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# Angular Distribution of Intensive Radiation from a Charge Rotating Around a Conductive Ball

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**Abstract.** The angular distribution of the radiation from a relativistic charged particle uniformly rotating about a conductive ball in its equatorial plane is studied. The magnetic permittivity of the ball is assumed to be one. The work is based on the corresponding exact analytic solutions of Maxwell's equations. The generalized Drude-Lorentz-Sommerfeld formula for the dielectric function of the conductive ball is used in numerical calculations. Earlier it was shown that localized and high-amplitude oscillations of the electromagnetic field can be generated at a given harmonic inside the ball at a certain (resonant) particle rotation frequency. Herewith, at large distances from the trajectory of the particle, these localized oscillations are accompanied by intense radiation at the same harmonic, which is many times more intense than the analogous radiation in the case when the ball is absent. The possibilities of using this phenomenon to develop sources of quasi-monochromatic electromagnetic radiation in the range from giga- to terahertz frequencies are discussed.

**Keywords:** synchrotron radiation, charged particle, conductive ball

## 1. Introduction

The presence of matter may essentially influence to the radiation of charged particle, which may have important practical applications. For example, in the presence of flat interfaces between media relativistic particle generates transition radiation [1,2]. Opportunities for radiation of a charged particle are expanded at the transition from open (flat) to a half-closed (cylindrically symmetric) or closed (spherically symmetric) boundary between two media.

It is important to mention that if the source of the field is moving near the boundary, in addition to the volume waves also surface waves (SW) can be generated on the boundary. Moreover, the phase velocity of the latter can be much smaller than that for the volume waves. This may have important practical applications.

This work is devoted to this topic. The electromagnetic field of the charged particles rotating around a metallic ball was studied in [3]. However, in [3] it was not investigated the possibility of SW generation inside a conductive ball. In [4] the radiation of a charged particle uniformly rotating around a conductive ball, in its equatorial plane has been investigated (see also [5]). The angular distribution of the radiation from a charged particle rotating along an equatorial orbit around a dielectric ball is studied in [6]. In this work the angular distribution of the radiation from a relativistic charged particle uniformly rotating about a conductive ball is studied.

## 2. The description of problem

Now consider the uniform rotation $\mathrm{v} = const$ of a relativistic electron in the equatorial plane of a conductive ball in the magnetic field, in empty space (see Fig.1).

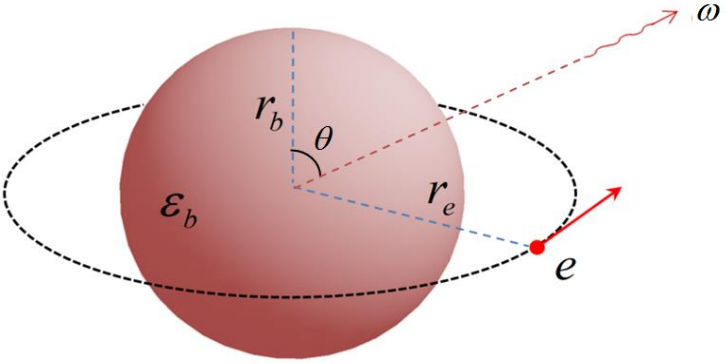


**Figure 1.** A relativistic electron rotating about a ball in its equatorial plane.

The permittivity of medium is the following step function of radial coordinate r:

$$\varepsilon(r) = \varepsilon_b + (1-\varepsilon_b)\Theta(r-r_b)\,, \tag{1}$$

where $r_b$ is the radius of ball and $\varepsilon_b = \varepsilon_b' + i\varepsilon_b''$ is the complex valued permittivity of the ball material and $\Theta(x)$ is the Heaviside step function. The magnetic permeability of the substance of the ball is taken to be unity. The uniform rotation of electron (with speed $\mathrm{v}$ and orbital radius $r_e$) entails radiation at some discrete cyclic frequencies (harmonics) $\omega_k = k\mathrm{v}/r_e$ with the harmonic number $k = 1,2,3\ldots$. It is convenient to introduce the following dimensionless quantity:

