# Conversion Efficiency of Electromagnetic Radiation in the Vacuum Diode

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#### Received 06 August 2020

**Abstract:** In this paper, we consider the efficiency of conversion of electromagnetic radiation in its nonlinear interaction with the space charge of a flat vacuum diode, depending on the radiation intensity and on the applied constant electric field. It is shown that the detected signal and the amplitude of the second harmonic field, as well as the conversion efficiency increase with an increase in the amplitude of the electromagnetic radiation field. The conversion efficiency can be improved by an optimal choice of the bias field, as well as by increasing the area of interaction of the electromagnetic wave with the space charge.

Keywords: nonlinear interaction, space charge, vacuum diode, terahertz detection.

#### 1. Introduction

Security issues and military defense technologies, biomedicines, etc. stimulated an intense interest in mobile systems from millimeter (*mm*) to terahertz (0.1 to 10*THz*) ranges. The use of terahertz technologies in biology and medicine is especially attractive due to non-ionizing nature and weak scattering of THz radiation in the fine-dispersed media.

The key nodes of THz systems are radiation sources and receivers (detectors). However, for a number of reasons [1-3], both microwave and optical generation and reception conventional methods are inefficient for this range, despite the fact that THz radiation occupies the region between the microwave and optical regions.

After the advent of femtosecond lasers, the frequency conversion of laser radiation using optical detection in a nonlinear medium is widely used to the generation of THz radiation [2-5]. To obtain powerful THz radiation, it is necessary to use nonlinear materials with a large margin of optical and mechanical strength and a large nonlinear coefficient. However, the crystals widely used for this purpose (lithium niobate, DAST, etc.) have a large absorption in the THz region, which limits the sizes of the crystals used, thereby limiting conversion efficiency and the ultimate power of the generated radiation. This is one of the main problems in the development of the THz range.

Another problem is the lack of high-speed, highly sensitive, and affordable detectors in this area [2–4]. Fast and sensitive optical receivers operating on the basis of an external or internal

photoelectric effect are unsuitable for detecting THz radiation due to the low energy of the terahertz photon. On the other hand, the sensitivity of widespread microwave Schottky diodes rapidly decreases in the THz region. As for thermal receivers, despite the fact that they are quite sensitive, but very inertial, and therefore they are not applicable for registration of fast processes. In addition, some of them require cooling to the temperature of liquid helium.

Therefore for successful development of terahertz frequency range, high-speed nonlinear elements are required both for generating and receiving THz radiation. In this regard, very promising can be vacuum electronic devices (VED), such as diodes, triodes, etc. They have been successfully used as nonlinear elements for the conversion of low-frequency electromagnetic waves [6-9]. This non-linearity is described by the Child–Langmuir Law (CL), discovered a century ago. However, despite the above, vacuum devices are still not widely used as non-linear elements for the detection and conversion of microwave, THz, and optical frequency ranges. This is mainly due to the problem of supplying high-frequency voltages to the lamp electrodes. However, this problem is automatically eliminated in the case when an electromagnetic wave propagates in the anode-cathode gap. Direct interaction of an electromagnetic wave with the inhomogeneously distributed space charge occurs. As a result of such an interaction, it is possible to implement the conversion of radiation - generation of harmonics, combination frequency generation, detection, etc.

In this paper, we consider the efficiency of the conversion of electromagnetic radiation in its nonlinear interaction with the space charge of a flat vacuum diode, depending on the radiation intensity and on the applied constant electric field.

## 2. The space charge in the field of electromagnetic wave.

The capacitive or the transit time model, can be used to approximate the space charge limited (SCL) current by I = Q/T, where  $Q = CU_A$  is the total bound surface charge on the cathode, C is the diode capacitance,  $U_A$  is the gap voltage, and T is the transit time of an electron to cross the gap subjected only to the vacuum field [6-11].

Due to the thermo-emitting cathode, there are free electrons in the space between the cathode and anode. Moreover, the more the number of free electrons, the less the field intensity in the vicinity of the cathode because of the increase of the screening effect of the electron cloud. The boundary condition of zero electric field on the cathode surface is known as the SCL condition. It determines the maximum injected current in steady-state [6], and charge conservation requires that current density  $j(x) = \rho(x)v(x)$  be constant across the entire gap from x = 0 to x = d ( $\rho(x)$  and v(x)) are the density and velocity of the charge in the gap, respectively).

