АСТРОФИЗИКА

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ACCRETION DISKS IN THE BIMETRIC THEORY OF GRAVITATION: THE STRONG GRAVITY OF THE INNER DISK EDGE

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Optically thin bremsstrahlung accretion disks, calculated in the bimetric theory of gravitation, are compared with the .corresponding ones in Einstein's theory. The sensibility of the inner disk edge located in the strong gravity region of the sufficiently compact central object suggests arguments for the confirmation that the accretion disk as a whole could be determined by strong gravity effects.

Introduction. Accretion disks, widely accepted to drive a great variety of galactic compact sources [1—4]. are very sensitive with respect to the inner boundary, behaviour and conditions, (already demonstrated in the basic works on accretion disk theory by Von Weizsaecker [5] and Lust [6]). These conditions, determining energetic efficiency and the structure of the whole disk, are located in the strong gravity region of the highly compact central object. Therefore, there seems to be a hope to find strong gravity affected processes which dominate in an astronomical phenomena as a whole [7, 8]. Unfortunately, in the relativistic standard disk models [9—11] an inner boundary value problem obstructs the clean physical understanding of the inner disk edge. This problem was attacked by Stoeger in [12, 13] neglecting pure hydrodynamical effects but illuminating possibilities for a solution in the framework of thin accretion disks. Another approach [14—17] is yielded from the thick accretion disk theory [18, 20]. A type of slender tourus approximation can be used for the inner edge region of the standard disk; interesting aspects of the thick disk theory arise from the instability investigations [21—23].

In recent years an innermost abnormal temperature profile was found by the refinement of the relativistic standard model [23, 24]. The temperature behaviour nearby the marginally stable test particle orbit r_{ms}, offers a possibility to solve the inner boundary value problem and to obtain some essentials of both approaches, previously mentioned 8]. Models using the condition of nondissipation in the supersonic stream region with the no angular momentum transport are the only consistent ones in the inner disk region up to now. But on the other hand, if the highly compact central body's radius R is less than the radius of the marginally bound test particle's orbit r_{ms}, or the radius of the marginally stable particle orbit a neccessary condition for the existence of this region appears in the refinement of the standard model [24, 7, 8].

The behaviour of this innermost cooling region visualizes the sensibility of the essentials of the whole disk from the strong gravitational field region [7]. Therefore the purpose of this paper is to give an example for this sensibility by the constriction of an optically thin bremsstrahlung disk [25, 26] in an alternative theory of gravitation with the following comparison of the inner edge with the corresponding one in Einstein's theory. The bimetric theory of gravitation [27, 28] is used here. For this theory the external static and stationary solutions are known [27—30]. Also, compact massive objects were discussed in the limits of this theory [31, 32]. The "weak case" of the bimetric gravitation theory (both masses in the line element (1) are equal, $\beta = M/M' = 1$) suggests strong gravity sensibility of accretion disks which is demonstrated by the existence of more massive, compact and softer radiating "bimetric disks" here than the corresponding ones in Einstein's theory. These different appearances of "bimetric" and "Einstein" disks must be strengthened, first, in the "strong" bimetric case $\beta = M/M' > 1$. and second in the case of the rotating central body. The first suggestion is proved in this paper.

2. *The equations of structure.* A thin accretion disk (thickness $h \ll$ radii R) without substantial self-gravitation orbits a compact, nonrotating object near and in the equational plane ($|\theta - \pi/2| \ll 1$). The external static spherical symmetric space-time of the central body is described by Line elements [27, 28, 34].

$$\begin{aligned} d\sigma^2 &= -dt^2 + dR^2 + R^2 d\varphi^2 + dz^2, \\ ds^2 &= -A^2(R)\,dt^2 + B^2(R)\,dR^2 + R^2 d\varphi^2 + dz^2, \end{aligned} \tag{1}$$

with $A^2 = e^{-M/r}$, $B^2 = \left(1 - \dfrac{M'}{r}\right)^{-2}$ and the isotropic radius $r = Re^{-M'/r}$.

