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Study of Two-Dimensional Phase Objects by Diffraction Focusing of Cylindrical X-Ray Wave on a Superlattice With Variable Period

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Abstract. The possibility of studying the internal structure of two-dimensional phase objects at diffraction focusing of X-ray cylindrical wave is investigated. As a crystal analyzer a superlattice with a period, variable along the axis of the cylinder was used in symmetric Laue geometry case. It is shown that by moving the phase object along the directions of the diffraction vector and the axis of the cylinder and recording the entire map of the obtained data, one can restore the additional phase of the wave, acquired in the phase object which in the end will afford an opportunity to restore the internal structure of the corresponding section of the phase object.

Keywords: cylindrical X-ray wave, focusing, superlattice, phase object

1. Introduction

The study of the internal structure of weakly absorbing noncrystalline materials, as well as of biological samples, is usually carried out using the method of $X-$ray phase contrast [1].

It is known that at incidence of monochromatic spherical $X-$ray wave on a crystal in the Laue geometry, the diffracted radiation is focused inside the crystal, as well as behind the crystal, in vacuum [2–5].

In [6] the possibility of reconstructing the internal structure of one-dimensional phase objects at diffraction focusing of $X-$ray spherical wave was investigated. As a crystal analyzer, a strongly absorbing wedge-shaped superlattice (SL) with a rib parallel to the diffraction vector was used in symmetric Laue geometry case.

2. Theoretical Approaches

Let prior to the incidence on the crystal the radiation from a point source S at L distance from the crystal passes through phase object (PO) (Fig.1), in which it is refracted according to the optical law $\sin\alpha/\sin\beta = n = 1-\delta$, where α is the angle of incidence, β is the angle of refraction, δ is the unit decrement of the refractive index n. The value of δ is directly proportional to the density of the material and for hard $X-$ray radiation is of the order of 10^{-5}. According to the presented law of refraction, it is not difficult to estimate the change in the direction of the refracted ray $\gamma = \beta - \alpha \approx \delta \tan\alpha$, which is also of the order of 10^{-5} for not very large angles of incidence.

It is assumed that phase object is a weakly absorbing medium, i. e., the intensity of beam at its propagation through that is practically changeless and only its phase is changed. The coordinate system was selected so that the origin of coordinates O on the surface of the crystal was the point of ray incidence at the exact Bragg angle θ_B, the entrance surface of the crystal coincided with $z = 0$ plane, and reflecting planes corresponded to $x = \text{const}$ planes, y-axis being perpendicular to the scattering plane (Fig.1).

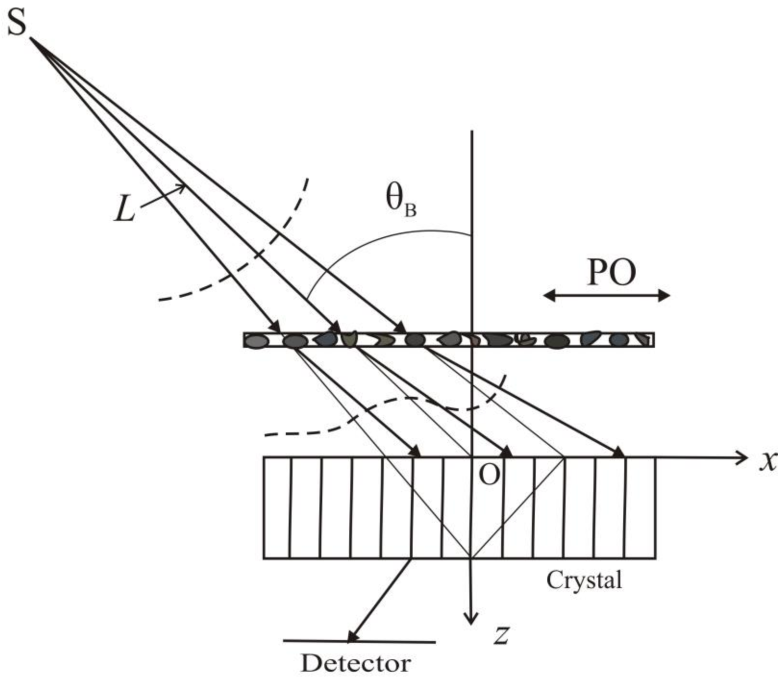

Fig. 1. Scheme of diffraction. The dotted lines represent the wave front before and after PO

It is known that a characteristic property of the diffraction of X – rays on SL and modulated structures is the presence of satellites around the main diffraction maximum, the position of which is determined by the lattice parameter averaged over the SL period (Fig.2).

