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Interaction of the TE_{01} Mode With a Particle In a Two-layer Metal-dielectric Cylindrical Waveguide

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Abstract. The possibility of focusing or channeling a relativistic particle in a two-layer metal-dielectric waveguide with copper outer wall using the synchronous TE_{01} waveguide mode is considered. The equations of motion are solved numerically and the trajectory of the particle is traced.

Keywords: Cylindrical waveguide, two-beam acceleration, dielectric loaded structures, TE_{01} mode field.

1. Introduction

In this work, we substantiate the principal possibility of focusing an electron beam in a metal cylindrical waveguide, the walls of which are covered from the inside with a thin dielectric layer**.** Dielectric loaded structures are widely used for two-beam acceleration [1-4], beam diagnostics [5], beam-based THz generation [6-12], beam manipulation [13-16] and self-wake beam energy modulation [17].

The most commonly used method of beam focusing is focusing with magnetic lenses [18]. Focusing is also done with a solenoid [19]. Along with them, methods of focusing using plasma lenses [20] and by laser-beam field [21] are also being developed. A general analysis of transverse deflecting fields in two-layer retarding structures is given in [22]. As shown in [23], the eigenmodes generated in a two-layer cylindrical metal-dielectric structure at certain frequencies are slowly propagating and their phase velocity can be synchronized with the motion of a particle propagating in the structure. It is also shown in [23] that when studying fields in a given structure, it is essential to take into account the finite conductivity of the metal of the outer wall. It will be shown below that taking into account the finite conductivity of the metal of the outer wall is also necessary when considering the process of the TE_{01} mode field influence on the beam passing through the structure.

2. Statement geometry of the problem

The charged particle and the TE_{01} waveguide mode, introduced from outside in a two-layer metal-dielectric cylindrical waveguide, propagate simultaneously. The particle initially directed parallel to the axis of the waveguide with some offset χ_0 (Fig.1). The structure is characterized by the following parameters: the inner radius of the waveguide a, the thickness of the dielectric layer

d, the dielectric constant of the layer $\varepsilon_1 = \varepsilon_1' + j\varepsilon_1''$, and the conductivity of the outer metal wall σ_2. The outer wall is supposed to be unbounded.

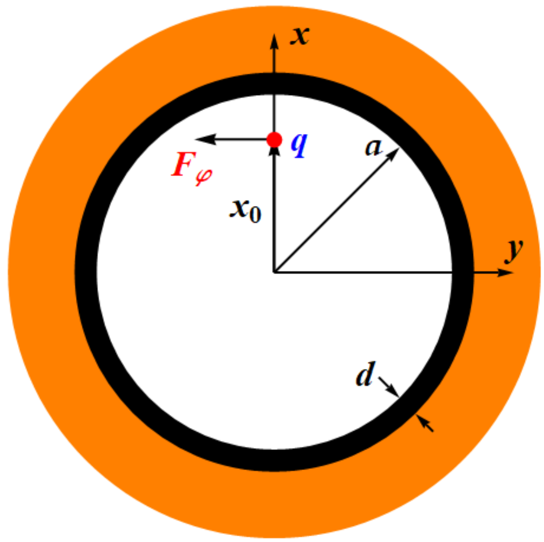

Figure 1. Geometry of the problem.

3. Simulation algorithm

In a cylindrical coordinate system r, φ, z, the TM_{01} mode field is written as follows:

$$
\begin{aligned}
E_\varphi &= -jaB\mu_0\omega \frac{J_1(vr/a)}{v} e^{j(\omega t - pz + \psi_0)} \\
H_r &= jaB0 \frac{J_1(vr/a)}{v} e^{j(\omega t - pz + \psi_0)} \\
H_z &= BJ_0(vr/a) e^{j(\omega t - pz + \psi_0)}
\end{aligned}
\tag{1}
$$

Here v dimensionless transverse eigenvalue of mode, $p = \sqrt{\omega^2/c^2 - v^2/a^2}$ – longitudinal wavenumber, a waveguide radius, ψ_0 is arbitrary constant initial phase, μ_0 is a magnetic permeability of vacuum, c is a speed of light in vacuum and B is an amplitude of mode field.