$$W_{kT} / \hbar\omega_k \equiv N_k \tag{2}$$

(the number of emitted quanta). Here $W_{kT}$ is the energy radiated at cyclic frequency $\omega_k$ during one period $T = 2\pi r_e / \mathrm{v}$ of electron gyration, and $\hbar\omega_k$ is the energy of corresponding electromagnetic wave quantum. Let us also introduce the angular distribution $n_k(\theta)$ of the number of emitted quanta determined by the equation

$$N_k = \int_0^\pi n_k(\theta) d\theta\,, \tag{3}$$

where $\theta$ is the corresponding polar angle.

It is known [7,8] that if the space as a whole is filled with a transparent and homogeneous substance (with constant dielectric permittivity $\varepsilon$) then

$$N_k(\infty;\mathrm{v},\varepsilon)=\frac{N_0}{\beta\sqrt{\varepsilon}}[2\beta^2 J'_{2k}(2k\beta)+(\beta^2-1)\int_0^{2k\beta}J_{2k}(x)dx], \tag{4}$$

$$n_k(\infty;\mathrm{v},\varepsilon)=\frac{N_0\cdot k}{\sqrt{\varepsilon}}\sin\theta\times[ctg^2(\theta)J_k^2(k\beta\sin\theta)+\beta^2 J_k'^2(k\beta\sin\theta)], \tag{5}$$

where $N_0=2\pi e^2/\hbar c\cong 0.0459$, $\beta=\mathrm{v}\sqrt{\varepsilon}/c$ and $J_k(x)$ is the Bessel function of integer order. The case $\varepsilon=1$ of these formulae corresponds to the synchrotron radiation in vacuum (see, e.g., [9,10]).

In [11-13] (see also [14,15]) the following expression was derived:

$$N_k(\mathrm{ball};\mathrm{v},x,\varepsilon_b)=\frac{2N_0}{k}\sum_{s=0}^{\infty}(|\,a_{kE}(s)\,|^2+|\,a_{kH}(s)\,|^2) \tag{6}$$

for the number of quanta emitted by the electron during one revolution around a dielectric ball. In this case the angular distribution of the number of emitted quanta is determined by the equation

$$n_k(\theta)=\frac{16\pi^2 N_0}{k}\sin\theta\cdot\left|\sum_{s=0}^{\infty}(-1)^s[a_{kE}(s)\vec{X}^{(2)}_{k+2s,k}(\theta,0)+i\cdot a_{kH}(s)\,\vec{X}^{(3)}_{k+2s+1,k}(\theta,0)]\right|^2, \tag{7}$$

where $\vec{X}^{(\mu)}_{lm}$ are spherical vectors of electric ($\mu=2$) and magnetic ($\mu=3$) types, $x=r_b/r_e<1$ and

$$a_{kE}(s)=kb_l(E)P_l^k(0)\sqrt{\frac{(l-k)!}{l(l+1)(2l+1)(l+k)!}}, \qquad l=k+2s,$$

$$a_{kH}(s)=b_l(H)\sqrt{\frac{(2l+1)(l-k)!}{l(l+1)(l+k)!}}\left.\frac{dP_l^k(y)}{dy}\right|_{y=0}, \qquad l=k+2s+1 \tag{8}$$

are dimensionless amplitudes describing the contributions of electric (E) and magnetic (H) type multipoles, respectively, $P_l^k(y)$ are the associated Legendre polynomials and $b_l(E)$, $b_l(H)$ are the following factors depending on $k,\mathrm{v},x$ and $\varepsilon_b$:

$$b_l(E)=(l+1)b_{l-1}(H)-lb_{l+1}(H)+x^{-2}(1-\varepsilon_b)[j_{\underline{l-1}}(xu_b)+j_{\underline{l+1}}(xu_b)][h_{\underline{l-1}}(u)+h_{\underline{l+1}}(u)]\frac{l(l+1)u_b\,j_l(xu_b)}{lz^l_{l-1}+(l+1)z^l_{l+1}}$$

$$b_l(H)=iu[j_l(u)-h_l(u)\frac{\{j_l(xu_b),j_l(xu)\}_x}{\{j_l(xu_b),h_l(xu)\}_x}], \qquad u=k\mathrm{v}/\mathrm{c}, \quad u_b=k\mathrm{v}\sqrt{\varepsilon_b}/c. \tag{9}$$