The angular distance $(\theta_B)_m$ between the main maximum and the mth satellite is determined by the formula $(\theta_B)_m = m/(kz_0)$, where m is an integer, z_0 is the SL period, and k is the wave number in vacuum. When $z_0 \ll \overline{\Lambda}$, where $\overline{\Lambda}$ is the extinction length of the crystal averaged over SL period, the diffraction pattern is a system of nonoverlapping satellites. By choosing the parameters of the SL, the condition $z_0 \ll \overline{\Lambda}$ may be weakened so that the nonoverlapping diffraction satellites are located as close as possible to each other to cover the largest possible angular interval.

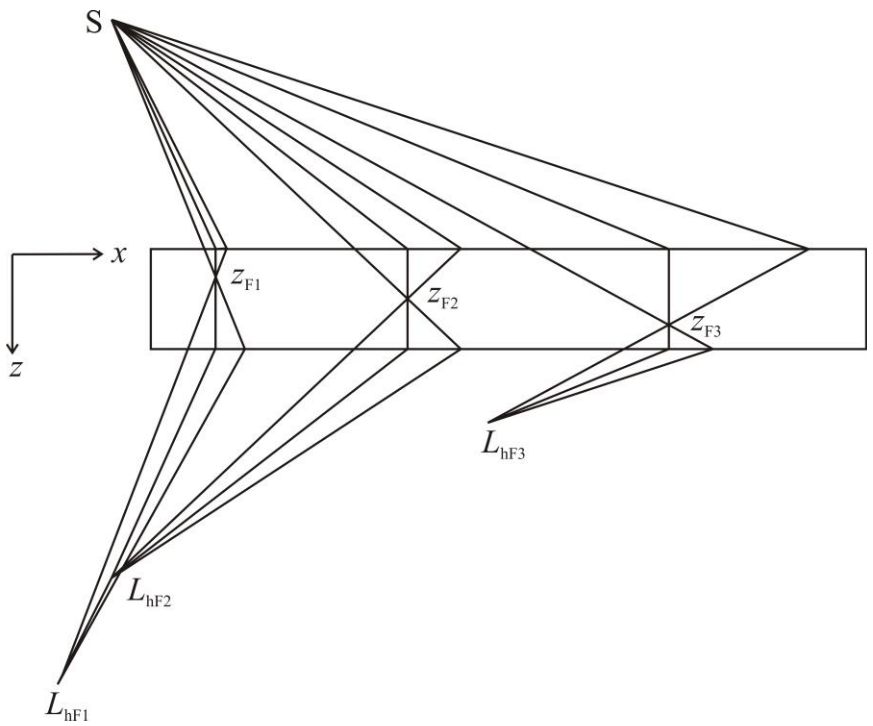

Fig. 2. Geometry of diffraction focusing of satellites with different structural factors

In this work the possibility of studying the internal structure of two-dimensional phase objects by means of diffraction focusing of a cylindrical $X-$ray wave in a superlattice with a variable period is investigated.

The choice of the cylindrical shape of the incident radiation is due to the fact that, on the one hand, owing to the two-dimensional character of diffraction in Laue geometry, the diffracted radiation of a monochromatic cylindrical wave, as well as a spherical wave, is focused both inside the crystal and behind the crystal in vacuum. On the other hand, in the case of a cylindrical wave, unlike a spherical one, the diffraction plane will not change, which is very important if moving the phase object along the cylinder axis.

It is proposed to use superlattice with a period, variable along the axis of the cylinder as a crystal analyzer (Fig.3). This choice is due to the following reason - the variable period of the SL will increase the variants of the angular distances of the satellites, which, at the transverse movement of the phase object, will provide possible matches of reflexes.