The particle is affected by the Lorentz force generated by the mode field:

$$\vec{F} = \vec{E} + \mu_0[\vec{V} \times \vec{H}] \tag{2}$$

with particle velocity $\vec{V}$. We assume that the interaction of the mode field with the particle begins at the moment of time $t = 0$ at the point $z = 0, r = x_0, \varphi = 0$. At the initial time of interaction, the particle velocity is parallel to the waveguide axis $\vec{V} = \{0, 0, V\}$. The Lorentz force at this initial time moment has only an angular component:

$$F_{\varphi} = -jaZ_0\beta B\Delta k \frac{J_1(\nu r/a)}{\nu} e^{j(\omega t - pz + \psi_0)}, Z_0 = \mu_0 c, \beta = \frac{v}{c} \tag{3}$$

As can be seen from (1) and (3), the radial magnetic and polar electric components of the TE_{01} mode field partially compensate of one another when forming the Lorentz force. As we see, the Lorentz force is proportional to the difference between the longitudinal wave numbers of the mode field and the particle own field

$$\Delta k = \sqrt{\omega^2/c^2 - \nu^2/a^2} - \omega/V \tag{4}$$

On the other hand, the condition of synchronization of the phase velocity of the wave with the particle motion requires that this difference be zero: $\Delta k = 0$. Consequently, the Lorentz force (3) also vanishes. Equality $\Delta k = 0$ as a condition of synchronization, can be performed in its pure form, if the transverse wave number ν is purely real or purely imaginary. In the metal-dielectric structure, this is the case if its outer wall is ideally conductive and the internal dielectric coating has no losses (Fig.2). For an ultrarelativistic particle, this takes place at $\nu = 0$. Equality $\Delta k = 0$ can also be fulfilled when $V < c$, but in the case of perfectly conducting outer wall it also leads to the vanishing of the Lorentz force (3).

Another picture takes place with a real conductor as an external wall: in this case, the transverse wave number ν is a complex number and the equality $\Delta k = 0$ cannot be fulfilled in its pure form. The “in phase” condition in this case is:

$$\Delta k = \mathrm{Re}\left\{\sqrt{\omega^2/c^2 - \nu^2/a^2}\right\} - \omega/V = 0 \tag{5}$$

Substituting the resulting form of equation (5) in the expression for the Lorentz force (3) does not nulling it and it can effectively influence on the particle.

Figure 2 shows the frequency dependence of the transverse wave numbers for the TM_{01} mode for different conductivities of the materials of the outer wall. As you can see, the curve for copper differs significantly from the curve for an ideal conductor.

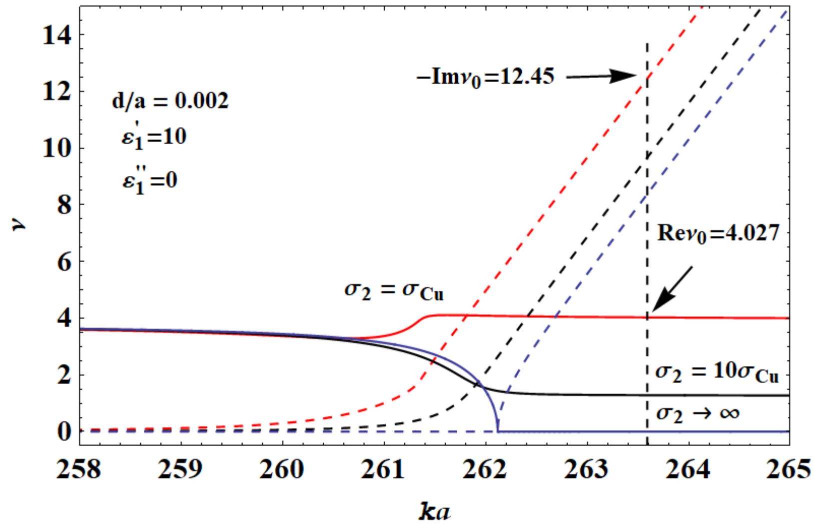

Figure 2. The real ($Re\ \nu$, solid) and imaginary ($-Im\nu$, dashed) parts of TE_{01} mode transverse wave number frequency distribution in a metal-dielectric waveguide for different values of the outer wall conductivities.

