

PROPAGATION OF MAGNETOELASTIC WAVES IN A PERIODIC MEDIA
Բաբյան Ա. Ա.

Key words: magetoelastic wave, perfectly conductive, periodic structure.

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РАСПРОСТРАНЕНИЕ МАГНИТОУПРУГИХ ВОЛН В
ПЕРИОДИЧЕСКОЙ СРЕДЕ

Ключевые слова: магнитоупругие волны, идеальный проводник, периодическая структура.

Рассматривается задача распространения волн в одномерной периодической электропроводящей среде, находящейся во внешнем постоянном магнитном поле. Задача решается на основе теории Флоке. Установлено влияние внешнего магнитного поля на условия существования запретных зон частот.

Պապյան Ա.Ա.
Պարբերական միջավայրում հաստատուն մագնիսական դաշտում մագնիսաառձգական
ալիքների տարածումը

Հիմնաբանը. մագնիսաառձգական ալիքներ, իդեալական հաղորդիչ, պարբերական միջավայրում:

Աշխատանքում ուսումնասիրվում է մագնիսաառձգական ալիքների տարածումը միաչափ պարբերական կառուցվածքով ունեցող միջավայրերում: Ֆլոկեի տեսության շրջանակում ստացվել են դիսպերսիոն հավասարումները, որոնք որոշում են ալիքների բաց թողնման եւ կասեցման հաճախականության տիրույթները: Կատարված են դիսպերսիոն առնչությունների վերլուծություններ:

We consider a problem of elastic wave propagation in a perfectly conductive periodic structure in a external magnetic field. In the framework of the Floquet theory the dispersion equations are obtained defining magnetoelastic wave frequency gap bandstructure. The influence of an external magnetic field on the magnetoelastic wave properties and forbidden frequencies are studied.

Introduction:

The presence frequency stop or forbidden gaps in a periodic unidirectional elastic structure was first noted in [1]. The detailed studies of elastic wave propagation in a periodic structures are given in [2-10]. In the [6,7,10] from a mathematical point of view the Floquet-Bloch waves in elastic periodic waveguides are investigated. The spectral theory of transverse vibrations of periodic elastic beams is given in [8-9]. The problems of a magetoelastic wave propagation in electroconducting solid deformable bodies and thin elastic structures are considered in [11,12].

Here we consider a problem of wave propagation in 1D bi-material periodic structure consisting of periodically alternating perfectly conductive materials in an external constant

magnetic field. We will investigate the effect of magnetic field on the properties of the magnetoelastic wave propagation in periodic structure.

The statement of problem:

The purpose of this paper is to investigate properties of magnetoelastic waves in an 1D infinite periodic elastic structure in an external constant magnetic field. The unit cell of a period d consists of two piecewise elastically bonded perfect conducting homogeneous materials (Figure 1).

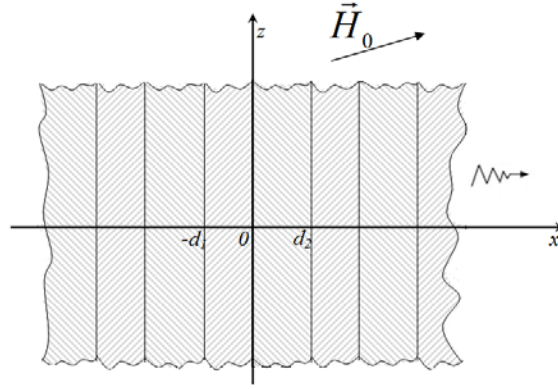


Fig. 1 1D periodic electro-conductive elastic structure in an external constant magnetic field $\vec{H}_0$.

