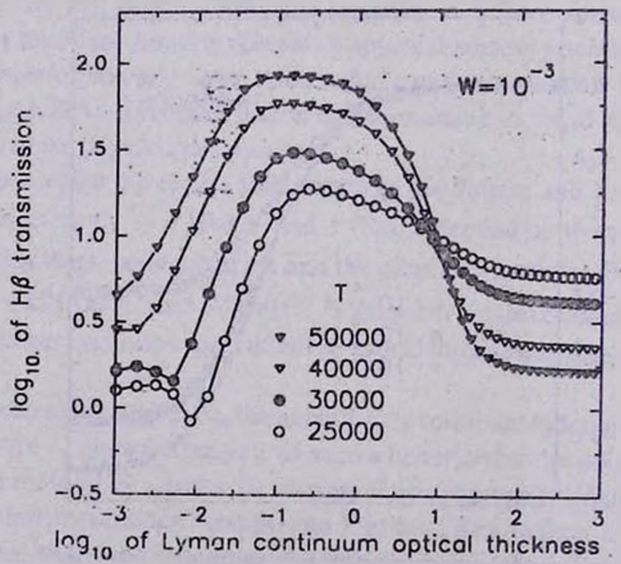

Fig. 14. Response of the $H\beta$ emission to changes in the temperature of the stellar core.

On the other hand the increasing excitation rate causes a rise in the line optical thicknesses, which tends to diminish the transmission coefficient and to block the passage way outwards. Figure 16 is illuminating in this respect. It shows the transmission factors in $H\beta$ as a function of the optical thickness of the hydrogen layer for various temperatures of the ionizing core. When the temperature rises the rate of emission at moderate optical thickness increases correspondingly. But for larger optical thicknesses the opacity effect overcomes this amplification factor so that the transmission coefficient drops. Eventually the transmission factor of $H\beta$ for infinite Lyman optical thicknesses proves to be a *decreasing* function of the temperature of the source. The resulting emitted intensity is found to be smaller for higher temperatures: from 25 000 K to 50 000 K the incident flux is multiplied by ~ 2 whereas the transmission factor is divided by ~ 4, so that the transmitted flux is finally divided by ~ 2.

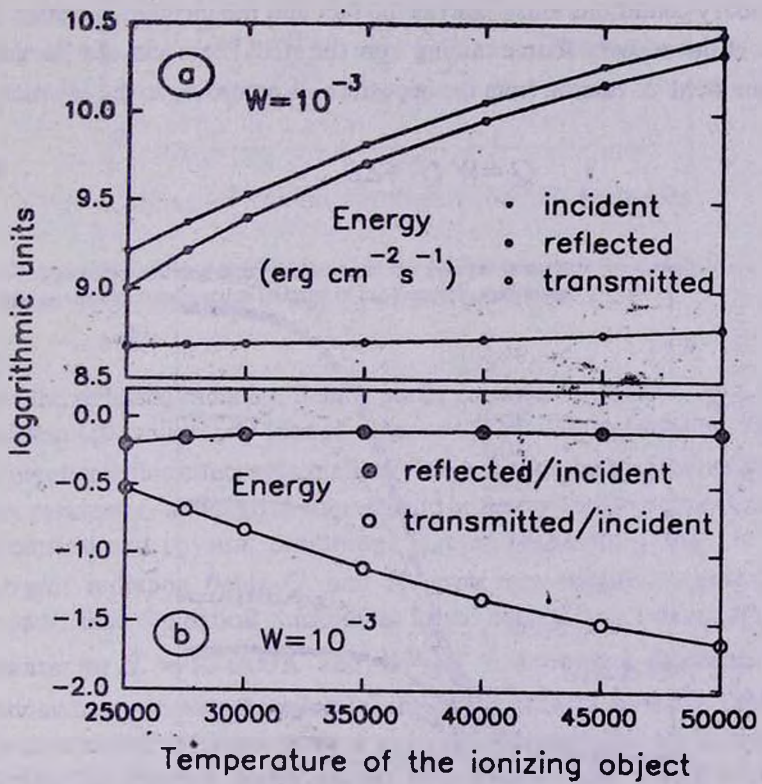

Fig. 15. Total energy transmitted or reflected. In response to a rise of the radiative flux falling onto the layer, the opacity increases and so does the amount of energy trapped below. The energies reported on the figure correspond to the emission of 1 cm^2 of the outer surface.