We follow closely the general methods developed for relativistic, thin accretion disks [9] and use the theoretical framework to calculate optically thin bremsstrahlung disks in general spherical symmetric space — time. The disk matter is described by the energy momentum tensor

$$T_{\mu\nu} = \rho_0 (1 + \pi) u_\mu u_\nu + t_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu \qquad (2)$$

with

$$q_\mu u^\mu = 0, \quad t_{\mu\nu} u^\nu = 0, \quad t^\mu_\mu = 0.$$

and ρ_0 rest mass density, π internal specific energy, $t_{\mu\nu}$ pressure tensor and q_μ heat flow.

First of all, the symmetric tensor $T_{\mu\nu}$ can be decomposed unarbitrarily with respect to a timelike unit vector in (2) plus a pressure term $P \cdot h_{\mu\nu} = P(g_{\mu\nu} + u_\mu u_\nu)$ [35, 36] ($h_{\mu\nu}$ is the projection tensor). This pressure term falls out during the integration of the equation of motion in the axial symmetric stationary case and with the matter assumptions considered here.

Second, in the standard theory of accretion disks the internal energy in $T_{\mu\nu}$ is neglected which is questionable at low temperatures. Temperature gradiants which follows from π in the equations of motion changes the type of differential equations qualitatively. Therefore, this neglection is not carried out here.

The disk matter orbiting the central body on nearly axial-symmetric paths with the velocity of orbit v_φ in the effective field

$$V^2(\tilde{L}^2, R) = A^2\left(1 + \frac{\tilde{L}^2}{R^2}\right), \qquad (3)$$

possesses specific angular momentum $\tilde{L} = \dfrac{R^2 \Omega}{\tilde{E}} \gamma$, specific energy $\tilde{E} = A\gamma$, and angular velocity $\Omega^2 = \dfrac{1}{2R}\dfrac{dA^2}{dR}\left(\gamma^2 = (1 - v_\varphi^2)^{-1} = \dfrac{A^2}{A^2 - \dfrac{R}{2}\dfrac{dA^2}{dR}}\right)$.

Using the disk structure equations for a completely ionized hydrogen plasma with viscosity [7, 8] and bremsstrahlung cooling [26], in a general spherical symmetric space time and substituting the strong gravity case of bimetric theory we obtain (in specified CGSE units) for the vertical energy flux

$$q^z(r) = 3.1807 \cdot 10^{46} \frac{\gamma^\omega \cdot t^2 \dot{M}_{17}^2}{A^4 R_\cdot^4 T^2 M_\cdot^3} \cdot \beta^{9/2}, \qquad (4)$$

for thickness

$$h(r) = 0.1899 \frac{T^{1/2}}{\gamma^\omega} \cdot \frac{M_\cdot}{\beta^{3/2}}, \qquad (5)$$

for rest mass density and surface density

$$\rho_0(r) = 4.1980 \cdot 10^{-11} \left[\frac{q^*}{hT^{1/2}} \right]^{1/2}, \quad \Sigma(r) = 2\rho_0 h, \tag{6}$$

for pressure and radial accretion flow velocity in orbiting reference

$$P(r) = 1.6508 \cdot 10^8 \cdot \rho_0 \cdot T,$$
$$\bar{v}^r = 1.7964 \cdot 10^{10} \frac{\dot{M}_{17} \cdot \beta}{M.R.A\rho_0 h}, \tag{7}$$

and for temperature profile

$$\frac{dT}{dr.} = 3.6294 \cdot 10^{12} \frac{\gamma^2 R.\beta^{1/2} i(r.)}{r.A^2 B} \left[\beta^{1/2}\omega_{,R} + 8.7269 \cdot 10^{20} \frac{\dot{M}_{17}\beta^2 \varpi Bi(r.)}{M.\alpha^2 A\gamma R.^3 T^2} \right]. \tag{8}$$

