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# Features of Radiation Generated by Bunches of Charged Particles Passing Through the Centre of a Ball

A.R. Mkrtchyan[1], P.A. Aleksandrov[2], L.Sh. Grigoryan[1], A.H. Mkrtchyan[1], H.F. Khachatryan[1], M.V. Kovalchuk[2]

[1] *Institute of Applied Problems of Physics NAS RA, 25 Hr. Nersissian Str., 0014 Yerevan, Republic of Armenia*
[2] *National research centre "Kurchatov institute", 1 Akademika Kurchatova pl., 23182 Moscow, Russia*
E-mail: levonshg@mail.ru

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**Abstract.** We investigate the radiation generated by a train of bunches crossing a ball. The expressions for the spectral and angular distributions of the radiation energy are obtained for a general case of the dispersion law for the material of the ball. The consideration is based on the corresponding exact analytical solutions of Maxwell's equations. The numerical results are given for a dielectric ball. The possibility for the appearance of strong peaks in the spectral distribution of the radiation energy is demonstrated. The wavelengths corresponding to these peaks are of the order of the ball radius. A visual explanation of this phenomenon is provided.

**Keywords:** Cherenkov radiation, charged particles, dielectric ball

## 1. Introduction

The presence of a medium may essentially influence on the characteristics of high-energy electromagnetic processes, giving rise to new types of phenomena [1-6]. Well known examples of this kind are the Cherenkov Radiation (CR) [1,2,7,8], the diffraction and transition [3,4] types of radiations. The operation of a large number of sources for generation of the electromagnetic radiation in various spectral ranges is based on the interaction of relativistic electrons with matter [5]. In particular, the interfaces separating different media can be used for the control of radiation fluxes emitted by various systems.

The Synchrotron Radiation (SR) of a charged particle rotating in a homogeneous medium has been considered in [7] (see also [2,8]), where it has been shown that the interference between SR and CR leads to nontrivial consequences. New features in the spectral-angular distribution of the radiation intensity appear in the case of a charge rotation in an inhomogeneous medium. Exactly solvable problems for the radiation from a charge rotating in spherically and cylindrically symmetric media have been considered in a series of papers, started in [9,10] (see also [11,12] and references therein).

The modern accelerators may generate relativistic and monoenergetic short bunches with submillimeter length and with a relatively large number of electrons ($\geq 10^9$). Such bunches allow to generate high-power quasi-coherent terahertz radiation, that is of considerable interest for applications in physics, chemistry and biology [13]. High intensity narrow-band and coherent terahertz CR generated by a bunch passing through a hollow channel inside a waveguide filled with a dielectric was directly observed in [14]. The use of a waveguide filled by dielectric plates with given separations between them leads to the further increase of the output power [15].

The theory predicts another possibility for a significant increase in the CR intensity. It is related to the fact that the recent accelerators allow to generate trains of bunches with submillimeter separations. Tuning the distance between bunches, in [16] a powerful resonant (or, so-called, parametric [17]) coherent terahertz CR has been observed on a single waveguide mode. A different choice for the separation of bunches, allowing the generation of resonant CR on a large number of neighboring waveguide modes, was proposed in [18]. A clear explanation of this phenomenon is given in [18].

The radiation from a relativistic electron, crossing a ball (with an arbitrary dielectric function $\varepsilon(\omega)$ and with magnetic permeability $\mu = 1$ ) through its center, was studied in [19,20] (see also the references therein). It was shown, that the spectral distribution of radiation generated by the relativistic particle inside a ball at specified frequencies is strongly influenced by the ball-vacuum boundary.

In the present paper, we investigate the radiation by a train of one-dimensional electron bunches passing through the center of a ball made of a dielectric, a conductor, or a composite material. The possibility for generation of quasi-coherent CR by a train of equidistant bunches of relativistic electrons is shown. The radiation energy is evaluated for the case of a dielectric ball.

## 2. Problem setup and the radiation intensity

We consider a train of electron bunches flying through the center of a ball with dielectric function $\varepsilon(\omega)$ and immersed in vacuum. The magnetic permeability for the material of the ball will be assumed to be unity (non-magnetic material). It is also assumed that there is an external field (for example, electric), which supports the uniform motion of particles inside the ball.