Here $h_l(y)=j_l(y)+i\mathrm{n}_l(y)$ and $j_l(y)$, $\mathrm{n}_l(y)$ are spherical Bessel and Neumann functions respectively. In (9) we used the following notations:

$$\{a(x\alpha),\ b(x\beta)\}_x \equiv a\frac{\partial b}{\partial x}-\frac{\partial a}{\partial x}b\,,\quad \underline{f}_l(x)\equiv f_l(x)/\{j_l(xu_b),h_l(xu)\}_x\,,$$

$$z_\nu^l \equiv \frac{uj_\nu(xu_b)h_l(xu)\varepsilon_b-u_b j_l(xu_b)h_\nu(xu)}{uj_\nu(xu_b)h_l(xu)-u_b j_l(xu_b)h_\nu(xu)}\,. \tag{10}$$

In the absence of dielectric ball ($\varepsilon_b=1$) the calculations by means of our formulae (6), (7) give the same results as those obtained using the well-known synchrotron radiation theory formulae (4), (5) where $\varepsilon=1$.

The simple analytical function $\varepsilon_b(\omega)$ often used to describe the dispersion law for the substance of conductive ball has the form

$$\varepsilon_b(\omega)=\varepsilon_0-\frac{\omega_p^2}{\omega^2+i\gamma\omega}=\varepsilon_b'(\omega)+i\varepsilon_b''(\omega) \tag{11}$$

(the generalized Drude-Lorentz-Sommerfeld formula). This expression satisfactorily describes the dielectric function of noble metals. For example, for gold [16]

$$\varepsilon_0^{Au}=9.84,\ \hbar\omega_p^{Au}=9.01eV,\quad \hbar\gamma^{Au}=0.072eV\,. \tag{12}$$

The effective parameter $\varepsilon_0>1$ describes the contribution of bound electrons, $\omega_p$ is the effective bulk plasma frequency which is associated with effective concentration of free electrons, $\gamma$ is the phenomenological damping constant of the electron motion.

We will consider electromagnetic oscillations in the frequency range for which

$$\varepsilon_b'(\omega)<0\,. \tag{13}$$

In this case, the generated electromagnetic oscillations inside the ball must be localized.

## 3. Numerical results in the gigahertz frequency range

We assumed that (a) the ball is made of a dielectric with a negligible mixture of gold, so that the plasma frequency of the free charge carriers is

$$\omega_p=3\cdot 10^{10}\,Hz \tag{14}$$

($\omega_p^{Au} = 1.4 \cdot 10^{16}\, Hz$). In this case, (b) the dielectric should have a weak dispersion and should absorb light slightly in the gigahertz (GHz) frequency range. For this purpose melted quartz can be used with the dielectric permittivity of [17]

$$\varepsilon_{SiO_2} = 3.78(1 + 0.0001i) \tag{15}$$

in the mentioned frequency range. The parameter $\varepsilon_0$ in (11) we identified by $\varepsilon_{SiO_2}$ because of the small concentration of gold in the substance of the ball. In numerical calculations, two estimated values of parameter $\gamma / \omega_p = 1/125; 1/500$ have been used, where $1/125 \approx \gamma^{Au} / \omega_p^{Au}$ (the ball is entirely made of gold).

Thus, the dielectric function $\varepsilon_b(\omega)$ was calculated from formula (11) with the following values of the parameters

$$\varepsilon_0 \approx \varepsilon_{SiO_2}, \quad \omega_p = 3 \cdot 10^{10}\, Hz, \qquad \gamma / \omega_p = 1/125; 1/500 . \tag{16}$$

The results of numerical calculations are shown in Figs. 2–5.