We have expressed the accretion rate $\dot{M}_0$, masses and the radii r, R in units typical for galactic X-ray sources;

$$\dot{M}_{17} = \dot{M}_0 / 10^{17} \frac{g}{s} = \dot{M}_0 / 10^{-9} \frac{M_\odot}{yr}, \quad M. = M/3M_\odot = \beta M.',$$
$$r. = r/M', \quad R. = R/M' = r.e^{-1/r.},$$

with this normalization

$$\gamma^2 = \frac{r.-1}{r.-2}, \quad \omega = \frac{M}{\beta^{3/2}} \Omega = \frac{e^{-\frac{1+\beta}{r.}}}{r.(r.-1)^{1/2}}, \quad A^2 = e^{-\frac{2\beta}{r.}},$$
$$B^2 = r.^2/(r.-1)^2, \tag{9}$$

and the integral

$$i = \int_{r_0}^{r.} e^{\frac{1-\beta}{r.}} \left\{ \frac{(r.-1)^{1/2}}{r.} - \frac{r.(r.-2+\beta)}{2(r.-1)^{3/2}(r.-2)} \right\} dr. \tag{10}$$

Which means that the algebraic functions describing the space time structure are only functions of the isotropic radius $r.$ independent on the parameter of structure α, $\dot{M}_0$, M for the accretion disk; the physical quantities from (4) up to (8) depend explicitly on T and $r.$. Integral (10) changes its sign caused by $\tilde{L}_{,R} \gtreqless 0$ for $R. \gtreqless R._{ms}$, $R._{ms}$ is the radius of marginally stable test particle orbit in $\tilde{V}^2$ (3) and can be found from

$$R._{ms} = r._{ms} e^{1/r._{ms}}, \quad r._{ms} = 2 + \beta + \sqrt{2 + 2\beta + \beta^2}$$

(following the conditions $d\tilde{V}^2/dR = d^2\tilde{V}^2/dR^2 = 0$, Fig. 1).

Thus the temperature gradient (8) possesses an innermost positive value leading to a cooling behaviour for decreasing r. ($T_{,r.} > 0$) and the usually negative outer value ($T_{,r.} < 0$). The integration of equation (8) can be started from the radius of marginally bound test particle orbit $R_{.mb} = r_{.mb}\, e^{-\frac{1}{r_{.mb}}}$ being the last possible bound motion of the disk matter $\left(\frac{d\tilde{V}^2}{dR}\right)_{r.\,mb} = 0$ and $(\tilde{V})_{r.\,mb} = 1$ obtained from

$$(r_{.mb} - 1)\cdot e^{-\frac{2\beta}{r_{.mb}}} = r_{.mb} - 1 - \beta.$$

Of course this boundary condition is chosen only with respect to strong gravity effects.

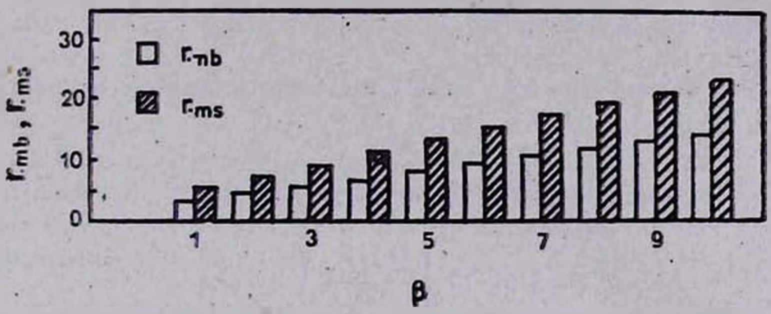

Fig. 1. The dependence of $r_{.ms}$, $r_{.mb}$ on "obsorption" parameter β.

In order to fix the inner edge $r_{.in}$ of the disk the boundary condition

$$M_r(r_s) = \frac{\bar{v}}{a_s} = 1,$$

$$a_s^2 = \frac{2}{m_p}\left[T + \rho_0 \frac{dT}{d\rho}\right], \tag{11}$$

is assumed which separates the hydrodynamically different super — and subsonic regions (a_s is the velocity of sound in the completely ionized hydrogen plasma). Condition (11) is based on the hydrodynamical fact that the turbulent viscosity causing the angular momentum transport breaks down by shock effects in the supersonic region

$$r. \leqslant r._s = r._{in}$$

(dissipation of the turbulent cells in the first approximation).