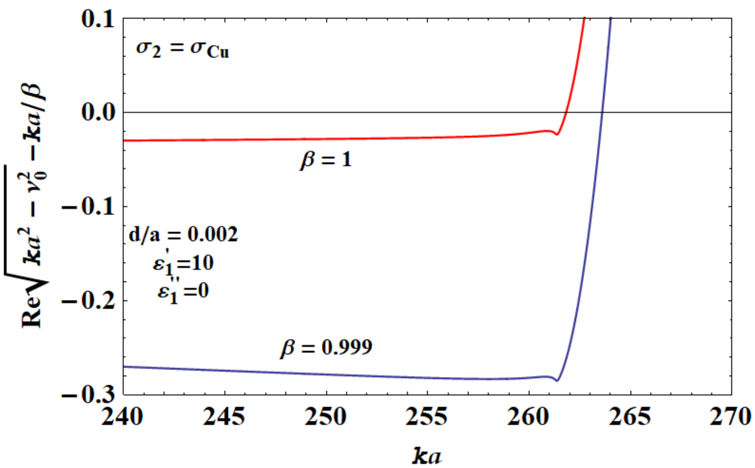

Figure 3. Matching diagram of dispersion curves with particle motion in a metal-dielectric waveguide with a copper outer wall; for ultra-relativistic particle (red); for the particle with initial velocity $V_z = 0.999c$ (blue).

The dimensionless transverse eigenvalue ν is calculated numerically by matching the mode transverse components at the boundaries between the inner vacuum region and the dielectric layer and between the dielectric layer and the outer metal wall. The equation for the transverse eigenvalue is derived from the requirement of zero equality of determinant of the homogeneous system of algebraic equations formed as a result of matching.

As already noted, TE_{01} mode, pumped into the waveguide, for effective interaction with the particle must be generated at a frequency at which its phase velocity would coincide with the velocity of the particle.

During the simulation, the following structure parameters were used: $a = 1mm, d = 2\mu m,$ $\sigma_2 = \sigma_{Cu} = 58 \cdot 10^6 \Omega^{-1} m^{-1}$ (copper), $\varepsilon_1' = 10, \varepsilon_1'' = 0$ (dielectric without losses). The initial velocity of the particle (at the moment $t = 0$ of the beginning of interaction with the TE_{01} mode) at the point $z = 0, \chi_0 = 25\mu m, y = 0$ is chosen to be $V = V_z = 0.999c\,(\gamma \approx 22.37)$.

The following equations of motion are solved:

$$\frac{d\vec{p}}{dt} = e\,\mathrm{Re}\left\{\vec{F}\right\}, \quad \frac{d\vec{r}}{dt} = C\,\frac{\vec{p}}{\varepsilon}. \tag{6}$$

with

$$\varepsilon = \sqrt{m^2c^2 + \vec{p}^{\,2}}, \quad \vec{V} = \frac{c\vec{p}}{\varepsilon}, \tag{7}$$

m, e electron mass and charge, $\vec{p}$ particle momentum. A $t = 0, \vec{p} = \{0, 0, p_z\}$, where

$$p_z = \frac{mV_z}{\sqrt{1 - \left(V_z/c\right)^2}} \tag{8}$$

The frequency at which the pumped mode is synchronized with the initial speed of the particle for the copper outer wall $\left(\sigma_2 = 58 \cdot 10^6 \Omega^{-1} m^{-1}\right)$ at $a = 1mm$ (Fig.2) is equal to $f \approx 126THz\,(k = 263591m^{-1})$. The dimensionless transverse eigenvalue is equal to $v = 4.027 - 12.45j$ (see vertical dashed lines on Fig.2 and Fig.3).

The system of equations (6 – 8) is solved numerically by discrete integration. The following algorithm is used:

$$\begin{aligned} &\vec{p}_{i+1} = \vec{p}_i + ec\Delta t\,\mathrm{Re}\left\{F(t_i, \vec{r}_i, \vec{p}_i)\right\} \\ &\vec{r}_{i+1} = \vec{r}_i + c\Delta t\,\vec{p}_i \big/ \varepsilon\left(\vec{p}_i\right), \\ &t_{i+1} = t_i + \Delta t,\ i = 0,1,2,... \end{aligned} \tag{9}$$

$$t_0 = 0, \vec{r}_0 = \{x_0, 0, 0\}, \vec{p}_0 = \{0, 0, p_z\}, B = 50\frac{MV}{m}, \psi_0 = \frac{\pi}{2}.. \tag{10}$$