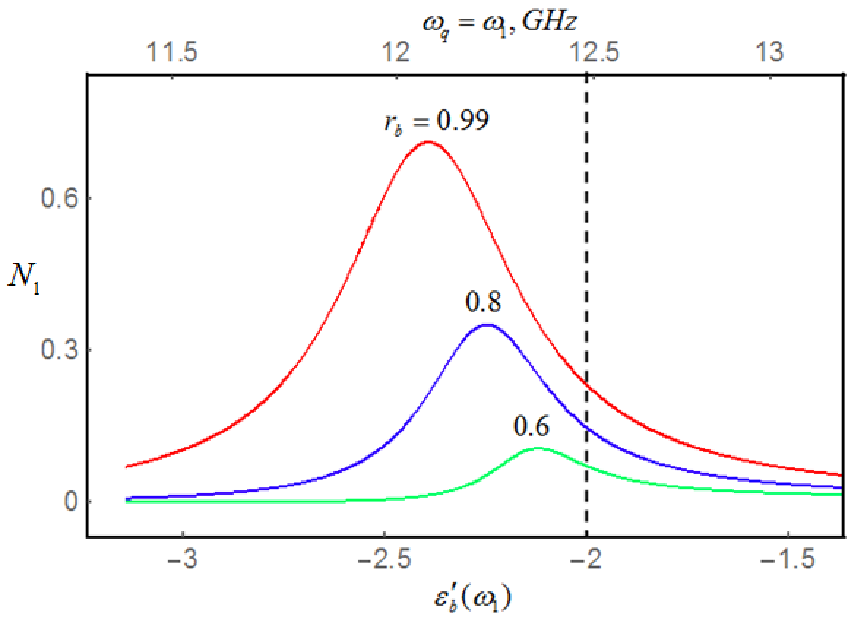


**Figure 2.** The number of quanta of the electromagnetic field $N_1$ emitted by the electron at the first harmonic during its one revolution about the conductive ball. Along the axis of abscissa (a) the values of $\omega_q = \omega_1$ (the upper part of the figure) and (b) the values of the real part $\varepsilon_b'(\omega_1)$ of the dielectric constant of the ball (the lower part of the figure) are plotted. The radius of the electron orbit is $r_q = 1$ cm, $\gamma / \omega_p = 1/125$. Near the curves, the values of radius of the ball are indicated.

Comparing the course of the curves with $r_b =$ 0.99, 0.8 and 0.6 cm in Fig. 2, we arrive at the following conclusions:

1) The electron generates very intense radiation ($N_1 >> e^2/c\hbar$) if it rotates about a conductive ball at a certain (resonant) frequencies;
2) The resonant frequency $\omega_q^{res}$ with which an electron rotates about a conductive ball depends on the ball radius $r_b$;
3) As the radius of the conductive ball $r_b$ (a) decreases, the maximum of the function $N_1$ decreases rapidly and (b) the value of the real part of the dielectric function of the ball $\varepsilon_b'$ corresponding to this maximum tends to -2.

In Fig.3 the dependence of angular distribution of the number of electromagnetic field quanta $n_1$ generated by the electron during its one rotation around the ball, on the rotation cyclic frequency $\omega_q$ is presented.

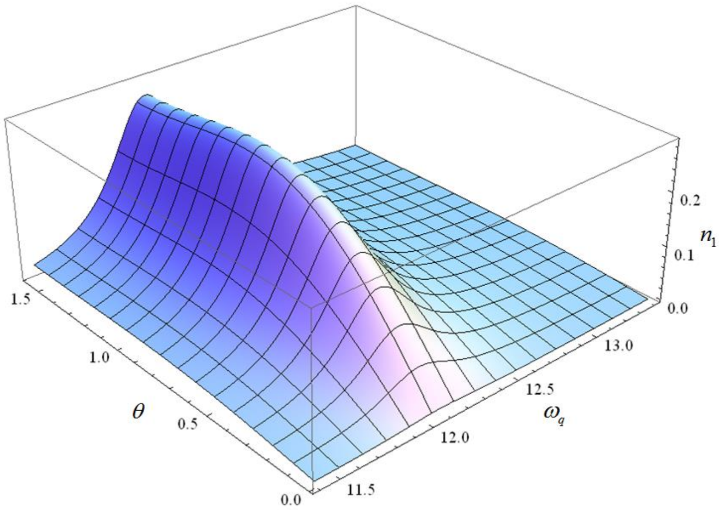


**Figure 3.** The angular distribution of the number of electromagnetic field quanta $n_1$ generated by the electron at the first harmonic (during its one rotation around the ball) as a function of rotation cyclic frequency $\omega_q$, $r_b = 0.99$.