But this angular momentum transport is needed for disks with gravitational potential energy release (not only for disks with a dominance of this release assumed here).

To find the inner disk edge behaviour we substitute power series

$$T = \Delta r_{.}^{\gamma} (b_1 + b_2 \Delta r_{.} + \cdots), \quad \Delta r_{.} = r_{.} - r_{.mb}, \tag{12}$$

in the equations of structure (4)—(8) and calculate q^z, h, ρ_0... in the neighbourhood of $r_{.mb}$ at $r_{.}$. By means of (11) one obtains $\Delta r_{.}$,

$$\Delta r_{.} = 6.80 \cdot 10^{-2} \left[\frac{\dot{M}_{17} \alpha^4 \beta^{5/2}}{M_{.}} \right]^{1/3} f_{\Delta}(r_{.}) \tag{13}$$

and $r_{.in} = r_{.mb} + \Delta r_{.}$. Where

$$f_{\Delta} = \frac{e^{-\frac{11}{r_{.mb}}} (r_{.mb} - 1)^2 r_{.mb}^{17/6} (r_{.mb} - 2)^{3/4}}{\{2 (r_{.mb} - 1)^2 (r_{.mb} - 2) - r_{.mb}^2 (\beta - 2 + r_{.mb})\}^{4/3}}.$$

Solving the equations of structure at the inner disk boundary we get:

$$q^z_{in} = 9.29 \cdot 10^{23} \left[\frac{\dot{M}_{17} \beta^{10}}{M_{.}^7 \alpha^2} \right]^{1/3} f_q(r_{.}), \tag{14}$$

$$h_{in} = 1.51 \cdot 10^4 \left[\frac{M_{.}^2 \dot{M}_{17} \alpha}{\beta^2} \right]^{1/3} f_h(r_{.}), \tag{15}$$

$$\rho_{0in} = 1.17 \cdot 10^{-3} \left[\frac{\dot{M}_{17} \beta^5}{M_{.}^4 \alpha^2} \right]^{1/3} f_{\rho}(r_{.}), \tag{16}$$

$$P_{in} = 1.22 \cdot 10^{15} \frac{\dot{M}_{17}}{M_{.}^2} \beta^3 f_p(r_{.}), \tag{17}$$

$$\Sigma_{in} = 35.33 \left[\frac{\dot{M}_{17}}{M_{.}^2 \alpha} \right]^{1/3} \beta \cdot f_h(r_{.}) f_{\rho}(r_{.}), \tag{18}$$

$$\bar{v}^r_{in} = 3.40 \cdot 10^{-2} \left[\frac{\dot{M}_{17} \alpha \beta^2}{M_{.}} \right]^{1/3} f_v(r_{.}), \tag{19}$$

and

$$T_{in} = 6.29 \cdot 10^9 \left[\frac{\dot{M}_{17} \alpha \beta^2}{M_{.}} \right]^{2/3} f_T(r_{.}). \tag{20}$$

Using the abbreviation $g = |2 (r_{.mb} - 1)^2 (r_{.mb} - 2) + r_{.mb}^2 (\beta - 2 + r_{.mb})|$, algebraic functions $f(r_{.})$ will be written in the form:

$$f_q = \frac{\exp\left(\frac{7-3\beta}{3r_{\cdot mb}}\right)\cdot g^{2/3}}{r_{\cdot mb}^{3}(r_{\cdot mb}-1)}, \qquad f_P = \frac{\exp\left(\frac{11-7\beta}{12\, r_{\cdot mb}}\right)}{r_{\cdot mb}^{7/6}(r_{\cdot mb}-2)^{1/12}}\, g^{3/4},$$

$$f_h = \frac{\exp\left(\frac{\beta-11}{r_{\cdot mb}}\right)(r_{\cdot mb}-1)^{1/2} r_{\cdot mb}^{7/12}(r_{\cdot mb}-2)^{1/24}}{g^{1/3}},$$

$$f_v = \frac{\exp\left(-\frac{2}{r_{\cdot mb}}\right) r_{\cdot mb}^{5/12}(r_{\cdot mb}-1)^{1/2}}{(r_{\cdot mb}-2)^{1/24}}\, g^{1/6},$$

$$f_\rho = \frac{\exp\left(\frac{37-7\beta}{12 r_{\cdot mb}}\right)}{r_{\cdot mb}^{2}(r_{\cdot mb}-1)}\, g^{5/12}, \qquad f_T = f_v^2.$$