The potential  $\varphi(x)$  must obey Poisson's equation

$$\nabla^2 \varphi(x) = -\rho(x)/\varepsilon_0 \tag{1}$$

Considering also the law of conservation of energy

$$mv^2(x)/2 = e\varphi(x) \tag{2}$$

Eq. (1) becomes a second-order nonlinear differential equation for the potential:

$$\frac{d^2\varphi(x)}{dx^2} = \frac{j(x)}{\varepsilon_0} \sqrt{\frac{m}{2e}} \frac{1}{\sqrt{\varphi(x)}},\tag{3}$$

The assumption of a zero electric field on the cathode surface is the condition for the limited space-charge emission; any additional injection of electrons will lead to a change in the direction of the field and, therefore, to the movement of electrons back to the cathode. In a steady-state, electrons leave the gap region, collecting at the anode, so that more electrons can escape from the cathode; this process gives rise to the space-charge-limited current density  $j_{SCL}$  [11].

Considering that j(x) is constant for all  $x(j(x) = j_{SCL})$ , it is easy to find a solution of equation (3) for  $\varphi(x)$ :

$$\varphi(x) = \left(\frac{3}{2}\right)^{4/3} \left(\frac{j_{SCL}}{\varepsilon_0}\right)^{2/3} \left(\frac{m}{2e}\right)^{1/3} x^{4/3} = ax^{4/3}$$
 (4)

so that

$$E(x) = -\varphi'(x) = -\frac{4}{3}ax^{1/3},\tag{5}$$

and

$$\rho(x) = \varepsilon_0 \varphi''(x) = \frac{4}{9} \varepsilon_0 a x^{-2/3}$$
 (6)

For simplicity, we restrict ourselves by describing one-dimensional case, when infinitely wide electron bunch traverses perpendicular to the electrodes (along the x axis)  $(\partial/\partial y = \partial/\partial z = 0)$ , while the electromagnetic wave is linearly polarized  $(E_z = E_{xz} \neq 0, E_{yz} = 0, E_{zz} = 0)$  and propagates along the z axis (see Fig. 1).

If we now apply the physical condition imposed by the anode electrode  $\varphi_A = \varphi(d)$ , we can determine the current density

$$j = j_{SCL} = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m}} \frac{\varphi_A^{3/2}}{d^2} = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m}} \frac{\varphi(x)^{3/2}}{x^2} = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m}} \frac{E(x)^{3/2}}{\sqrt{x}}$$
(7)

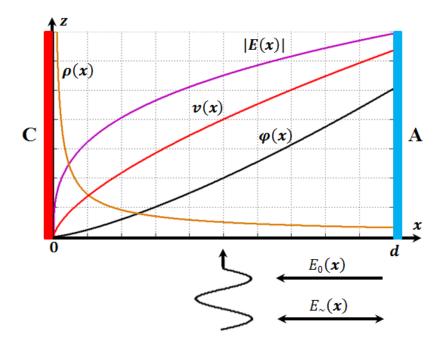


Fig. 1. Profiles of the potential  $\varphi(x)$ , electric field E(x), velocity v(x), and charge density  $\rho(x)$  in the gap  $0 \div d$  at anode voltage  $\varphi_A = \varphi(d)$ , and with zero electric field on the cathode surface.

Physically, any further increase of injection current leads to a negative potential near the cathode which would prevent further release of electrons (with assumed zero initial velocity) into the diode. Note that this space charge limited current density, Eq. (7), is independent of material properties. Regardless of the cathode material or cathode temperature, Eq. (7) gives the bound imposed by the Poisson equation [6].

When considering the interaction of an electromagnetic wave with the space charge of a diode, it is necessary to take into account that the period of oscillation of the electric field can be comparable with electrons transit time from the cathode to the anode or even less than it.

Let us estimate the transit time of the charge of the interelectrode space.