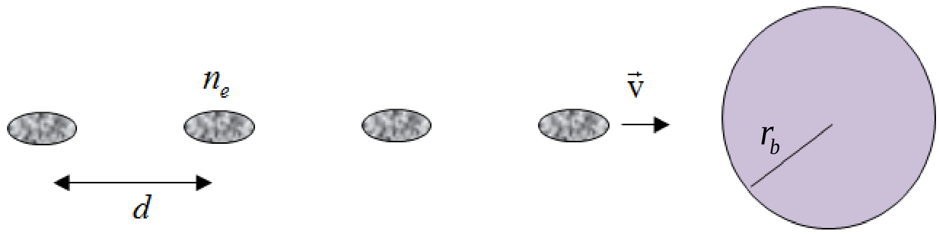


**Figure 1.** A train of electron bunches flying through the center of a ball.

In [9] a method is proposed for the evaluation of the Green function of the electromagnetic field in a medium consisting of $N \geq 1$ spherically-symmetric layers having a common center and different dielectric permittivities. Based on this method, a formula is derived for the energy of radiation from a charged particle moving along an arbitrary trajectory in such medium. The special cases of a charged particle rotating around or inside a dielectric ball have been investigated in [21,11] (see also the references given therein). Another example of a charged particle uniformly moving along a straight line through the center of a ball with the radius $r_b$ and the dielectric permittivity $\varepsilon_b$, immersed in a homogeneous medium with permittivity $\varepsilon_1$, is discussed in [19] (see Fig. 2).

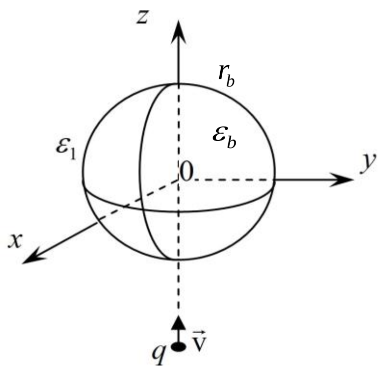


**Figure 2.** A particle flying from a medium into a ball.

Assuming that the trajectory of the particle is described by the equation $z = \mathrm{v}t$, the corresponding current density can be written as

$$\vec{j}(\vec{r},t) = \frac{q\vec{\mathrm{v}}}{\pi r^2 \sin\theta}\begin{cases} \delta(r-\mathrm{v}t)\delta(\theta) & \text{for} \quad t>0 \\ \delta(r+\mathrm{v}t)\delta(\theta-\pi) & \text{for} \quad t<0 \end{cases}, \tag{1}$$

where $q$ and $\mathrm{v}$ are the charge and velocity of the particle, and $\theta$ is the polar angle of spherical system of coordinates $r,\theta,\varphi$. For the corresponding Fourier transform one has

$$\vec{j}(\vec{r},\omega) = \frac{1}{2\pi}\int \vec{j}(\vec{r},t)e^{i\omega t}dt = \frac{q\vec{\mathrm{v}}}{2\pi^2 \mathrm{v}r^2\sin\theta}[\delta(\theta)e^{i\frac{\omega}{\mathrm{v}}r} + \delta(\theta-\pi)e^{-i\frac{\omega}{\mathrm{v}}r}]. \tag{2}$$

Expanding $\vec{j}(\vec{r},\omega)$ in spherical vectors $\vec{X}_{lm}^{(\mu)}$ [22], we can evaluate the corresponding expansion coefficients:

$$j_{\mu}^{lm}(r) \equiv \int \vec{j}(\vec{r},\omega)\vec{X}_{lm}^{(\mu)*}d\Omega. \tag{3}$$

As a result we find

$$j_1^{lm} = \frac{q\delta_{m0}}{2\pi r^2}\sqrt{\frac{2l+1}{4\pi}}[e^{i\frac{\omega}{\mathrm{v}}r} - (-1)^l e^{-i\frac{\omega}{\mathrm{v}}r}], \qquad j_2^{lm} = j_3^{lm} = 0. \tag{4}$$