β depends on the equation of state for the matter of the central body. On the other hand M' represents the sum of rest masses of the particles of the central body, while M is the mass weighed by circular orbiting test particles. This means β measures the absorption (positive and negative, respectively) effect of gravitation. In order to visualize the influence of the absorption parameter on the inner disk edge behaviour the quantities (14 up to 20) are normalized on the value without absorption ($\beta = 1$):

$$\begin{aligned} &\Delta\bar{r}_{\cdot}(\beta) = 2.69\,\beta^{5/6} f_\Delta, && \bar{\rho}_{0in}(\beta) = 2.81\,\beta^{5/3} f_\rho, \\ &\bar{h}_{in}(\beta) = 0.48\,\beta^{-2/3} f_h, && \bar{q}^*_{in}(\beta) = 14.16\beta^{10/3} f_q, \\ &\bar{P}_{in}(\beta) = 32.36\,\beta^{3} f_P, && \bar{v}_{in}(\beta) = 3.40\,\beta^{2/3} f_v, \\ &\bar{T}_{in}(\beta) = 11.48\,\beta^{4/3} f_T. \end{aligned} \tag{21}$$

In order to compare the inner boundary behaviour of the bimetric disk with the corresponding one in Einstein's theory we have written down the quantities (14 up to 20) in units of the corresponding values found for the "Einstein disk":

$$\begin{aligned} &\frac{(q^*_{in})^B}{(q^*_{in})^E} = 45.10\cdot\beta^{10/3} f_q, && \frac{(h_{in})^B}{(h_{in})^E} = 2.98\cdot\beta^{-2/3} f_h, \\ &\frac{(\rho_{0in})^B}{(\rho_{0in})^E} = 1.07\cdot\beta^{5/3} f_\rho, && \frac{(P_{in})^B}{(P_{in})^E} = 22.71\cdot\beta^{3} f_P. \end{aligned} \tag{22}$$

$$\frac{(v_{in}^{r})^{B}}{(v_{in}^{r})^{E}} = 11.90\cdot\beta^{2/3} f_{v}, \qquad \frac{(T_{in})^{B}}{(T_{in})^{E}} = 141.59\,\beta^{4/3} f_{T},$$

$$\frac{(\Sigma_{in})^{B}}{(\Sigma_{in})^{E}} = 3.19\,\beta f_{\rho} f_{h}.$$

(B and E stand for functions calculated in bimetric and Einstein theories, respectively). The inner disk edge reflecting quantities (14 up to 22) were calculated by assuming the condition $\Delta r_{*} \ll 1$ or $\Delta r \ll M'$ respectively. This condition is fulfilled for a wide class of astrophysical objects

$$\frac{\dot{M}_{17}\alpha^{4}}{M_{*}} \ll \frac{3\cdot 10^{3}}{f\Delta^{3}\beta^{5/2}} \tag{23}$$

(At the Eddington limit we have $\dot{M}_{17} \simeq 10\, M_{*}$; $\alpha < 1$).

Of course, the deduced properties of the inner disk boundary are necessary conditions for the functions of structure (4)—(8), only, and valid under the condition (23). In the next higher order of Δr_{*} (12), (13) the values of the structure functions on the inner disk edge have to be corrected in the next higher approximation. With respect to the parameter of structure the class of accretion disks taken into consideration is certainly enlarged.