The ballistic or space-chargefree transit time so is given by the simple relation  $\tau=d/\overline{V}$ , where  $\overline{V}$  is the time-averaged velocity of the charge during the transit. We assume that the charge leaves the cathode with negligible initial velocity, and, from energy conservation, we know that upon reaching the anode, the final velocity of the charge  $V_{\rm max}=\sqrt{2e\phi(d)/m}$ . Because in this case, the acceleration is constant, then, we may approximate  $\overline{V}$  by taking the simple arithmetic mean of the initial and final velocities [11]:

$$\overline{V} = \frac{1}{2} \sqrt{\frac{2e\varphi_A}{m}}$$
, consequently  $\tau = \sqrt{\frac{2md}{eE}}$  (8)

Note that, as expected, the transit time decreases as the field at the surface E increases.

To determine the charge transit time  $\tau_1$  at the space charge limited condition in the interelectrode space, one must first determine the motion velocity. Using the charge conservation  $j_{SCL} = \rho(x)v(x)$  and the solution for  $\rho(x)$ , Eq. (6), we can write:

$$V(x) = V_{\text{max}} \left(\frac{x}{d}\right)^{2/3} = \sqrt{\frac{2e\varphi_A}{m}} \left(\frac{x}{d}\right)^{2/3} = \frac{dx}{dt}$$
(9)

(8) can be easily integrated to find the position as a function of time x(t). Assuming that x(0) = 0, we get:

$$3x^{1/3} = V_{\text{max}}d^{-2/3}t\tag{10}$$

Thus, when space charge is present, the transit time  $\tau_1$  can be determined from (9) by looking at the anode boundary, that is,  $x(\tau_1) = d$ .

$$\tau_1 = \frac{3d}{V_{\text{max}}} = 1.5\tau \tag{11}$$

The actual time-averaged velocity must be

$$\overline{V} = \frac{V_{\text{max}}}{3} = \frac{1}{3} \sqrt{\frac{2e\varphi_A}{m}} \tag{12}$$

Thus, when considering the interaction of an electromagnetic wave with a non-uniformly distributed space charge, based on (12), it is possible to estimate the amplitude of charge displacement  $\Delta x$  in the anode-cathode gap during one half-period T/2 of the wave:

$$\Delta x \approx \frac{\overline{V}T}{2} \approx \frac{T}{6} \sqrt{\frac{2e\varphi_A}{m}} \tag{13}$$

Numerical estimates show that for sufficiently high intensity of the electromagnetic wave even if electric field in the interelectrode gap reaches up to  $10^6 V/m$ , at frequencies of the order of 10THz,  $\Delta x$  is only  $10\mu m$ , and decreases with increasing frequency.

Since this displacement is much smaller than the anode-cathode gap spacing, the change in the space charge distribution can be neglected. In this case, the change in the diode current in time obeys the CL law, considering that on the charge acts the total electric field  $E(x,t) = E_0(x) + E_{\infty}(x,t)$ , where  $E_0(x)$  is the static field created by the anode voltage (bias field), and  $E_{\infty}(x,t)$  is the electric field of the electromagnetic wave.

Thus, due to the inhomogeneous distribution of space charge in the interaction of electromagnetic waves, the charge performs anharmonic oscillations, as a result harmonics of oscillations, combination frequencies, and a DC component can arise.

It should be noted that this capacitance model proves to be very useful in predicting the maximum current density that can be injected in a very short time, much less than the electron's transit time across the anode-cathode (AK) gap. Despite the fact the total injected charge density is still bounded by  $Q = CU_A$  for a short pulse in a planar gap, the maximum current density in fact can be much higher than  $j_{SCL}$ . But even in this case, the nature of the dependence of the current on electric field corresponds to the Child – Langmuir law, without requiring that the electric field be driven to zero at the cathode surface [6,10,11].

Because of its simplicity and accuracy, this capacitance or transit time model is very usefull for study a short pulse diode, and more recently the quantum regime [6,11].

# 3. The efficiency of nonlinear interaction

In this section by using numerical methods, the efficiency of the detection and the second harmonic generation of electromagnetic radiation at the interaction with the space charge, depending on the bias field and on the radiation intensity is investigated.

The results of calculations of the efficiency of detection and second harmonic generation depending on the bias field at  $E_{0\omega} = 10^5 \, V/m$  are shown in Figs. 2a and 2b.