Since a cylindrical wave incident on the SL in the absence of PO, in the plane of incidence comprises every possible direction near the angles of the main maximum and satellites, it will focus in these directions.

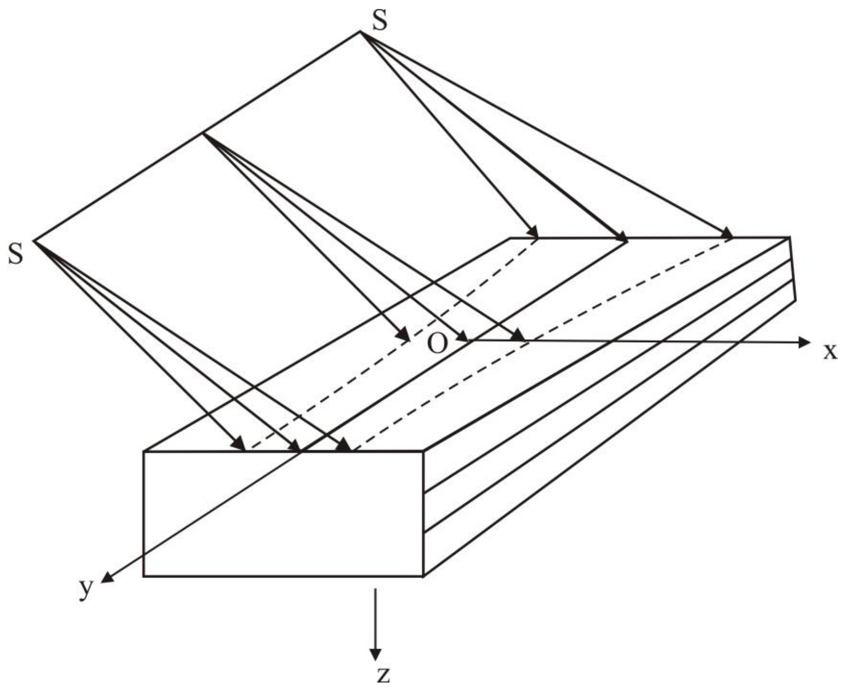

Fig. 3. The geometry of the linear X-ray source SS relative to the SL.
The dotted lines show the central trajectories of the satellites on the surface of the SL

If the PO is placed in front of the SL, then for the incident radiation the focusing conditions are violated, since as a result of refraction on inhomogeneities the rays change their directions and the wave front is deformed. The change of the direction of rays is mathematically represented by the presence of $\partial\varphi(x,y)/\partial x$ term in the wave phase, and the deformation of front – by $\partial^2\varphi(x,y)/\partial x^2$, where x is the coordinate in the direction of the diffraction vector, y is the coordinate in the transverse direction, $\varphi(x,y)$ is the additional phase function of the wave, that was acquired at the propagation through the PO. Here $\partial\varphi(x,y)/\partial x = k\Delta\theta(x,y)$, where $\Delta\theta_{\mathrm{m}}(\mathrm{x})$ is local angular displacement of the incident radiation due to the refraction on inhomogeneities of PO, $\partial^2\varphi(x,y)/\partial x^2$ is related to the local variation of the wave front curvature owing to the refraction in PO. The change in the direction of the refracted ray $\gamma \approx \delta \tan\alpha$ is directly proportional to the quantity δ, which in turn is directly proportional to the density of local inhomogeneity in the PO.

It was shown in [7], that for a short-period SL the diffraction pattern is a system of non-overlapping satellites and within the limits of the $m-$th reflection one may consider the SL as an ideal crystal with a modified Fourier component of the crystal polarizability:

$$\chi_{hm} = M_m \overline{\chi}_h \tag{1}$$

where m is the satellite number, $\bar{\chi}_h$ is the Fourier component of the polarizability of crystal averaged over SL period and M_m is the model-dependent structural factor of the SL.