Besides the boundary properties, for a detailed comparison of the "bimetric disk" and the "Einstein disk" the equations of structure (4)—(8) must be integrated in the innermost disk region. This will be carried out later. Strong restrictions, hounding the region of validity of the model used here, came from the thin disk assumption $h \ll R$ and the subsonic condition $\mu_{r} \leqslant 1$. On the inner edge, the first condition leads to

$$\frac{\dot{M}_{17}\alpha}{M_{*}} \ll \frac{2.5\cdot 10^{4}}{\beta\cdot f_{h}^{3}}. \tag{24}$$

(with (15), (13)) while the second condition is fulfilled by the definition. From the thin disk assumption ($h \ll R$) it follows that

$$T \ll 5.4\cdot 10^{12}\,\frac{e^{-\frac{2\beta}{r_{*}}}}{(r_{*}-2)}\beta; \tag{25}$$

This is fulfilled in the inner edge region ($r_{*mb} < r_{*in} \leqslant r_{*} < r_{*ms}$).

3. *Discussion*. With the increase of mass of the central body, massivity Σ_{in}, pressure P_{in}, temperature T_{in}, and energy flux q^{z}_{in} decrease, because the inner gap h_{in} of the accretion disk opens with the corresponding decrease of radial accretion flow $\bar{v}^{r}_{in}$ in this case. The inner gap h_{in} widens with the increase of the accretion rate $\dot{M}_{17}$, too. But in this case the accretion flow velocity grows and the inner disk edge becomes hotter and hardly radiates with the corresponding growth of massivity, pressure and compactness. With the increase of "viscosity" (parameter) the inner disk boundary becomes hotter but radiates weaker. This can be understood through the growth of the accretion flow velocity on the one hand, and on the falling off of massivity and the compactness on the other.

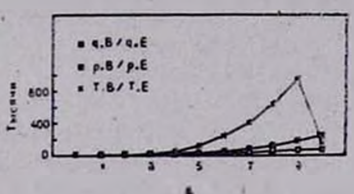

Fig. 2. The dependence of vertical energy flux, rest mass density, temperature on "obsorption" parameter β.

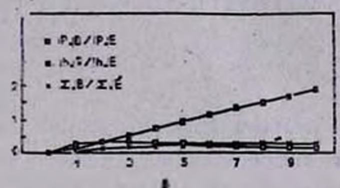

Fig. 3. The dependence of pressure, thickness and surface density on "obsorption" parameter β.

This behaviour of the inner edge known already for the weak bimetric case $\beta = 1$, is modified by the absorption parameter $\beta = M/M'$. Large β represents the strong gravity case in bimetric theories. It is interesting to note that with the growth of gravity $\beta \lesssim 10$) qualitatively the same effect can be observed as with increasing accretion rate. Although, in this case, the gap is much more closed. This means the inner disk edge radiates stronger and harder from a more massive, but less compact inner disk boundary r_{*in} at growing gravity β. In addition, the absorption parameter from 2 up to 3 orders of magnitude is stronger than the accretion rate and the "viscosity". Thereby, an important conclusion can be drawn that the theory depending effects determine stronger the appearance, of the disk rather than the internal parameters of structure ($\dot{M}_{17}$ α) of the disk. This conclusion is supported by comparison of the inner boundary behaviour of corresponding disks in bimetric and Einstein's theory.

From (22) one can conclude that the "bimetric disk" radiates harder and stronger from a more massive and less compact inner edge than the "Einstein disk" ($\beta \gg 1$). Independently from the disk parameters M_{17}, M and a this appearance is strengthened by the increasing gravity (increasing "absorption parameter") in the bimetric case.

The existence of a more massive and compact "bimetric disk", which radiates softer with higher luminosity from the innermost region, can be considered as a hint that accumulating accretion disk can be used to test gravitation theories.

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АККРЕЦИОННЫЕ ДИСКИ В БИМЕТРИЧЕСКОЙ ТЕОРИИ ГРАВИТАЦИИ: СИЛЬНАЯ ГРАВИТАЦИОННАЯ ЧУВСТВИТЕЛЬНОСТЬ ВНУТРЕННЕЙ ГРАНИЦЫ ДИСКА

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В рамках Биметрической теории гравитации рассчитаны оптически тонкие аккреционные диски с тормозным излучением. Проведено сравнение полученных результатов с соответствующими в эйнштейновской теории. Гравитационная чувствительность расположенной в сильном поле центрального компактного объекта внутренней границы диска является аргументом в пользу утверждения, что аккреционный диск возможно в целом определяется сильными гравитационными эффектами.