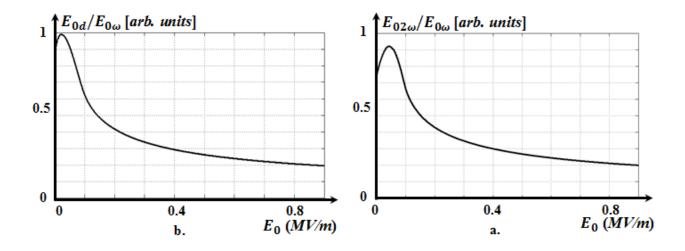


Fig. 2. Efficiency of detection  $(E_{0d}/E_{0\omega})$  (a.), and second harmonic generation  $(E_{02\omega}/E_{0\omega})$  (b.) dependencies on the bias field at  $E_{0\omega}=10^5~V/m$ 

From 2a and 2b it follows that the most effective detection is achieved at  $E_0 \approx 0.24 E_{0\omega}$ , and the maximum efficiency of the second harmonic generation is obtained at the value of the bias field  $E_0 \approx 0.445 E_{0\omega}$ . It should be noted that these ratios are preserved when the amplitude of the field of the fundamental harmonic  $E_{0\omega}$  changes.

The dependences of the detected signal and the amplitude of the second harmonic field on the amplitude of the electromagnetic radiation field at different values of bias field are presented in fig. 3a. Fig. 3b depicts the efficiency of detection and second harmonic generation dependencies on the amplitude of the electromagnetic radiation field for the same values of bias field.

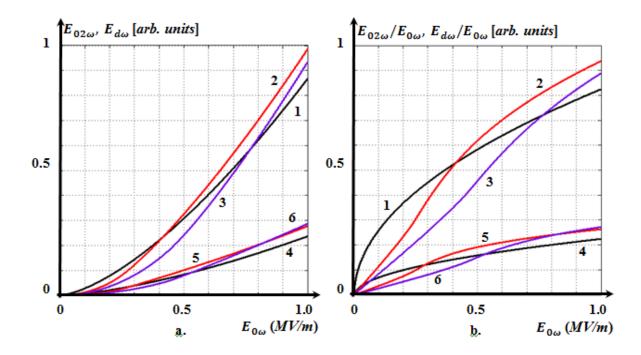


Fig. 3. The detected signal (curves 1;2;3) and the amplitude of the second harmonic field (curves 4;5;6) (a.), efficiency of detection (curves 1;2;3) and second harmonic generation (curves 4;5;6) (b.) dependencies on the amplitude of the field  $E_{0\omega}$  of electromagnetic radiation at different values of bias field  $E_0$ : curves 1;4 (black)  $E_0 = 0.24 \, MV/m$ , curves 3;6 (blue)  $E_0 = 0.445 \, MV/m$ .

As expected, with an increase in the amplitude of the electromagnetic radiation field  $E_{0\omega}$ , both the amplitude of the second harmonic and the detected signal increase. Because the growth of amplitudes is faster than the linear law, then with an increase in  $E_{0\omega}$ , the conversion efficiency also increases. As seen from Fig. 2a, b and 3b, at a certain value of the amplitude of the fundamental frequency field, by choosing the bias field, the conversion efficiency can be increased by more than 15%.

The results of numerical calculations well correlate with the experimental results given in [12].

#### 4. Conclusion

Thus, under certain conditions, it is possible to obtain an effective nonlinear interaction of electromagnetic radiation with an inhomogeneously distributed space charge. Therefore, because vacuum devices are capable of withstanding high intensities, they can be used as non-linear elements to convert powerful electromagnetic radiation.

The conversion efficiency can be improved by an optimal choice of the bias field, as well as by increasing the area of interaction of the electromagnetic wave with the space charge.

## Acknowledgements

The author is grateful to R. Martirosyan, Yu. Avetisyan, Kh. Nerkararyan, A. Hakhumyan, H. Haroyan and V. Mekhitaryan for very useful discussions.

## **Funding**

This work was supported in part by the Terahertz Waves Laboratory of the department of chair of Radiophysics and Electronics of Yerevan State University.

### **Conflict of Interest**

There is no conflict of interest.

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