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KINEMATICS AND VELOCITY ELLIPSOID OF HALO RED GIANTS

M.I.NOUH[1,2], W.H.ELSANHOURY[2,3]

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Here, we aim to determine the kinematical properties, velocity ellipsoid and Oort constants using a sample of halo red giants. The study is based mainly on the space and radial velocities of 1583 red giant stars collected from the SEGUE-1 and SEGUE-2 surveys. We divided the sample into three subsamples: the inner halo, the outer halo and the stars near the galactic plane. The fitting of the radial velocity equation gives a mean of Oort constants $A = 15.6 \pm 1.6$ km s^{-1} kpc^{-1} and $B = -13.9 \pm 1.8$ km s^{-1} kp^{-1} and angular velocity $|A - B| = 29.5 \pm 0.2$ km s^{-1} kpc^{-1}, which implies a rotational velocity of 221.25 ± 26.66 km s^{-1} if we take the distance to the galactic center as 7.5 kpc.

Keywords: *stellar halo: solar elements: kinematical analysis: Oort constants*

1. *Introduction*. Stellar kinematics is an essential ingredient in the study of galactic structure and evolution. Halo stars in particular may be exploited to probe the formative phases of our galaxy [1].

Various studies have provided evidence that the Milky Way's halo may not comprise a single population by analyzing spatial profiles (or inferred spatial profiles) of halo objects [2-7]. A recent example of such an analysis is the observation of two different spatial density profiles for the distinct Oosterhoff classes of RR Lyrae variable stars in the halo [8]. Additionally, tentative claims for a net retrograde motion of halo objects support the existence of a likely dual-component halo [9-15]. Using astrophysical simulations, the galactic halo has been divided into two components, the inner halo and the outer halo [16]. The inner halo is dominated by stars that formed within the galaxy, where the outer halo mainly comprises stars accreted through merger events.

Red giant stars are important because they are the most luminous of old stars and so are particularly useful to study the early history of the Milky Way. Therefore, researchers use these stars like fossils because in many cases their chemistry and motions have been unchanged since they were formed more than 10 Gyr ago. According to the Sloan Digital Sky survey's SEGUE project, there are over 5000 giant stars, some of them as far away as 100 kiloparsecs (kpc; for comparison, the Milky Way's brightest satellite companion galaxies, the Magellanic Clouds, are only 50 kpc away). In the present paper, we calculate the kinematical parameters and the rotational constants for a sample of halo red giants.

The structure of the paper is as follows: Section 2 describes the observational data. Section 3 is devoted to calculating the kinematical parameters of the sample. The galactic rotational constants are determined in section 4. The conclusion is given in Section 5.

2. *Observational data*. The sample of halo red giant stars in the halo fields is selected by [17] from the SEGUE-1 and SEGUE-2 surveys [18]; [19] and SDSS-III/SEGUE-2 [20]; [21] surveys. Both SEGUE surveys were spectroscopic extensions of SDSS, with the goal of acquiring broad-wavelength coverage, moderate-resolution $R \sim 2000$ optical spectra of stars in specific galactic populations. Carollo [22] used proper motions in combination with distance estimates and radial velocities to provide the information required to calculate the full space motions (the components of which are referred to as U, V, W) of our program stars with respect to the local standard of rest.

We retrieved data for 1444 stars including complete records of space velocities, radial velocity, proper motion, distance and metallicity. The effective temperature of the program stars ranges from 4266 K to 6330 K, the metallicity ranges from -2.29 to -0.69 and distances are up to 40 kpc from the Sun. Carollo [15] applied the corrections for the Sun's motion with respect to the local standard of rest when calculating the full space motions by adopting values from Mihalas & Binney [31]