It can be seen from Fig.3 that the radiation intensity is just about evenly distributed in the region $\pi/4 < \theta \le \pi/2$ and tends to zero at $\theta \to 0$.

Similar numerical calculations have been done for the second harmonic and for $\gamma/\omega_p = 1/500$. The results are presented in Figs. 4 and 5.

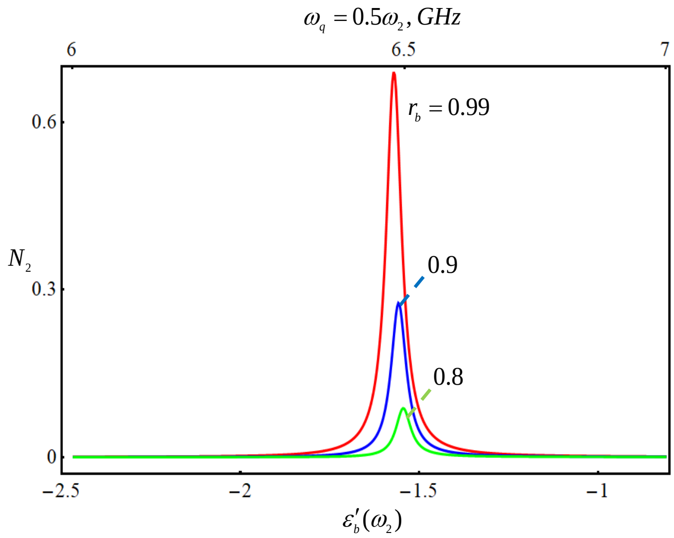


**Figure 4.** The same as in Fig.2 for the 2-nd harmonic, $\gamma/\omega_p = 1/500$.

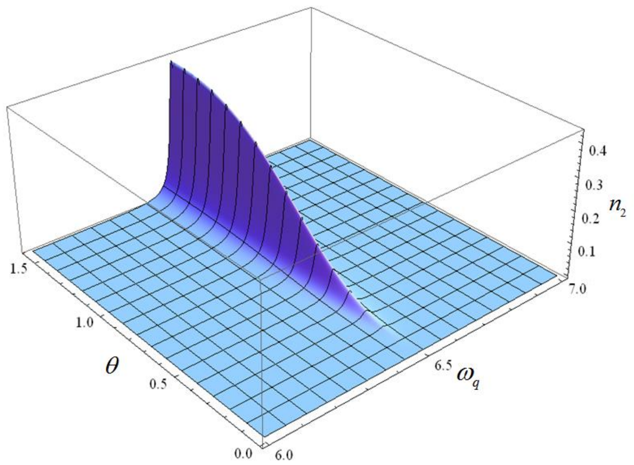


**Figure 5.** The same as in Fig.3 for the 2-nd harmonic, $\gamma/\omega_p = 1/500$, $r_b = 0.99$.

Comparing Figs. 4 and 5 with Figs. 2 and 3, we arrive at the following conclusions:

4) For the second harmonic, the peaks are significantly narrower and at decreasing ball radius $r_b$, the maximum of the function $N_2$ decreases more rapidly (Fig.4).
5) The angular density of radiation intensity for the 2-nd harmonic is maximal for $\theta = \pi/2$ and rapidly tends to zero at $\theta \to 0$.