The structure of a periodically repeating unit cell is characterized by the material density and the modulus of elasticity as follows:

$$\begin{aligned} \rho_1, E_1 &\rightarrow -(n-1)d - d_1 < x < (n-1)d, \\ \rho_2, E_2 &\rightarrow (n-1)d < x < nd - d_1 \end{aligned} \quad (1)$$

In the rectangular Cartesian coordinate system $Oxyz$ axis x is directed along wave direction, when axis z can be chosen so, that the displacements to the axis x the component of the magnetic field coincide with the axis z , when $\vec{H}_0 = H_{0x} \vec{i} + H_{0z} \vec{k}$

The ponderomotive force vector for a perfect conductive material is as follows [11].

$$\vec{R} = \frac{\mu}{4\pi} \left\{ \text{rot rot} (\vec{u} \times \vec{H}_0) \right\} \times \vec{H}_0 \quad (2)$$

where $\vec{u}$ is the vector of elastic displacement, μ magnetic permeability of the material.

The components of ponderomotive force vector are as follows:

$$\begin{aligned}
R_x &= \frac{\mu}{4\pi} \left(\frac{\partial^2 u}{\partial x^2} H_{0z}^2 - \frac{\partial^2 w}{\partial x^2} H_{0x} H_{0z} \right), \quad R_y = \frac{\mu}{4\pi} \frac{\partial^2 v}{\partial x^2} H_{0x}^2 \\
R_z &= \frac{\mu}{4\pi} \left(\frac{\partial^2 w}{\partial x^2} H_{0x}^2 - \frac{\partial^2 u}{\partial x^2} H_{0x} H_{0z} \right)
\end{aligned} \tag{3}$$

According to the Floquet theory, we consider solutions only in the elementary cell $-d_1 < x < d_2$ ($n=1$), applying the Floquet–Bloch quasi-periodicity conditions at the ends of the elementary cell.

The motion equation in for 1D waves can be written in the following form

$$\frac{\partial \sigma_{11}}{\partial x} R_x = \rho \frac{\partial^2 U_1}{\partial t^2}, \quad \frac{\partial \sigma_{12}}{\partial x} + R_y = \rho \frac{\partial^2 U_2}{\partial t^2}, \quad \frac{\partial \sigma_{13}}{\partial x} + R_z = \rho \frac{\partial^2 U_3}{\partial t^2}, \tag{4}$$

where σ_{ij} are components of stress tensor.

The motion equation(2) in displacements in the interval $-d_1 < x < d_2$, ($n=1$) can be written as

$$\begin{aligned}
c_{l(s)}^2 \frac{\partial^2 u_{(s)}}{\partial x^2} + \frac{\mu_{(s)}}{4\pi\rho_{(s)}} \left(\frac{\partial^2 u_{(s)}}{\partial x^2} H_{0z}^2 - \frac{\partial^2 w_{(s)}}{\partial x^2} H_{0x} H_{0z} \right) &= \frac{\partial^2 u_{(s)}}{\partial t^2} \\
c_{t(s)}^2 \frac{\partial^2 w_{(s)}}{\partial x^2} + \frac{\mu_{(s)}}{4\pi\rho_{(s)}} \left(\frac{\partial^2 w_{(s)}}{\partial x^2} H_{0x}^2 - \frac{\partial^2 u_{(s)}}{\partial x^2} H_{0x} H_{0z} \right) &= \frac{\partial^2 w_{(s)}}{\partial t^2}
\end{aligned} \tag{5}$$

where indexes $s=1,2$ characterize materials of periodic structure c_l^2 , c_t^2 are the speeds of the longitudinal and transversal waves, correspondingly.

We have the conditions of elastic contact on the boundary of separation of materials

$$\begin{aligned}
u_{(1)}(0,t) &= u_{(2)}(0,t), \quad w_{(1)}(0,t) = w_{(2)}(0,t), \\
\sigma_{xx(1)}(0,t) + t_{xx(1)}(0,t) &= \sigma_{xx(2)}(0,t) + t_{xx(2)}(0,t), \\
\sigma_{xz(1)}(0,t) + t_{xz(1)}(0,t) &= \sigma_{xz(2)}(0,t) + t_{xz(2)}(0,t),
\end{aligned} \tag{6}$$