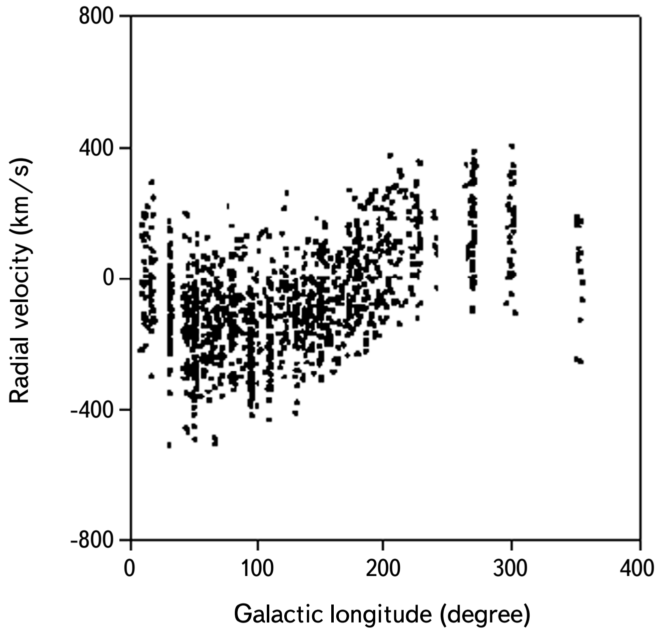

Fig.1. Distribution of observed radial velocities of the red giant stars at different galactic longitudes.

$(U, V, W) = (-9, 12, 7)\,\mathrm{km\,s^{-1}}$. Errors in the distances are taken to be 10% of the stated distance, the average error in the proper motions is ~1 $\mathrm{mas\,yr^{-1}}$ and the error in the radial velocities ranges between 0.7 and 4.8 $\mathrm{km\,s^{-1}}$.

We followed the procedure introduced by [15] and divided our sample into three small samples, an inner halo of stars $d \le 15$ kpc away, an outer halo for stars $d = 15$ - 20 kpc away and add another small sample for the stars near the galactic plane; $7 \le R \le 10$ kpc. Fig.1 displays the distribution of the radial velocities of the program stars with galactic longitudes. In Fig.2, we plot the space velocities U, V, and W as a function of the metallicity.

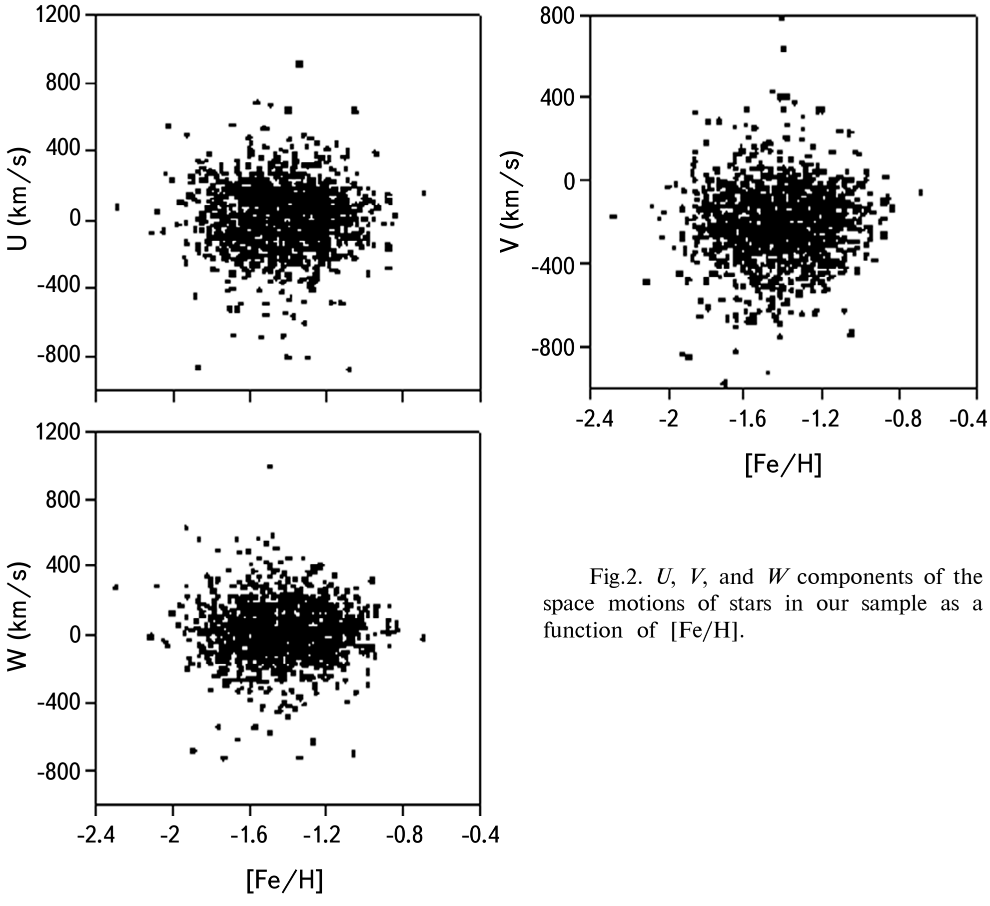

Fig.2. U, V, and W components of the space motions of stars in our sample as a function of [Fe/H].