The same phenomenon affects the total flux, as shown by Fig. 15, which exhibits the variation of the flux emerging from the layer as a function of the incident flux. When the temperature of the ionizing core increases from 25 000 K to 50 000 K, the transmitted net flux remains almost constant, although the incident flux becomes nearly 20 times higher. This means that the opacity has also increased and that the amount of radiative energy blocked inside has grown in proportion. So it is interesting to notice that an *increase* of the luminosity from the stellar core results in *constricting* the outward flow of radiative energy. However this result should be reexamined with more realistic assumptions about the frequency redistribution in the lines.

5.2. The role of the boundary conditions. If the hydrogen layer under study is intended to model a spherical envelope located at a certain distance from a star, the radiation S leaving the internal boundary of the layer may cross the void between the star and the envelope and fall onto the opposite side. Under such circumstances the internal boundary conditions must express the fact that the incident radiation field Q is the sum of the primary source coming from the stellar core and of a fraction Z of the radiation field S coming from the opposite side according to the equation

$$Q = W\,Q^{*} + ZS\ , \tag{10}$$

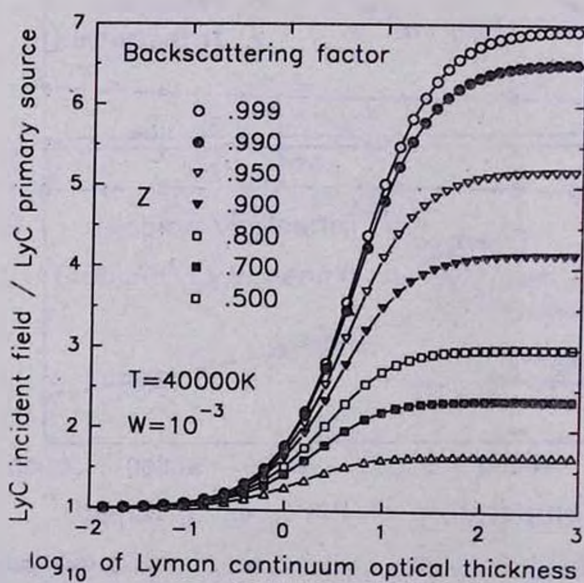

Fig. 16. Amplification of the radiation field falling onto the layer by the geometrical backscattering. The Lyman continuum is taken as an example.

where W is the dilution parameter (see Magnan [7] for the numerical treatment of such a boundary condition in the present context).

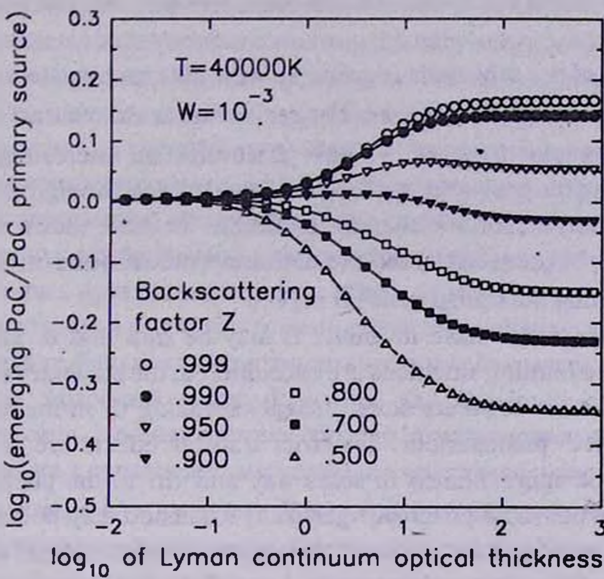

Fig. 17. The backscattering of the radiation at the internal boundary causes an increase of the emerging intensity as shown for instance in the case of the Paschen continuum.

