АСТРОФИЗИКА

TOM 64

ФЕВРАЛЬ, 2021

ВЫПУСК 1

ORBITS AND INDIVIDUAL MASSES OF SOME VISUAL BINARIES

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The orbits of visual binary systems still attract the interest of many working groups in astronomy. These orbits are the most important and reliable sources of stellar masses. In the present paper, we computed the orbits and dynamical masses of some visual binaries using an independent code. We used the Kowalsky method to compute the geometrical elements, while the dynamical elements (the orbital period and the time of periastron passage) were computed by implementing the double areal constant. We used the developed code to calculate the orbits for four visual binaries: WDS J02262+3428, WDS J14310-0548, WDS J17466-0354, and WDS J12422+2622. We introduce a new orbit for the neglected visual binary WDS J17466-0354 and modified the orbits for the remaining three binaries. Using Gaia DR2 parallaxes, we computed the total and individual dynamical masses of the systems. Comparing the adopted masses with those derived from the mass-spectral type relation revealed good agreement.

Keywords: methods: Kowalsky, stars: visual binaries: orbital elements, stellar masses

1. Introduction. The study of visual binaries is one of the most crucial sources of existing knowledge regarding stellar masses. In addition, the utilization of these masses led to the discovery of the mass-luminosity relation, which in turn supports many theories of stellar evolution. On the other hand, the correlation between binary parameters provides important data for theories concerning star formation. Many authors have developed automated orbit determination techniques with the advent of modern computers. For example, in the same manner as the Thiele-Innes-van den Bos method [1-3], a 3.5 observing points method was proposed in which all observations are used simultaneously in the final stage only.

A sufficiently accurate set of orbital parameters is required for a first approximation by many techniques and methods. Eichhorn and Xu [4] developed a method that requires a precise initial orbit, and only in the improvement phase are all the observations used at the same time. Catovic and Olevic [5] suggested a falsified observation approach for the least-squares solution, which should be chosen as an ellipse; as a result, many elliptical orbits can be created by changing the position, and the final choice of the best orbit is left to the computer. Pourbaix [6] and Pourbaix and Lampens [7] developed a function-based method to quantify the distance from the observed position to the calculated position; the simulated

annealing method has been used to minimize this distance successfully, thus minimizing the function of the best orbit. Nouh et al. [8] introduced an algorithm implementing an optimum point (ρ_a, θ_a) that minimizes the average length of a particular function between the smallest and least-squares solutions.

The aim of the present paper is two-fold. The first is to introduce computational algorithms for the determination of visual orbits using the Kowalsky method. The second is to implement the proposed algorithm to determine the orbital elements and individual masses of selected visual binary systems. To confirm the calculation efficiency, we predict the ephemerides of the systems and compare the present orbits and masses with those in the sixth orbit catalog of visual binary stars.

The structure of the remainder of this paper is as follows. Section 2 is devoted to the computational method used to calculate the orbits. In section 3, we compute the orbital elements of the visual systems under study. Section 4 describes the computational algorithm. Section 5 presents the results. In section 6, we outline the conclusion.

2. Computational method. The orbital elements are extracted from observations of celestial motions. To characterize the movements of the components of visual binaries in their orbits, seven quantities are required: the orbital period P (usually expressed in years for visual binaries); the inclination i of the orbital plane measured with respect to the tangent plane of the celestial sphere centered on the star; the position angle Ω (measured clockwise from north) of the line of nodes joining the points of intersection between the orbital and tangent planes; the longitude of the periastron ω ; the angle between the direction pointing to the ascending node (at which the star crosses the tangent plane while receding from the observer) and the direction oriented toward the point of closest approach of the two stars (periastron) (this angle is measured in the orbital plane in the direction of orbital motion); the length of the semimajor axis of orbit a, usually expressed in kilometers or astronomical units; the eccentricity of orbit e (a dimensionless number between zero and unity); and the time of periastron passage T (the time at which the two stars pass through the periastron).

The Kowalsky method was described first in [9], after which Smart [10] offered formulae and an elegant proof. An analytical formulation and a computational algorithm for the Kowalsky scheme are presented here briefly. Given the apparent separation ρ in arcseconds and the position angle θ in degrees, the equation of the apparent ellipse is given by

$$Ax^{2} + 2Hxy + By^{2} + 2Gx + 2Fy + 1 = 0,$$
 (1)

where $x = \rho \cos\theta$ and $y = \rho \sin\theta$.