4. Simulation results

On Figure 4, the results of performed simulation are presented. The position of the particle along the waveguide axis is plotted on the horizontal axis. The vertical axis describes the deviation of the particle trajectory from the axis of the waveguide. Particle motion traced over approximately

$12cm$. As can be seen from the Figure, at the initial stage of interaction with a particle, a slow deviation of the particle trajectory from a straight line towards the waveguide axis occurs. Having reached a certain minimum distance from the axis, it experiences several kicks, after each of which it returns to the vicinity of the waveguide axis. Then comes a period of relative stabilization: the movement of the particle occurs in close proximity to the axis. Here it also periodically experiences small shocks, but invariably returns to the waveguide axis (Fig.5). The dependence of the particle distance from the axis of the waveguide on the longitudinal coordinate z on stable sections has a sinusoidal shape with a slowly increasing or decreasing amplitude (Figs. 6.1, 6.3, 6.4, right).

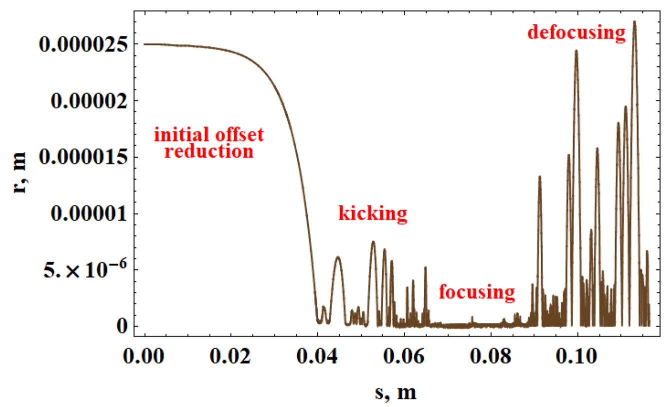

Figure 4. The distribution of the particle deviation from the axis as a result of its interaction with the field of the TE_{01} mode. Full simulation area.

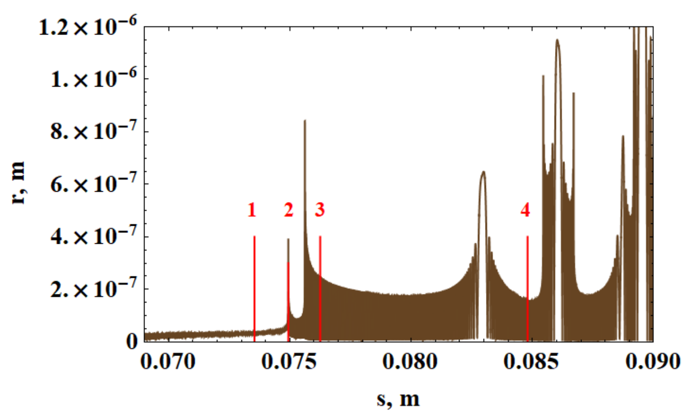

Figure 5. The distribution of the particle deviation from the axis as a result of its interaction with the field of the TE_{01} mode. Paraxial stabilization area.