3. *The kinematical model.* We follow the computational algorithm developed by [23] to compute the velocity ellipsoid parameters for the above data sample and solar elements. A brief explanation of the algorithm follows. The coordinates of the i-th star with respect to axes parallel to the original axes, but shifted to the center of the distribution, that is, into the points $\overline{U}$, $\overline{V}$ and $\overline{W}$,

will be $\left(U_i-\overline{U}\right)$, $\left(V_i-\overline{V}\right)$, $\left(W_i-\overline{W}\right)$ where U, V and W are the components of the space velocities $\overline{U}$, $\overline{V}$ and $\overline{W}$ are the mean velocities. This is defined as:

$$\overline{U}=\frac{1}{N}\sum_{i=1}^{N}U_i\,;\quad \overline{V}=\frac{1}{N}\sum_{i=1}^{N}V_i\,;\quad \overline{W}=\frac{1}{N}\sum_{i=1}^{N}W_i\,, \tag{1}$$

where N is the total number of stars and the components U, V and W can be computed by transformation formulae [24]. Then,

$$U=-0.0518807421V_x-0.8722226427V_y-0.4863497200V_z\,, \tag{2}$$

$$V=0.4846922369\,V_x-0.4477920852\,V_y+0.7513692061\,V_z\,, \tag{3}$$

$$W=-0.8731447899V_x-0.1967483417V_y+0.4459913295V_z\,. \tag{4}$$

Let ξ be an arbitrary axis, its zero point coincident with the center of the distribution, and let l, m and n be the direction cosines of the axis with respect to the shifted axis. The coordinates Q_i of the point i, with respect to the ξ axis are then given by:

$$Q_i=l\left(U_i-\overline{U}\right)+m\left(V_i-\overline{V}\right)+n\left(W_i-\overline{W}\right). \tag{5}$$

Let us adopt a generalization of the mean square deviation, as the measured of the scatter components Q_i, defined by

$$\sigma^2=\frac{1}{N}\sum_{i=1}^{N}Q_i^2\,. \tag{6}$$

From Equations (1), (5) and (6), we deduce that

$$\sigma^2=\underline{x}^T B\,\underline{x}\,, \tag{10}$$

where $\underline{x}$ is the (3×1) direction cosine vector and B is (3×3) symmetric matrix μ_{ij} with element

$$\begin{aligned}
\mu_{11}&=\frac{1}{N}\sum_{i=1}^{N}U_i^2-\overline{U}^2\,; &\quad \mu_{12}&=\frac{1}{N}\sum_{i=1}^{N}U_iV_i-\overline{U}\,\overline{V}\,;\\
\mu_{13}&=\frac{1}{N}\sum_{i=1}^{N}U_iW_i-\overline{U}\,\overline{W}\,; &\quad \mu_{22}&=\frac{1}{N}\sum_{i=1}^{N}V_i^2-\overline{V}^2\,;\\
\mu_{23}&=\frac{1}{N}\sum_{i=1}^{N}V_iW_i-\overline{V}\,\overline{W}\,; &\quad \mu_{33}&=\frac{1}{N}\sum_{i=1}^{N}W_i^2-\overline{W}^2\,.
\end{aligned} \tag{11}$$

The necessary conditions for an extremum are now

$$\left(B-\lambda\,I\right)\underline{x}=0\,. \tag{12}$$

These are three homogenous equations in three unknowns, which have a nontrivial solution if and only if

$$D(\lambda)=\left|B-\lambda\,I\right|=0\,, \tag{13}$$

where λ is the eigenvalue and $\underline{x}$ and B are:

$$\underline{x} = \begin{bmatrix} l \\ m \\ n \end{bmatrix} \quad \text{and} \quad B = \begin{vmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{23} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{vmatrix}.$$