Similarly, one can expand the vector potential $\vec{A}(\vec{r},\omega)$ of the electromagnetic field in spherical vectors $\vec{X}_{lm}^{(\mu)}$. In the Lorentz gauge the expansion coefficients $A_{\mu}^{lm}$ are given by the expressions

$$\frac{(2l+1)cr_b^2}{4\pi}\begin{bmatrix}A_1^{lm}(r)\\A_2^{lm}(r)\end{bmatrix}=\begin{bmatrix}\frac{lu_{\underline{l-1}}+(l+1)u_{\underline{l+1}}}{\sqrt{l(l+1)}(u_{\underline{l-1}}-u_{\underline{l+1}})}\end{bmatrix}_r^{lm}+\gamma_l[lB_{\underline{l-1}}^{lm}-(l+1)B_{\underline{l+1}}^{lm}]\begin{bmatrix}\frac{lP_{\underline{l-1}}+(l+1)P_{\underline{l+1}}}{\sqrt{l(l+1)}(P_{\underline{l-1}}-P_{\underline{l+1}})}\end{bmatrix}_{(r,r_b)} \tag{5}$$

(for details see [9,11]). The magnetic type multipoles ( $\mu=3$) are not generated and $A_3^{lm}(r)=0$. Here

$$u_{\underline{l_1}}^{lm}(r)\equiv\int_0^\infty P_{\underline{l_1}}(r,x)\,j_1^{lm}(x)x^2dx,$$
$$B_{\underline{l_1}}^{lm}=\frac{q\delta_{m0}}{2\pi}\sqrt{\frac{2l+1}{4\pi}}\{\lambda_0 j_l(\lambda_0 r_b)\int_{r_1}^\infty h_{\underline{l_1}}^{(1)}(\lambda_1 x)[e^{i\frac{\omega}{\mathrm{v}}x}-(-1)^l e^{-i\frac{\omega}{\mathrm{v}}x}]dx+$$
$$+\lambda_1 h_l^{(1)}(\lambda_1 r_b)\int_0^{r_b} j_{\underline{l_1}}(\lambda_0 x)[e^{i\frac{\omega}{\mathrm{v}}x}-(-1)^l e^{-i\frac{\omega}{\mathrm{v}}x}]dx\}, \tag{6}$$

with the notations

$$\gamma_l=\frac{1/\varepsilon_b-1/\varepsilon_1}{l z_{l-1}^l+(l+1)z_{l+1}^l},\quad \lambda_0=\frac{\omega}{c}\sqrt{\varepsilon_b},\quad \lambda_1=\frac{\omega}{c}\sqrt{\varepsilon_1},$$
$$z_\upsilon^l=\frac{\lambda_1 j_\upsilon(\lambda_0 r_b)h_l^{(1)}(\lambda_1 r_b)/\varepsilon_1-\lambda_0 j_l(\lambda_0 r_b)h_\upsilon^{(1)}(\lambda_1 r_b)/\varepsilon_b}{\lambda_1 j_\upsilon(\lambda_0 r_b)h_l^{(1)}(\lambda_1 r_b)-\lambda_0 j_l(\lambda_0 r_b)h_\upsilon^{(1)}(\lambda_1 r_b)} \tag{7}$$

and $h_l^{(1)}(y)=j_l(y)+in_l(y)$ ( $j_l(y)$ and $n_l(y)$ are the spherical Bessel and Neumann functions respectively). In (6) the following notations are used

$$f_{\underline{l}}(\tau)\equiv f_l(\tau)/[\lambda_0 j_{l-1}(\lambda_0 r_b)h_l^{(1)}(\lambda_1 r_b)-\lambda_1 j_l(\lambda_0 r_b)h_{l-1}^{(1)}(\lambda_1 r_b)] \tag{8}$$

and for the fields outside the ball ( $r>r_b$) one has

$$P_l(r,r_b)=h_l^{(1)}(\lambda_1 r)\,j_l(\lambda_0 r_b). \tag{9}$$

From the relations

$$h_l^{(1)}(x)\approx(-i)^{l+1}\frac{e^{ix}}{x},\qquad j_l(x)\approx\frac{1}{x}\sin(x-l\pi/2),\quad x>>l, \tag{10}$$

for the spherical Bessel and Hankel functions [23,24] it follows that at large distances from the ball one has

$$A_\mu^{lm}(r)\approx\delta_{m0}a_\mu^l\frac{e^{i\lambda_1 r}}{\lambda_1 r}+\ldots,\qquad r\to\infty. \tag{11}$$