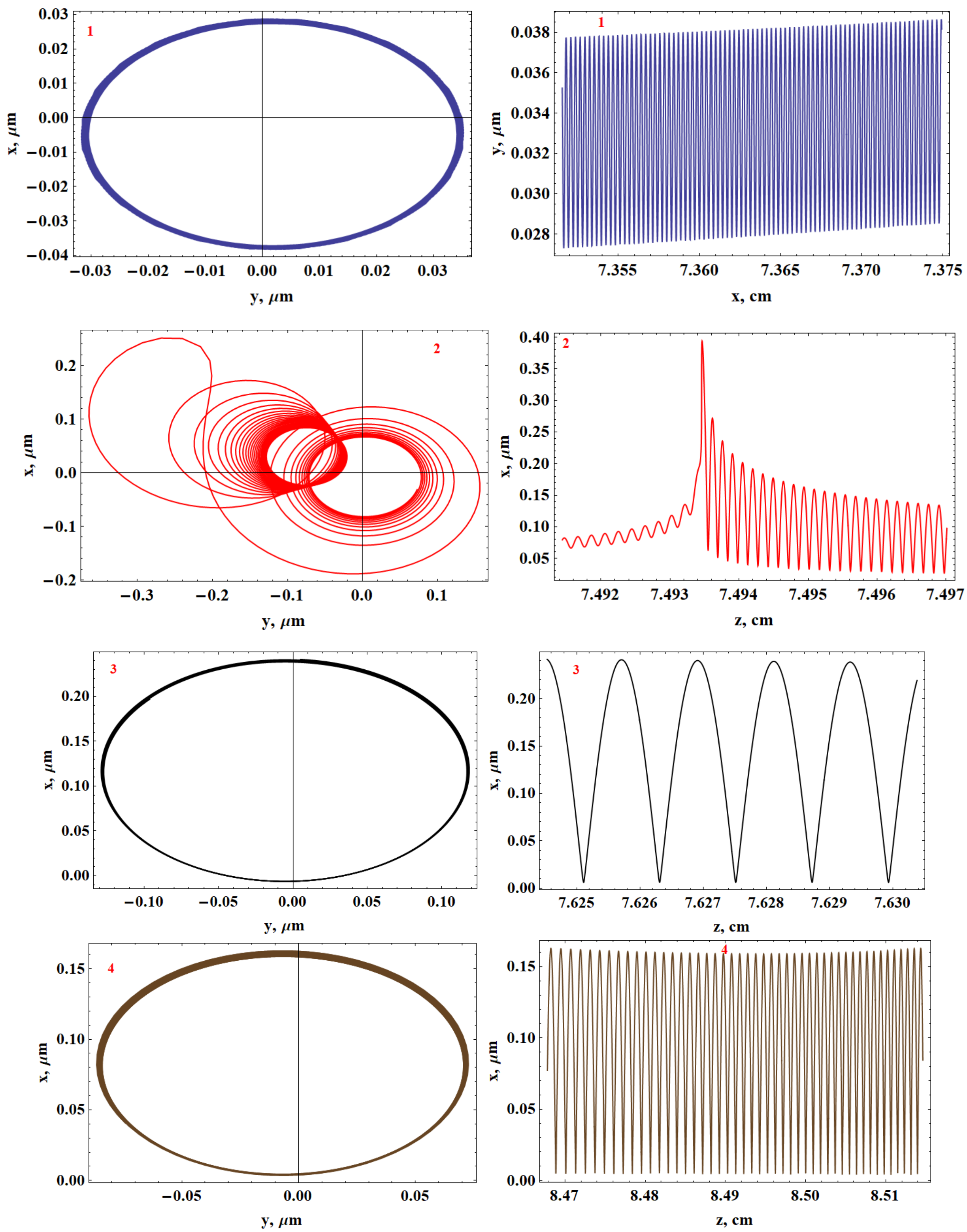

Figure 6. Paraxial stabilization mode. The distribution of the trajectory in the cross section (left) and the longitudinal distribution (right) in certain areas, marked with numbers 1–4 in Figure 5.

In the same areas, the projection of the trajectory on the cross section plane has a quasi-elliptic character: the ellipses with their decreasing or with increasing dimensions are turned towards the axis of the waveguide or include it in themselves (Fig. 6, left). The trajectory of the particle is complicated in the jump region (Fig. 6.2). A transition from one quasistable elliptical orbit to another is observed here (Fig. 6.2, left). The longitudinal profile of the trajectory changes accordingly (Fig. 6.2, right).

The defocusing mode, accompanied by an increase in the amplitude of the deviation from the axis, can be explained by the attenuation of the mode power with a distance, due to the finite conductivity of the outer copper wall. There is a tendency to complicate the trajectory, but in the focusing area, it still occurs in the immediate vicinity of the axis of the waveguide. In any case, in the observed area, the focusing is obvious.

5. Conclusion

Thus, in this work, for the first time, an attempt is made to trace the transformation of the trajectory of a charged particle in a two-layer metal-dielectric waveguide, under the influence of a waveguide TE_{01} mode introduced from outside. The trajectory is traced in a relatively small area $(12cm)$, but the main patterns of trajectory development are visible. From the presented results of numerical simulation, it follows that, with an appropriate selection of the power of the generated mode, focusing or channeling of the electron beam can take place on a certain section of the waveguide. The practical implementation of the described method is due to the presence of a TE_{01} mode generator at the synchronization frequency and its matching with the two-layer waveguide structure. The synchronization frequency can be adjusted by changing the inner radius of the waveguide and the thickness and dielectric constant of the inner layer.

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