The constants A, B, ..., F of Equation (1) for the apparent orbit are first

derived from the least-squares method. After some manipulation, the following equations are obtained:

$$F^{2} - G^{2} + A - B = \frac{\cos 2\Omega \tan^{2} i}{p^{2}},$$
(2)

where

$$p = a\left(1 - e^2\right),\tag{3}$$

$$FG-H = -\frac{\sin 2\Omega \tan^2 i}{2 p^2}.$$
 (4)

Combining Equations (2) and (4) gives

$$(F^{2}-G^{2}+A-B)\sin 2\Omega + 2(FG-H)\cos 2\Omega = 0,$$
(5)

from which Ω can be determined.

Using the value of Ω found above, the value of $\tan^2 i/p^2$ is found by either Equation (2) or Equation (4). Additionally, we have

$$F^{2} + G^{2} - (A + B) = \frac{2}{p^{2}} + \frac{\tan^{2} i}{p^{2}}.$$
 (6)

However, $\tan^2 i/p^2$ has already been determined; hence, the value of p^2 can be determined from Equation (6). When p has been found, the value of $\tan^2 i$ (and hence, the inclination) can be calculated. Finally, we can compute ω from

$$\tan \omega = \frac{(F \cos \Omega - \sin \Omega G) \cos i}{F \sin \Omega + G \cos \Omega}.$$
 (7)

Using the above equations, we can determine the geometrical elements $(a, e, i, \Omega, \omega)$; the details of the computational steps will be described in section 2.2. The remaining two of the seven orbital elements, namely, the orbital period and the time of periastron passage, are computed as follows. The double of the areal constant *C* computed for the number *N* of observed positions (ρ, θ) takes the form

$$C = \frac{1}{N-1} \sum_{j=1}^{N-1} S_j , \qquad (8)$$

where

 $S_j = (x_j \Delta y_j - y_j \Delta x_j) / \Delta t_{j+1}.$

The period P is then given by

$$P = \frac{2\pi a^2 \sqrt{1 - e^2}}{C} \cos i \,. \tag{9}$$

The mean motion μ and eccentric anomaly can be written as

$$\mu = \frac{2\pi}{P},\tag{10}$$

and

$$E = 2\tan^{-1} \left[\sqrt{\frac{1-e}{1+e}} \tan(\nu/2) \right].$$
 (11)

The true anomaly v is computed from

$$\tan(\nu + \omega) = \tan(\theta - \Omega)\cos i.$$
(12)

The mean anomaly M is computed from the Kepler equation:

$$M = E - e \sin E \,. \tag{13}$$

Finally, the time of periastron passage T is given by

$$T = (\mu t - M)/\mu. \tag{14}$$

3. *Total and individual masses*. The total and individual dynamical masses can be computed using the well-known formula

$$M_a + M_b = \frac{d^3 a^3}{P^2},$$
 (15)

where d is the distance to the system in parsecs, a is the length of the semimajor axis in arcseconds, and P is the period in years. We determine the mass ratio M_b/M_a from the relation [11]

$$\frac{M_b}{M_a} = 10^{-0.1157(m_b - m_a)},\tag{16}$$

where m_a and m_b are the apparent magnitudes of the primary and secondary components, respectively. Consequently, the individual masses are derived by solving Equations (15) and (16). The errors in the masses can be obtained from the equation

$$\frac{\sigma_M}{M} = \sqrt{9\left(\frac{\sigma_\pi}{\pi}\right) + 9\left(\frac{\sigma_a}{\pi}\right) + 4\left(\frac{\sigma_P}{P}\right)}.$$
(17)

4. Computational algorithm. To calculate the seven orbital elements $(a, e, i, \Omega, \omega, P, T)$, we traverse the following computational sequence.

- Input: ρ'' and θ^{o}
- 1. Compute the following for i = 1, ..., N: $x_i = \rho_i \cos \theta_i, y_i = \rho_i \sin \theta_i$.

2. Solve Equation (1) by the least-squares method, which yields A, B, F, G, H.

- 3. Compute the quantities X_1, X_2, X_3 from $X_1 = -2(FG H)$,
- $X_2 = F^2 G_2 + A B$, $X_3 = F^2 + G^2 (A + B)$.
- 4. Compute Ω (the angle of the ascending node) from $\Omega = (1/2) \tan^{-1} [X_1/X_2]$.
- 5. Compute Y_1 and Y_2 from $Y_1 = X_2 / \cos 2\Omega$, $Y_2 = 2/(X_3 Y_1)$.

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6. Compute the semiparameter p from $p = \sqrt{Y_2}$.