Equation (13) is characteristic equation for the matrix B. The required roots (that is, the eigenvalues) are

$$\lambda_1 = 2\rho^{1/3}\cos\frac{\phi}{3} - \frac{k_1}{3}; \quad \lambda_2 = -\rho^{1/3}\left\{\cos\frac{\phi}{3} + \sqrt{3}\sin\frac{\phi}{3}\right\} - \frac{k_1}{3};$$
$$\lambda_3 = -\rho^{1/3}\left\{\cos\frac{\phi}{3} - \sqrt{3}\sin\frac{\phi}{3}\right\} - \frac{k_1}{3}. \tag{14}$$

where

$$k_1 = -(\mu_{11} + \mu_{22} + \mu_{33}), \quad k_2 = \mu_{11}\mu_{22} + \mu_{11}\mu_{33} + \mu_{22}\mu_{33} - \left(\mu_{12}^2 + \mu_{13}^2 + \mu_{23}^2\right),$$
$$k_3 = \mu_{12}^2\mu_{33} + \mu_{13}^2\mu_{22} + \mu_{23}^2\mu_{11} - \mu_{11}\mu_{22}\mu_{33} - 2\mu_{12}\mu_{13}\mu_{23}, \tag{15}$$

$$q = \frac{1}{3}k_2 - \frac{1}{9}k_1^2; \quad r = \frac{1}{6}(k_1k_2 - 3k_3) - \frac{1}{27}k_1^3, \tag{16}$$

$$\rho = \sqrt{-q^3}, \tag{17}$$

$$x = \rho^2 - r^2, \tag{18}$$

and

$$\phi = \tan^{-1}\left(\frac{\sqrt{x}}{r}\right). \tag{19}$$

Depending on the matrix that control the eigenvalue problem (9) for the velocity ellipsoid, we establish analytical expressions of some parameters for the correlation studies in terms of the matrix elements μ_{ij} of the eigenvalue problem for the velocity ellipsoid. The velocity dispersions σ_j; $j = 1, 2, 3$ could be given by

$$\sigma_j = \sqrt{\lambda_j}. \tag{20}$$

The center of the cluster can be derived by simply finding the equatorial coordinates of the center of mass for the number N_i of discrete objects, that is

$$x_c = \left[\sum_{i=1}^{N} r_i \cos\alpha_i \cos\delta_i\right] \Big/ N, \quad y_c = \left[\sum_{i=1}^{N} r_i \sin\alpha_i \cos\delta_i\right] \Big/ N, \quad z_c = \left[\sum_{i=1}^{N} r_i \cos\delta_i\right] \Big/ N. \tag{21}$$

The solar motion can be defined as the absolute value of the Sun's velocity relative to the group of stars under consideration,

$$S_{\odot} = \left(\overline{U}^2 + \overline{V}^2 + \overline{W}^2\right)^{1/2} \ \mathrm{km s^{-1}}. \tag{22}$$

The galactic longitude l_A and galactic latitude b_A of the solar apex are

$$l_A = \tan^{-1}\left(-\overline{V}/\overline{U}\right), \tag{23}$$

$$b_A = \sin^{-1}\left(-\overline{W}/S_{\odot}\right). \tag{24}$$

These three parameters may be called elements of solar motion with respect to a group under consideration.

We computed the kinematical parameters and solar motion for the three subsamples (the inner halo, outer halo and stars near the galactic plane). The results are listed in Table 1, in which row 1 is the total number of stars in each class; rows 2, 3 and 4 are the average space velocities due to galactic coordinates; rows 5, 6 and 7 are the eigenvalues; rows 8, 9 and 10 are devoted to dispersion velocities; rows 11, 12 and 13 are the direction cosines and rows 14, 15 and 16 give the solar elements.