REFERENCES

1. *J. E. Pringle*, Ann. Rev. Astrophys., 19, 137, 1981.
2. *C. L. Shapiro, S. A. Teukolsky*, Black Holes, White Dwarfs and Neutron Stars, John Willy and Sons, N. Y., 1983.
3. *J. Frank, A. R. King, D. J. Raine*, Accertion power in astrophysics, Cambridge University Press, Cambridge, 1985.
4. *H. G. Paul, G. Rudiger*. Stern (in press).
5. *C. F. V. Weizsacker*, Z. Naturforschg., 3a, 524, 1948.
6. *R. Lust*, Z. Naturforschg, 7a, 87, 1952.
7. *H. G. Paul*, Astron. Nachr., 306, 117, 1985.
8. *H. G. Paul*, Astron. Nachr., (in press).
9. *J. M. Bardeen, W. H. Press, S. A. Teukolsky*, Astrophys. J., 178, 347, 1972.
10. *I. D. Novikov, K. S. Thorne*, in "Black Holes", eds. C. DeWitt, B. DeWitt, Gordon a Breach, 1973.
11. *D. N. Page, K. S. Thorne*, Astrophys. J., 191, 499, 1974.
12. *W. R. Stoeger*, Astron. and Astrophys., 53, 267, 1976.
13. *W. R. Stoeger*, Astrophys. J., 235, 216, 1980.
14. *B. Muchotrzeb, B. Paczynski*, Acta. Astron., 32, 1, 1982.
15. *Z. Loska*, Acta Astron., 32, 13, 1982.

16. *B. Muchotrzeb*, Acta Astron., 33, 79, 1983.
17. *B. Muchotrzeb-Cerny, M. Zbysrewska*, Contr. papers to GR—10, Padova 1983.
18. *M. Korlowski, M. Jariszynski, M. A. Abramowicz*, Astron. and Astrophys., 63, 209, 1978.
19. *B. Paczynski, P. Wiifa*. Astron. and Astrophys., 88, 23, 1980.
20. *M. Jaroszynski, M. A. Abramowicz, B. Paszyński*, Acta Astron., 30, 1, 1980.
21. *M. A. Abramowicz, M. Calvani, L. Nobili*, Astrophys. J., 242, 772, 1980.
22. *S. Hacyan*, preprint Instituto de Astronomia, Universidad Mexico, 1982.
23. *H. G. Paul*, proseeding of COSPAR/IAU Simp., 1987, Sofia.
24. *J. Zhang, S. Jiang*, Act. Astrophys. Sin., 2, 277, 1982.
25. *J. E. Pringle, J. M. Rees, A. G. Pacholczyk*, Astron. and Astrophys., 29, 179, 1973.
26. *D. G. Payne, D. M. Eardley*, Astrophys. Lett., 19, 39, 1977.
27. *N. Rosen*, Gen. Rel. and Grav., 4, 435, 1973.
28. *N. Rosen, J. Rosen*, Astrophys. J., 212, 605, 1977.
29. *G. S. Sahakian, A. V. Sarkissian, B. V. Khachatrian, E. V. Chubarian*, Astrofizika, 14, 489, 1978.
30. *A. V. Sarkissian, B. V. Khachatrian, E. V. Chubarian*, Astrofizika, 15, 503, 1979.
31. *A. V. Sarkissian, E. V. Chubarian*, Astrophys. and Space Sci., 98, 1, 1984.
32. *E. V. Chubarian, A. V. Sarkissian*, Contr. papers to GR-11, Stockholm, 1986.
33. *H. G. Paul*, Contr. papers to 14-th seminar on Rel. Phys., Jena, 1982.
34. *D. E. Liebscher*, Ann. Phys. Lei., 34, 402, 1977.
35. *C. Eckart*, Phys. Rev., 58, 919, 1940.
36. *P. Jordan, J. Ehlers, R. K. Sachs*, Akad. Weis. Lit. Mainz. Abh. Math.-Nat. Kl., 1, 1961.
37. *N. J. Shakura, R. A. Sunyaev*, Astron. Astrophys., 24, 337, 1973.