and the Floquet quasi-periodicity conditions

$$\begin{aligned}
u_{(1)}(-d_1,t) &= \lambda u_{(2)}(d_2,t), \quad w_{(1)}(-d_1,t) = \lambda w_{(2)}(d_2,t), \\
\sigma_{xx(1)}(-d_1,t) + t_{xx(1)}(-d_1,t) &= \lambda \left(\sigma_{xx(2)}(d_2,t) + t_{xx(2)}(d_2,t) \right), \\
\sigma_{xz(1)}(-d_1,t) + t_{xz(1)}(-d_1,t) &= \lambda \left(\sigma_{xz(2)}(d_2,t) + t_{xz(2)}(d_2,t) \right),
\end{aligned} \tag{7}$$

Here $\lambda = \exp(ikd)$, k - Floquet's wave number.

t_{xx}, t_{xz} are components of the Maxwell tensor, that are as follows[11]:

$$\begin{aligned} t_{xx(s)} &= \frac{\mu_{(s)}}{4\pi} \left(\frac{\partial u_{(s)}}{\partial x} H_{0z}^2 - \frac{\partial w_{(s)}}{\partial x} H_{0x}^2 \right) \\ t_{xz(s)} &= \frac{\mu_{(s)}}{4\pi} \left(\frac{\partial w_{(s)}}{\partial x} H_{0x}^2 - \frac{\partial u_{(s)}}{\partial x} H_{0x} H_{0z} \right) \end{aligned} \quad (8)$$

Presenting solutions of equations (3) and (4) in the form

$$\begin{aligned} u_1(x, t) &= u_{10}(x) \exp(i\omega t), \quad u_2(x, t) = u_{20}(x) \exp(i\omega t), \\ w_1(x, t) &= w_{10}(x) \exp(i\omega t), \quad w_2(x, t) = w_{20}(x) \exp(i\omega t), \end{aligned} \quad (9)$$

We have

$$\begin{aligned} u_{(s)0}'' - p_{(s)} w_{(s)0}'' + q_{(s)} u_{(s)0} &= 0 \\ u_{(s)0}'' - \alpha_{(s)} p_{(s)} w_{(s)0}'' + \alpha_{(s)} q_{(s)} u_{(s)0} &= 0 \end{aligned} \quad (10)$$

Here the following notations are made:

$$\begin{aligned} a_{3(s)}^2 &= \frac{\mu_{(s)}}{4\pi\rho_{(s)}} H_{0z}^2, \quad a_{1(s)}^2 = \frac{\mu_{(s)}}{4\pi\rho_{(s)}} H_{0x}^2, \quad a_{3(s)}, a_{1(s)} = \frac{\mu_{(s)}}{4\pi\rho_{(s)}} H_{0x} H_{0z} \\ \alpha_{(s)} &= \frac{c_{l(s)}^2 + a_{3(s)}^2}{c_{l(s)}^2 + a_{1(s)}^2}, \quad p_{(s)} = \frac{a_{3(s)} a_{1(s)}}{c_{l(s)}^2 + a_{3(s)}^2}, \quad q_{(s)} = \frac{\omega^2}{c_{l(s)}^2 + a_{3(s)}^2}; \end{aligned} \quad (11)$$

The solutions of the differential system equations (10) satisfying the conditions of elastic contact and on the boundary of separation materials and the Floquet conditions, we obtain the dispersion equation for

$$\cos(kd) = F(\eta, H_{01}, H_{03}) \quad (12)$$

The dispersion equation (12) determines the wave number k depending on the frequency of η (non dimensionless frequency) and external magnetic field.

Special case:

Let consider a special case when the magnetic field is directed along the axis Z ,

$$\vec{H}_0 = H_{0z} \vec{k}$$

In this case the system of differential equations are separated from each other regard to and we have only one the electro-magneto active equation regard to longitudinal displacement

$$u_{(s)0}''$$

If magnetic field is directed perpendicular to wave propagation direction we will have the electro-magneto active equation regard to transversal displacement $w_{(s)0}$.