7. Compute *i* (the inclination) from $i = \tan^{-1} \left[p \sqrt{Y_1} \right]$.

8. Compute ϕ_1 and ϕ_2 from $\phi_1 = [F \cos\Omega - G \sin\Omega] \cos i$, $\phi_2 = [F \sin\Omega + G \cos\Omega]$. 9. Compute ω (the angle of the descending node) from $\omega = \tan^{-1}[\phi_1/\phi_2]$.

10. Compute Y_{3} from $Y_{3} = \phi_{1}^{2} + \phi_{2}^{2}$.

- 11. Compute *e* (the eccentricity) from $e = \sqrt{Y_3}$.
- 12. Compute *a* (the length of the semimajor axis) from $a = p/(1-Y_3^2)$.
- 13. For all i = 1(1)N, compute the true anomalies v_i from

$$v_{i} = \tan^{-1} \left[\frac{\tan(\theta_{j} - \Omega)}{\cos i} \right] - \omega; \quad j = 1(1)N.$$
14. For all $i = 1(1)N$, compute the eccentric anomalies E_{i} from
$$E_{i} = 2\tan^{-1} \left[\sqrt{\frac{1-e}{1+e}} \tan \frac{v_{i}}{2} \right].$$
15. For all $i = 1(1)N$, compute the mean anomalies M from $M = 1$

15. For all i = 1(1)N, compute the mean anomalies M_i from $M_i = E_i - e \sin E_i$. 16. Compute the double areal constant from

$$C = \frac{1}{N-1} \sum_{j=1}^{N-1} S_j , \quad S_j = (x_j \Delta y_j - y_j \Delta x_j) / \Delta t_{j+1} .$$

17. Compute the period P from $P = \frac{2\pi a^2 \sqrt{1-e^2}}{C} \cos i$.

18. Compute the time of periastron passage T from $T = (\mu t - M)/\mu$,

$$t = \frac{1}{N-1} \sum_{i=1}^{N} t_i , \quad M = \frac{1}{N-1} \sum_{i=1}^{N} M_i , \quad T = \frac{1}{N-1} \sum_{i=1}^{N} T_i , \quad \mu = \frac{2\pi}{P}.$$

To determine the true anomaly v, we employ the following sequence. First, we determine the kind of motion: if the motion is retrograde, $R1 = v + \omega$; otherwise, if the motion is direct, $R1 = 2\pi - v - \omega$. Then, we determine the quadrant of R1. Second, we set $R = \theta - \omega$ and determine the quadrant of R. Third, we compare R and R1 as follows: if the quadrant of R1 is greater than the quadrant of R, then v must be reduced to $v - \pi/2$; if the quadrant of R is greater than the quadrant of R1, then v becomes $v = v + \pi/2$.

5. Results. Based on the Kowalsky method described in section 2, we developed an independent FORTRAN code utilizing the computational algorithm described in section 4 to calculate the orbits of the four visual binaries whose information is listed in Table 1. The apparent magnitudes m_a and m_b in columns 2 and 3, respectively, are derived from the Washington Double Star Catalog [12], the distances d in parsecs listed in column 4 are derived from Gaia DR2 [13] except for the system WDS J17466-0354, and the spectral types in column 5 are obtained from [12]. Columns 6, 7, and 8 represent the masses, effective temperatures, and absolute magnitudes, respectively, derived from the mass-spectral type relation presented by [14]. The epochs of the observations t, the position angles

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WDS name	<i>m</i> _a	m _b	<i>d</i> (pc)	Sp. type	Masses	$T_{e\!f\!f}$	$M_{_V}$
02262+3428	8.70	9.14	44.41 ± 0.21	G8+G9	0.99, 0.95	5559, 5450	4.92, 5.25
14310-0548	8.81	8.39	41061 ± 0.43	G5+G5	1.031, 1.031	5741, 5741	4.64, 4.64
17466-0354	9.34	10.22		F8	1.222, 1.222	6152, 6152	3.8, 3.8
12422+2622	10.09	10.8	41.0846	K4V	0.8, 0.8	4400, 4400	7,7

OBSERVATIONAL PARAMETERS OF THE FOUR VISUAL BINARIES

 θ^{o} , and the angular separations ρ'' are retrieved from the Fourth Catalog of Interferometric Measurements of Binary Stars (astro.gsu.edu/wds/int4.html). The updated list for each system is formed using additional observed positions collected from the literature and added to the list retrieved from the Fourth Catalog of Interferometric Measurements of Binary Stars.