## 4. Conclusions

The angular distribution of the intensive radiation of a charged particle uniformly rotating around a conductive ball, in its equatorial plane has been investigated, taking into account the dispersion of electromagnetic waves inside a conductive ball. It is shown that

1. Charged particle can generate powerful radiation if revolves around a conductive ball at a specific resonant frequency. In [4] it was shown that generation of powerful radiation is due to the fact that at a certain (resonant) particle rotation frequency, localized oscillations of a high-amplitude electromagnetic field (Surface Plasma Waves (SPW)) can be generated inside the ball. Herewith, at large distances from the trajectory of the particle, these localized oscillations are accompanied by intense radiation, which is many times more intense than the analogous radiation in the case when the ball is absent.

2. The angular density of this "resonant" radiation intensity is maximal for $\theta \cong \pi/2$ and tends to zero at $\theta \to 0$.

3. The linear dimensions of the resonators for SPW may be many times smaller than the linear dimensions of the cavities for the bulk electromagnetic waves. This fact may be used to develop powerful sources of electromagnetic radiation of small sizes.

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## Author Contributions

The authors equally contributed to all steps of the paper preparation.

## Conflicts of interest

The authors declare no conflict of interest.

### References

[1] G.M. Garibian, Yan Shi, X-Ray Transition Radiation (AN Arm. SSR Press, Yerevan, 1983) (in Russian).
[2] V.L. Ginzburg, V.N. Tsytovich, Transition Radiation and Transition Scattering (Adam Hilger, Bristol, 1990).
[3] M.R. Magomedov, Izv. Akad. Nauk Arm. SSR, Fiz., 4 (1969) 271 (in Russian).
[4] A.H. Mkrtchyan, L.Sh.Grigoryan, H.F. Khachatryan, M.L. Grigoryan, A.V. Sargsyan, Resource-Efficient Technologies **3** (2018) 1.
[5] M. Torabi, B. Shokri, Physics of Plasmas **24** (2017) 013114.
[6] L.Sh. Grigoryan, A.A. Saharian, H.F. Khachatryan, M.L. Grigoryan, A.V. Sargsyan, T.A. Petrosyan, JINST **15** (2020) C04035.
[7] V.P. Zrelov, Vavilov-Cherenkov Radiation and its Applications in High Energy Physics (Atomizdat, Moscow, 1968) (in Russian).
[8] V.N. Tsytovich, Westnik MGU **11** (1951) 27 (in Russian).
[9] I.M. Ternov, V.V. Mikhailin, V.R. Khalilov, Synchrotron Radiation and its Applications (Harwood Academic, Amsterdam, 1985).
[10] A.A. Sokolov, I.M. Ternov, Radiation from Relativistic Electron (ATP, New York, 1986).
[11] S.R. Arzumanyan, L.Sh. Grigoryan, A.A. Saharian, Izv. Nats. Akad. Nauk Arm., Fiz. (Engl. Transl.: J. Contemp. Phys.) **30** (1995) 99.
[12] S.R. Arzumanyan, L.Sh. Grigoryan, A.A. Saharian, Kh.V. Kotanjian, Izv. Nats. Akad. Nauk Arm., Fiz. (Engl. Transl.: J. Contemp. Phys.) **30** (1995) 106.

[13] L.Sh. Grigoryan, H.F. Khachatryan, S.R. Arzumanyan, Izv. Nats. Akad. Nauk Arm., Fiz. (Engl. Transl.: J. Contemp. Phys.) **33** (1998) 267.
[14] L.Sh. Grigoryan, H.F. Khachatryan, S.R. Arzumanyan, Izv. Nats. Akad. Nauk Arm., Fiz. (Engl. Transl.: J. Contemp. Phys.) **37** (2002) 327.
[15] L.Sh. Grigoryan, H.F. Khachatryan, S.R. Arzumanyan, M.L. Grigoryan, Nucl. Instr. and Meth. B **252** (2006) 50.
[16] K. Kolwas, A.J. Derkachova, Quant. Spectrosc. & Radiat. Transfer. **114** (2013) 45.
[17] E.M. Voronkova, B.N. Grechushnikov, G.I. Distler, I.P. Petrov, Optical Materials for Infrared Technology (Nauka, Moscow, 1965) (in Russian).