Table 1

VELOCITY ELLIPSOID AND SOLAR VELOCITY FOR THE THREE SUBSAMPLES

Parameters	Inner $d \le 15$ kpc 926 stars	Outer $d = 15 - 20$ kpc 518 stars	Galactic plane $10 \ge R \ge 7$ kpc 160 stars
$\overline{U}$ (km/s)	15.05 ± 3.88	-7.19 ± 2.68	22.45 ± 4.74
$\overline{V}$ (km/s)	-212.20 ± 14.57	-208.93 ± 14.45	-210.36 ± 14.50
$\overline{W}$ (km/s)	16.28 ± 4.03	21.31 ± 4.62	32.93 ± 5.74
λ_1 (km/s)	70681.7	81799.7	79029.4
λ_2 (km/s)	35148.0	47588.5	36182.6
λ_3 (km/s)	19123.3	30402.6	19373.3
σ_1 (km/s)	265.86	286.01	281.12
σ_2 (km/s)	187.48	218.15	190.22
σ_3 (km/s)	138.29	174.36	139.19
$(l_1, m_1, n_1)_{\deg}$	0.056, -1.00, 0.086	0.00, -0.990, 0.144	0.103, -0.992, -0.070
$(l_2, m_2, n_2)_{\deg}$	-0.92, -0.085, -0.390	-0.906, -0.062, -0.419	-0.697, -0.021, -0.717
$(l_3, m_3, n_3)_{\deg}$	0.40, -0.058, -0.917	0.423, -0.130, -0.897	0.710, -0.123, -0.693
$S_{\odot}$ (km/s)	213.36 ± 14.61	210.14 ± 14.50	214.10 ± 14.63
l_A	85.94	-88.03	83.91
b_A	-4.37	-5.82	-8.85

Table 1 shows that the velocity dispersions ($\sigma_1, \sigma_2, \sigma_3$) obey the inequalities $\sigma_1 > \sigma_2 > \sigma_3$ and they behave in a radially-elongated velocity ellipsoid. Chiba & Beers [25] obtained the same behavior in terms of the halo's kinematics. The longitude of the vertex of the velocity ellipsoid l_A^o calculated for our sample indicates that the principal axis points toward the galactic center.

4. *The galactic rotation constants*. The Oort constants can be related to circular velocity and thus the galaxy's potential is an axisymmetric approximation

[26]. The first proof of the existence of the differential galactic rotation was made by Oort [27,28]. Several calculations have since been made for the two Oort's constants, A and B.

To determine the rotation constant A, we follow two methods. The first uses the radial velocity V_r to show a double sine-wave variation with the galactic longitude with an amplitude that increases linearly with distance [29]:

$$V_r = -2A(R - R_0)\sin l \cos b + K, \tag{25}$$

where l and b are the longitude and latitude of the individual star, respectively; R_0 is the distance from the Sun to the galactic center and K can be interpreted as systematic motions of large stellar groupings and systematic errors in the radial velocities due to gravitational redshift, motions within stellar atmospheres and erroneous wavelength systems [30].

The radial distance of the star from the galactic center R (the cylindrical radius vector) is given by

$$R^2 = R_0^2 + d^2 - 2R_0 d \cos l. \tag{26}$$

We calculated the Oort constant A for the three subsamples (the inner halo, outer halo and stars near the galactic plane). The results are listed in Table 2, in which column 1 is the first Oort constant computed from the least squares fit to Equation (25), column 2 is the K term and column 3 is the second Oort constant computed using the relation $(\sigma_2/\sigma_1)^2 = -B/(A-B)$ [31].

Table 2

ROTATION CONSTANTS FOR THE THREE SUBSAMPLES

Parameters	A (km s^{-1} kpc^{-1})	K-term (km s^{-1})	B (km s^{-1} kpc^{-1})	σ_2/σ_1
Galactic plane	16.723 ± 1.81	-2.30 ± 0.37	-13.610	0.68
Inner halo	14.592 ± 1.76	-2.861 ± 0.37	-14.360	0.70
Outer halo	14.930 ± 1.82	-2.78 ± 0.37	-20.420	0.76

The Oort constants can be connected to the local angular velocity through the relation $|A-B|$. According to the present result, $|A-B| = 29.5 \pm 0.2$ km s^{-1} kpc^{-1}. This result agrees with [31] for the red giants $|A-B| = 29.6 \pm 1$ km s^{-1} kpc^{-1} but differs from the results in the works listed in Table 3. Table 3 lists the Oort constants calculated by different authors. The rotational velocities in column 4 are calculated assuming that $R_0 = 7.5$ kpc. The negative K-terms for the three program stars do not differ significantly from zero. These values differ from many authors' findings for early-type stars and showed significant values of the K-term.