To determine the errors accompanying the orbital elements, we use the trial and error technique, while the standard deviation is calculated in the usual way. The best orbit is chosen according to the following criteria. The dynamical masses for the computed orbits are determined by employing the parallax of Gaia DR2; therefore, these masses are compared to the masses derived from the mass-spectral type relation. Moreover, to compute the ephemerides from the calculated orbits and determine the orbit with the smallest $\Delta\theta$ and $\Delta\rho$, we used the algorithm developed by [15-16]. In the following subsection, we present the results for the four selected visual binaries.

5.1. *WDS J02262+3428*. The visual binary WDS J02262+3428 (HD 15013, HIP 11352, GR2 326940164774368384) is a late-type star (G8+G9) with apparent magnitudes $m_a = 8.74$ and $m_b = 9.14$. The Gaia DR2 parallax $\pi = 22.515$

Table 2

Element	Present work	Sixth orbit catalog
a "	0.101 ± 0.003	0.099
е	0.27 ± 0.02	0.291
i°	51.47 ± 0.50	49.9
Ω^{o}	13.80 ± 1.5	16.0
ω°	4.13 ± 0.22	2.8
P (years)	6.63 ± 0.30	6.937
Т	1998.98 ± 0.20	2015.82
$M_{t}(M_{\odot})$	2.05 ± 0.14	1.87
$M_a(M_{\odot})$	1.08 ± 0.07	0.99
M_{b} (M_{\odot})	0.96 ± 0.06	0.88

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mas, and the spectral type of both stars is G5V. The orbit in the sixth orbit catalog (http://www.astro.gsu.edu/wds/orb6.html) is calculated by [17]. Table 2 lists the two sets of orbital elements (our orbit and the orbit calculated by [17]) and the dynamical masses calculated using Equations (15)-(17). The difference between our time of periastron passage and that computed by [17] is remarkable. By investigating the observed angular separation, one can easily obtain that the minimum value of the separation is approximately $\rho = 0.057$ arcseconds, and this separation occurred at epoch 2000, which ensures the validity of our result. The total and individual masses agree well (within error) with the masses derived from the empirical relation by [14] listed in Table 1. In addition to the observed positions



Fig.1. The orbit of the visual binary WDS J02262+3428.

plotted in Fig.1, we illustrate the positions computed from the present orbit and those computed from the orbit calculated by [17].

3.2. WDS J14310-0548. The two components of the system WDS J14310-0548 (HIP 70973, HD 127352, IDS 14258-0522, Gaia DR2 3641365877340584064) are late-type stars (G5V) with apparent magnitudes of $m_a = 8.81$ and $m_b = 8.39$, and the Gaia DR2 parallax $\pi = 24.354$ mas. The orbit in the sixth orbit catalog computed by [18] and the present orbit are described in Table 3. The total and individual masses computed from the present orbit using the distance derived from the Gaia DR2 catalog (d = 41.061 pc) are in good agreement (the mass of the primary is slightly overestimated) with the masses derived from the mass-spectral

Elements	Present work	Sixth orbit catalog
<i>a</i> "	0.24 ± 0.01	0.243
е	0.48 ± 0.02	0.499
i°	50.44 ± 0.50	49.1
Ω^{o}	11.62 ± 0.50	13.8
ω°	122.82 ± 0.88	121
P (years)	21.43 ± 0.42	22.98
Т	1996.18 ± 0.60	1993.62
$M_{t}(M_{\odot})$	2.13 ± 0.21	1.881
$M_a(M_{\odot})$	1.12 ± 0.11	0.993602
M_{b} (M_{\odot})	1.01 ± 0.11	0.884202

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type relation by [14]: $M_a = M_b = 1.03$. In Fig.2, we plot the positions computed from the present orbit and those computed from the orbit calculated by [18].

3.3. WDS J17466-0354. The spectral type of the system WDS J17466-0354 (ADS 10780, HD 161588, IDS 17413-0352, CCDM J17466-0354) in Simbad is between F3 and F8, and the apparent visual magnitudes are $m_a = 9.34$ and $m_b = 10.22$. There is no parallax for the system in the Hipparcos or Gaia



Fig.2. The orbit of the visual binary WDS J14310-0548.