The intensity of diffracted radiation in vacuum at L_{2m} distance from the crystal is expressed by the following formula:

$$I_{hm}(x,y,L_{2m}) = \frac{\exp\left(-\frac{\bar{\mu}D}{\cos\theta_m}(1-CM_m\frac{|\bar{\chi}_{hi}|}{\bar{\chi}_{0i}})\right)}{4(L_{1m}+L_{2m})\Gamma_m\left((\frac{L_{1m}+L_{2m}}{\Gamma_m}-D+\varphi''_{xx}(x,y)D\frac{\lambda(L_{1m}+L_{2m})}{2\pi\cos^2\theta_m})^2+(D\left|\frac{\bar{\chi}_{hi}}{\bar{\chi}_{hr}}\right|)^2\right)^{1/2}}$$
$$\times\exp\left(-\pi\frac{DCM_m|\bar{\chi}_{hi}|}{\lambda\cos\theta_m}\frac{(x+\varphi'_x(x,y)\frac{\lambda(L_{1m}+L_{2m})}{2\pi\cos^2\theta_m})^2\cot^2\theta_m}{(\frac{L_{1m}+L_{2m}}{\Gamma_m}-D+\varphi''_{xx}(x,y)D\frac{\lambda(L_{1m}+L_{2m})}{2\pi\cos^2\theta_m})^2+(D\left|\frac{\bar{\chi}_{hi}}{\bar{\chi}_{hr}}\right|)^2}\right) \tag{2}$$

where $\bar{\mu}=2\pi\bar{\chi}_{0i}/\lambda$ is the coefficient of linear absorption of $X-$rays in the crystal, averaged over the period of the SL, L_{1m} is the source-crystal distance for the $m-$th satellite, L_{2m} is the distance of the $m-$th satellite in vacuum, D is the thickness of the crystal, C is the polarization factor, λ is the incident wavelength,

$$\Gamma_m = \frac{\sin\theta_m \sin 2\theta_m}{M_m C|\bar{\chi}_{hr}|} \tag{2}$$

Suppose that individual inhomogeneities in the scattering plane are of the order of the extinction length averaged over the period of the SL, i.e. $\bar{\Lambda}\sim 10^{-3}\div 10^{-2}cm$. Densities of individual heterogeneities can be different, but within the limits of one local heterogeneity it can be considered unchanged. Then, the value γ within the limits of an individual reflex also will not change, which in spatial regions having dimensions of the order of $\bar{\Lambda}$, will ensure the presence of all possible directions of incident radiation near the direction of the corresponding diffraction maximum, and therefore the focusing of radiation in this direction.

For SL with $z_0\sim 10^4 cm$ the angular distance of neighboring satellites for hard $X-$ray radiation will be of the order of 10^{-4}. As noted above, in the absence of phase object, the radiation incident from a point source has all possible directions near the Bragg angles θ_B and diffraction satellites θ_m so the focusing conditions are satisfied in all these directions. At the propagation of radiation through phase object, the conditions of focusing are violated, since as a result of refraction on inhomogeneities the rays change their directions and the wave front is deformed.

If for any section the change in the direction of the refracted ray is equal to γ, then moving the PO in the opposite direction by this angle will restore the focusing condition for this section. At the same time, in the angular interval between neighboring satellites in places where there was no focusing in the absence of PO, the presence of phase object can lead to the appearance of new sections that can satisfy one of the noticed above reflexes and will be recorded. It should be noted that the size of the recorded inhomogeneity in the scattering plane should be no less than the averaged extinction length, which will ensure the diverging character of the radiation necessary for focusing.

At the propagation of radiation behind the crystal, the diffracted beam would once again focus in vacuum. By measuring the transverse width and coordinates of the reflex behind the crystal at two different distances, we can determine which satellite it corresponds to and calculate its focal length.

3. Conclusions

At the dynamical diffraction of X – ray cylindrical wave on short period SL with variable period, the focusing of satellites occurs both at different depths inside the crystal, and in vacuum at different distances from the crystal depending on the structural factors of the superlattice. The presence of a phase object can both violate the focusing condition of some satellites and lead to the appearance of new sections that can satisfy one of the noticed above reflexes and will be recorded.

Moving the PO by certain steps along the directions x and y and by registering the entire map of the obtained data, one can restore the additional phase of the wave, acquired in the PO. According to the obtained data, it is possible to restore the internal structure of the corresponding section of the phase object.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

The authors equally contributed to all steps of the paper preparation.

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