5. *Conclusion*. In this work, we calculate the kinematical parameters and

Table 3

ADOPTED OORT CONSTANTS AND COMPARISON WITH PREVIOUS WORKS

Origin	A km s^{-1} kpc^{-1}	B km s^{-1} kpc^{-1}	$\|A - B\|$ km s^{-1} kpc^{-1}	Rotational velocity km s^{-1}
Oort [27,28]	19	-24	33	247.5
Kerr & Lynden-Bell [33]	14.4 ± 1.2	-12.0 ± 2.8	26.4	198.5
Comeron et al. [34]	12.9 ± 0.7	-16.9 ± 1.1	29.8	223.5
Feast a Whitelock [35]	14.82 ± 0.84	-12.37 ± 0.64	27.19	203.9
Olling & Dehnen [36]	15.9 ± 2	-16.9 ± 2	32.8	246
R.Branham [37]	16.08 ± 0.72	-10.74 ± 0.65	26.78	200.8
R.Branham, [38]	14.85 ± 7.47	-10.85 ± 6.83	25.43	190.7
Bovy [39]	15.3 ± 0.4	-11.9 ± 0.4	27.2	204
Chengdong et al. [32]	15.1 ± 0.1	-13.4 ± 0.1	28.5	213.7
This work	15.6 ± 1.6	-13.9 ± 1.8	29.5	221.2

the Oort constants with a sample of 1444 red giants from SEGUE-1 and SEGUE-2 surveys. We divided the sample into three subsamples: the inner halo, the outer halo and the stars near the galactic plane. The velocity dispersions, projected distances and solar velocities for each subsample are computed. The derived velocity dispersions $(\sigma_1, \sigma_2, \sigma_3)$ for the two halo components are as follows: inner halo $= (265.86,\ 187.48,\ 138.29)$ km s^{-1}, outer halo $= (286.01,\ 218.15,\ 174.36)$ km s^{-1} and stars near the galactic plane $= (281.12,\ 190.22,\ 139.19)$ km s^{-1}. The solar velocities in km s^{-1} are 213.4 ± 14.6, 210.14 ± 14.5 and 214.1 ± 14.6 for the inner halo, outer halo and galactic plane, respectively.

We adopt the Oort constants $A = 15.6 \pm 1.6$ km s^{-1} kpc^{-1} and $B = -13.9 \pm 1.8$ km s^{-1} kpc^{-1} and angular velocity $|A - B| = 29.5 \pm 0.2$ km s^{-1} kpc^{-1}, which implies a rotational velocity of 221.25 ± 26.66 km s^{-1} assuming the distance to the Galactic center as 7.5 kpc. Our results indicate that the rotation curve $-(A + B)$ equals -1.7 ± 0.3 km s^{-1} kpc^{-1}, which indicates that the gradient of the rotation curve is negative and the circular velocity decreases in the galactic halo.

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[1] Physics Dept., College of Science, Northern Border University, Arar, Saudi Arabia, e-mail: abdo_nouh@hotmail.com
[2] Astronomy Dept., National Research Institute of Astronomy and Geophysics (NRIAG), 11421, Helwan, Cairo, Egypt (Affiliation ID: 60030681)
[3] Physics Dept., Faculty of Science and Arts, Northern Border University, Rafha Branch, Saudi Arabia, e-mail: welsanhoury@gmail.com

КИНЕМАТИКА И ЭЛЛИПСОИД СКОРОСТЕЙ КРАСНЫХ ГИГАНТОВ ГАЛО

М.И.НОУХ, В.Х.ЭЛЬСАНХУРИ

В настоящей статье определены кинематические свойства, эллипсоид скоростей и константы Оорта с помощью выборки красных гигантов гало. Исследование основано главным образом на пространственных и лучевых скоростях около 1583 красных гигантских звезд, отобранных из обзоров SEGUE-1 и SEGUE-2. Мы разделили выборку на три подвыборки: звезды внутреннего гало, звезды внешного гало и звезды около плоскости галактики. Подгонка уравнения лучевой скорости дает среднее значение для постоянных Оорта, $A = 15.6 \pm 1.6$ км $с^{-1}$ $кпк^{-1}$ и $B = -13.9 \pm 1.8$ км $с^{-1}$ $кпк^{-1}$, угловая скорость $|A - B| = 29.5 \pm 0.2$ км $с^{-1}$ $кпк^{-1}$, что соответствует скорости вращения 221.25 ± 26.66 км $с^{-1}$, если мы примем расстояние до центра Галактики 7.5 кпк.

Ключевые слова: *звездное гало: элементы Солнца: кинематический анализ: постоянные Оорта*

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