Table 4

ElementPresent work a'' 0.37 ± 0.03 e 0.23 ± 0.01 i° 67.58 ± 0.72 Ω° 62.56 ± 1.71 ω° 167.82 ± 0.40 P (years) 56.28 ± 0.02 T 1891.73 ± 0.05		
$\begin{array}{cccc} a'' & 0.37 \pm 0.03 \\ e & 0.23 \pm 0.01 \\ i^{\circ} & 67.58 \pm 0.72 \\ \Omega^{\circ} & 62.56 \pm 1.71 \\ \omega^{\circ} & 167.82 \pm 0.40 \\ P \text{ (years)} & 56.28 \pm 0.02 \\ T & 1891.73 \pm 0.05 \end{array}$	Element	Present work
	$a'' \\ e \\ i^{\circ} \\ \Omega^{\circ} \\ \omega^{\circ} \\ P (years) \\ T$	$\begin{array}{c} 0.37 \pm 0.03 \\ 0.23 \pm 0.01 \\ 67.58 \pm 0.72 \\ 62.56 \pm 1.71 \\ 167.82 \pm 0.40 \\ 56.28 \pm 0.02 \\ 1891.73 \pm 0.05 \end{array}$

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DR2 surveys. This neglected system has no orbital elements in the sixth orbit catalog. We list our computed orbital elements in Table 4. The computed and observed positions are plotted in Fig.3 for the sake of comparison.



Fig.3. The orbit of the visual system WDS J17466-0354.

3.4. WDS J12422+2622. The components of the system WDS J12422+2622 (ADS 8635, HD 110465, HIP 61986) have apparent magnitudes of $m_a = 10.09$ and $m_b = 10.8$. The present orbital elements and the orbital elements from the sixth orbit catalog calculated by [19] are listed in Table 5. The total and individual masses computed from the present orbit using the distance supplied by the Gaia DR2 catalog (d = 41.0846 pc) are slightly different from those derived from the

Element	Present work	Sixth orbit catalog
a "	0.41 ± 0.05	0.415
е	0.27 ± 0.01	0.252
i°	31.13 ± 1.52	26
Ω°	104.61 ± 3.42	129.8
ω°	383.22 ± 5.38	319.5
P (years)	63.71 ± 2.3	61.3
Т	1958.09 ± 3.21	1959.3
$M_t (M_{\odot})$	1.17 ± 0.42	1.332
M_a (M_{\odot})	0.64 ± 0.02	0.66
M_b (M_{\odot})	0.53 ± 0.04	0.66

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Fig.4. The orbit of the visual system WDS J12422+2622.

mass-spectral type relation ($M_a = M_b = 0.8$) [14] but are in good agreement with the masses computed using the orbital elements provided by [19].

6. *Conclusion*. In the present work, we introduced a computational algorithm to determine the orbital elements of visual binaries. The geometrical elements are computed using the Kowalsky method, while the orbital period and the time of periastron passage are determined by utilizing the double areal constant.

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We derived the orbital elements, total masses, and individual masses for four visual binaries, namely, WDS J02262+3428, WDS J14310-0548, WDS J17466-0354, and WDS J12422+2622. Comparisons between our orbits with those listed in the sixth orbit catalog present good agreement. A comparison between the total dynamical masses derived from the present orbits and those computed from the empirical mass-spectral type relation also show good agreement. Finally, we introduced a new orbit for the neglected visual binary WDS J17466-0354.

Acknowledgments. The authors gratefully acknowledge the approval and the support of this research study by grant number SCI/2019/1/10/F/8282 from the Deanship of Scientific Research at Northern Border University, Arar, Saudi Arabia.

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ОРБИТЫ И ИНДИВИДУАЛЬНЫЕ МАССЫ НЕКОТОРЫХ ВИЗУАЛЬНО-ДВОЙНЫХ ЗВЕЗД

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Орбиты визуально-двойных систем по-прежнему вызывают интерес у многих рабочих групп по астрономии. Эти орбиты являются наиболее важными и надежными источниками звездных масс. В данной статье вычислены орбиты и динамические массы некоторых визуально-двойных звезд с помощью независимого кода. Для вычисления геометрических элементов использован метод Ковальского, в то время как динамические элементы (период обращения и время прохождения периастра) определены с использованием постоянной двойной площади. Разработанный код использован для расчета орбит для четырех визуальных двойных систем: WDS J02262+3428, WDS J14310-0548, WDS J17466-0354 и WDS J12422+2622. Получена новая орбита визуально-двойной системы WDS J17466-0354 и модифицированы орбиты для остальных трех двойных систем. Используя параллаксы Gaia DR2, вычислены полные и индивидуальные динамические массы систем. Сравнение принятых масс с

массами, полученными из соотношения масс-спектральный тип, показало хорошее согласие.

Ключевые слова: методы: Ковальский, звезды: визуально-двойные системы: элементы орбиты: звездные массы

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