As the reflected radiation field is partly re-injected into the layer, the boundary condition (10) will clearly tend to enhance the excitation conditions in the medium. To investigate this effect systematically we just take the backscattering factor Z as a free parameter - although its value should be determined in each case by the relevant geometrical and physical conditions. Figures 16-17 reveal that the incident and emergent radiation fields Q and R grow very roughly in proportion to Z. At the same time the optical thicknesses in the subordinate transitions increase. For instance, for $T = 40\,000$K and $W=10^{-3}$, the optical thickness in the Balmer continuum now reaches the value 7.7 instead of 4.8 as reported in Table 1. Although in real extended envelopes there is no "real" void between the central star and the envelope the internal backscattering effect should play a role in amplifying the radiative energy falling onto the medium. We conclude that the spherical geometry is an important factor of the problem.

6. *Conclusions and perspectives.* In the present paper we have considered a generalization of the Schuster monochromatic problem to the multifrequency case in investigating the radiative scattering mechanisms inside a medium composed of multilevel atoms and illuminated on one side by a specified radiation field. The main rule for building the model was to link in a self-consistent way the population and the transfer effects.

In the framework of our schematic exercice we were able to compute the structure of the entire medium from the ionized HII region up to the neutral HI region, including the HI^{*} transition region. We have discovered an interesting feed-back mechanism regulating the populations of the subordinate levels through the combined effects of Lyman absorption and excitation conditions. We have shown that intense emission feature may be produced by recombination mechanisms and can escape from the medium by crossing the transparent HI region.

Which applications do we have in mind? It may be said that in astrophysical transfer problems two limiting situations are encountered: the planetary nebulae and the photospheres of main-sequence stars. Broadly speaking (i) in the nebular case non-LTE effects are predominant but strict transfer effects are of secondary importance or can be approximated in some way and (ii) in the photospheres of main-sequence stars non-LTE effects are generally small and may be considered as *departures* from a prevailing state of local thermodynamic equilibrium (though such departures may prove large in certain circumstances). However some objects, such as the extended atmospheres of certain stars, are not amenable to those limiting cases and require the treatment of extreme non-LTE effects in very opaque media. Actually the density of such envelopes is too low to insure LTE populations of the atomic levels and the medium is too thick to allow the simplified treatment of gaseous nebulae. For such cases the model presented here could prove useful.

The main limitation of our model for practical applications is the assumption of rectangular profiles in the lines. But most of the effects we have found are so prominent and so physically consistent that they could hardly disappear when turning to realistic calculations. We thus feel confident that such effects are present in the real extended envelopes of stars: calculations now under development with Voigt broadening in the lines do display the same basic mechanisms even if the spatial scales and the details of the phenomena differ.

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ПРОСТАЯ ЗАДАЧА ПЕРЕНОСА ИЗЛУЧЕНИЯ МНОГОУРОВНЕВЫМИ АТОМАМИ

С.МАНЬЯН, П.ДЕ ЛАВЕРНИ

Рассматривается задача определения полей излучения, отраженного и пропущенного слоем, содержащим многоуровневые атомы водорода и освещенным с одной стороны заданным полем излучения. Мы имеем дело с экстремальной не–ЛТР ситуацией, в которой населенности различных уровней определяются процессом переноса излучения. Эффекты населенности и переноса учитываются путем самосогласования, решая уравнения переноса во всех линиях и континуумах совместно с уравнениями статистического равновесия для всех уровней. Мы ограничились идеальным случаем, допуская прямоугольные профили в линиях и континуумах и принимая модель атома с четырьмя уровнями и континуумом. В условиях, близких к термодинамическому равновесию, мы эмпирически вывели закон, подобный закону Шустера, для континуумов с проходящими полями излучения, изменяющимися обратно пропорционально оптической толщине. Обращаясь к неравновесным условиям, мы подчеркиваем решающую роль вероятности потери $Ly\alpha$ фотонов. Благодаря быстрому убыванию степени возбуждения и ионизации в среде и противоположно консервативному случаю, оптические толщины субординатных переходов остаются теперь конечными даже тогда, когда населенность основного уровня вдоль луча зрения становится бесконечной. В результате этой относительной прозрачности, сильные эмиссионные линии, образованные рекомбинационными механизмами, могут выйти из среды. Хотя из–за ряда принятых упрощений поставленная задача является в большой степени академической, мы предлагаем некоторые возможные применения и развития.

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