We consider now the equation

$$u_{(s)0}'' + q_{(s)} u_{(s)0} = 0 \quad (13)$$

with the conditions of elastic contact on the boundary of separation of materials:

$$u_{(1)}(0, t) = u_{(2)}(0, t), \quad (14)$$

$$\sigma_{xx(1)}(0, t) + t_{xx(1)}(0, t) = \sigma_{xx(2)}(0, t) + t_{xx(2)}(0, t),$$

and Floquet quasi-periodicity conditions

$$u_{(1)}(-d_1, t) = \lambda u_{(2)}(d_2, t), \quad (15)$$

$$\sigma_{xx(1)}(-d_1, t) + t_{xx(1)}(-d_1, t) = \lambda (\sigma_{xx(2)}(d_2, t) + t_{xx(2)}(d_2, t)),$$

The solutions of differential equation (13) can be written as:

$$\begin{aligned} u_{10}(x) &= (B_1 \sin(\sqrt{q_1}x) + B_2 \cos(\sqrt{q_1}x)) \\ \sigma_{10}(x) &= \tilde{E}_1 \sqrt{q_1} (B_1 \cos(\sqrt{q_1}x) - B_2 \sin(\sqrt{q_1}x)) \\ u_{20}(x) &= (C_1 \sin(\sqrt{q_2}x) + C_2 \cos(\sqrt{q_2}x)) \\ \sigma_{20}(x) &= (\lambda_2 + 2G_2) \sqrt{q_2} (C_1 \cos(\sqrt{q_1}x) - C_2 \sin(\sqrt{q_1}x)) \end{aligned} \quad (16)$$

where $\tilde{E}_{(s)} = \lambda_{(s)} + 2G_{(s)}$, λ, G are Lamé constants.

Satisfying the solution (14) conditions of the elastic contact on the line of separation materials and Floquet condition, we obtain homogeneous system equations regard to constants B_1, B_2, C_1, C_2 .

Equating to zero the determinant of this system, we obtain the following dispersion equation

$$\begin{aligned} \cos(kd) &= \cos(d_1 \sqrt{q_1} + d_2 \sqrt{q_2}) \\ &- \frac{1}{2} \left\{ \frac{\tilde{E}_1 + \frac{\mu_1 H_{03}^2}{4\pi}}{\tilde{E}_2 + \frac{\mu_2 H_{03}^2}{4\pi}} \frac{\sqrt{q_1}}{\sqrt{q_2}} + \frac{\tilde{E}_2 + \frac{\mu_2 H_{03}^2}{4\pi}}{\tilde{E}_1 + \frac{\mu_1 H_{03}^2}{4\pi}} \frac{\sqrt{q_2}}{\sqrt{q_1}} \right\} \sin(d_1 \sqrt{q_1}) \sin(d_2 \sqrt{q_2}) \end{aligned} \quad (17)$$

Let note that when magnetic field is directed perpendicular to wave propagation direction we get the same dispersion equation changing $c_{l(s)} \rightarrow c_{t(s)}$.

Discussions and numerical results:

The dispersion equations (17) define ranges of frequencies associated with waves that can propagate in perfectly conductive periodic solid (pass bands), alternated with ranges of frequencies of waves that cannot be transmitted (stop band gaps).

In the case when the material electromagnetic impedances are equal

$$\left(\tilde{E}_1 + \frac{\mu_1 H_{0z}^2}{4\pi} \right) \sqrt{\frac{1}{c_{l1}^2 + a_{3(1)}^2}} = \left(\tilde{E}_2 + \frac{\mu_2 H_{0z}^2}{4\pi} \right) \sqrt{\frac{1}{c_{l2}^2 + a_{3(2)}^2}}$$

we have

$$\cos(kd) = \cos\left(d_1\sqrt{q_1} + d_2\sqrt{q_2}\right) \quad (18)$$

which means that in this case the stop band gaps donot occur.

When the magnetic field is vanished we get the classic result considered in [2]:

The dispersion curves $\eta(kd)$ of dimensionless frequency $\eta = \omega d_1 c_{l2}^{-1}$ in the first Brillouin zone $0 < kd < \pi$ defining the stop band gaps, are illustrated in the Figure.2, Figure. 3 for certain values of dimensionless parameters.

$$\alpha^2 = \frac{a_{3(1)}^2}{c_{l(1)}^2}, \quad \gamma = \frac{c_{l(1)}^2}{c_{l(2)}^2}, \quad \beta = \frac{a_{3(2)}^2}{a_{3(1)}^2}, \quad \theta = \frac{d_2}{d_1}, \quad \zeta = \frac{\rho_1}{\rho_2};$$

characterizing mechanical and geometric properties of periodic structure and the magnitude of the external magnetic field. The dashed curves correspond to magnetoelastic case, the solid curves correspond to the classic elastic case.

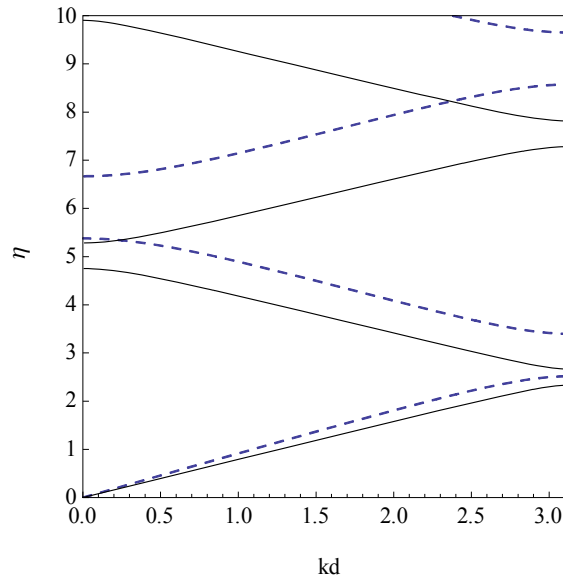


Fig1. Dispersion curves $\eta(kd)$, $\alpha=0.75$; $\xi=0.5$; $\beta=0.55$; $\gamma=0.25$; $\theta=0.25$

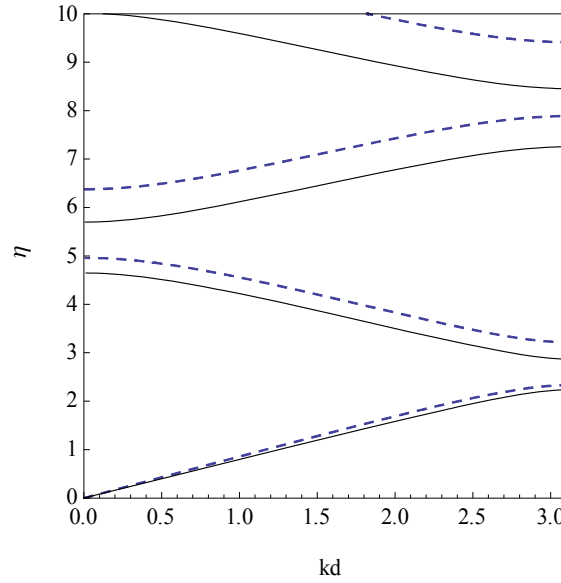


Fig. 2 Dispersion curves $\eta(kd)$, $\alpha=0.5$; $\xi=0.5$; $\beta=0.25$; $\gamma=0.2$; $\theta=0.2$

Analysis of the dispersion curves shows that the external magnetic field change the position and essentially diminish the width of band gaps.

Conclusions: In the framework of the magneto-elasticity equations and the Floquet theory the magnetoelastic wave propagation in 1D perfectly conductive piecewise periodic media is studied. The corresponding dispersion equation are obtained in the case when the direction of magnetic field is perpendicular to the wave propagation direction. The dispersion curves illustrating the magneto elastic wave properties are presented. Analysis of the dispersion curves in the first Brillouin zone shows that the external magnetic field essentially change the positions and diminish the widths of band gaps.

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