Here

$$\begin{bmatrix}a_1^l\\a_2^l\end{bmatrix}=\frac{(-i)^l q}{cr_1^2\sqrt{\pi(2l+1)}}\begin{bmatrix}\frac{l\mathrm{w}_{\underline{l-1}}-(l+1)\mathrm{w}_{\underline{l+1}}}{\sqrt{l(l+1)}(\mathrm{w}_{\underline{l-1}}+\mathrm{w}_{\underline{l+1}})}\end{bmatrix},\qquad a_3^l=0 \tag{12}$$

and

$$\mathrm{w}_{l_1} \equiv J_{l_1}(<) + i\lambda_1 r_b^2 \left[a_{l_1}^{(12)} J_{l_1}(>) + a_{l_1}^{(22)} H_{l_1}\right] + \gamma_l j_{l_1}(\lambda_0 r_b)\left[l Q_{\underline{l-1}} - (l+1) Q_{\underline{l+1}}\right]. \tag{13}$$

In (13) the following notations are introduced

$$J_{l_1}(<) \equiv \int_0^{r_b} j_{l_1}(\lambda_0 x)[e^{i\frac{\omega}{\mathrm{v}}x} - (-1)^l e^{-i\frac{\omega}{\mathrm{v}}x}]dx, \quad J_{l_1}(>) \equiv \int_{r_b}^{\infty} j_{l_1}(\lambda_1 x)[e^{i\frac{\omega}{\mathrm{v}}x} - (-1)^l e^{-i\frac{\omega}{\mathrm{v}}x}]dx,$$

$$H_{l_1} \equiv \int_{r_b}^{\infty} h_{l_1}^{(1)}(\lambda_1 x)[e^{i\frac{\omega}{\mathrm{v}}x} - (-1)^l e^{-i\frac{\omega}{\mathrm{v}}x}]dx, \qquad Q_{l_1} = \lambda_0 j_l(\lambda_0 r_b) H_{l_1} + \lambda_1 h_l^{(1)}(\lambda_1 r_b) J_{l_1}(<)\,. \tag{14}$$

According to [9] the spectral-angular and spectral distributions of the radiation energy (during the all time of the charge motion) are determined by the expressions

$$\frac{dI_1}{d\omega\, d\Omega} = \frac{c}{\sqrt{\varepsilon_1}} \left| \sum_l a_2^l \vec{X}_{l0}^{(2)} \right|^2 \tag{15}$$

and

$$\frac{dI_1}{d\omega} = \frac{c}{\sqrt{\varepsilon_1}} \sum_l \left| a_2^l \right|^2 \quad , \tag{16}$$

respectively.

We now turn to the study of the spectral distribution of energy emitted by a train of electron bunches over the entire time of its movement:

$$\int F(\omega) I_1(\omega) d\omega \equiv \int I(\omega) d\omega\,. \tag{17}$$

Here, $I_1(\omega)$ is the spectral density of the energy emitted by a single electron and $F(\omega)$ is the structure factor of the train of electron bunches. In what follows we will assume that the transverse size of bunches is much smaller compared with the radius of the ball and with the radiation wavelength.

The structure factor in (17) is presented in the form [25, 26]

$$F = n_e[1 - f_e(\omega) f_{tr}(\omega)\,] n_b + n_e^2 f_e(\omega) n_b^2 f_{tr}(\omega)\ . \tag{18}$$

It is determined by the coherence factor of electrons inside bunches:

$$f_e = \exp(-\omega^2 \sigma^2 / \mathrm{v}^2) \tag{19}$$

(we assume Gaussian distribution of electrons with standard deviation $\sigma$), and by the coherence factor for the radiation of bunches inside the train:

$$f_{tr} = \frac{\sin^2(\omega d n_b / 2\mathrm{v})}{n_b^2 \sin^2(\omega d / 2\mathrm{v})}\,. \tag{20}$$

Here, $d$ is the distance between bunches, $n_e$ is the number of electrons in the bunch, $n_b$ is the number of bunches in the train. From (20) it follows that the train of bunches (with characteristic size essentially smaller than the radiation wavelength: $\sigma << 2\pi c/\omega$) radiates coherently $f_e(\omega) \approx 1$, $f_{tr}(\omega) = 1$ on discrete frequencies

$$\omega_m = \frac{2\pi v}{d} m, \qquad m = 1, 2... \tag{21}$$

(the frequency of emitted electromagnetic waves is proportional to the repetition frequency), and quasi-coherently $f_e(\omega) \approx 1$, $0.5 < f_{tr}(\omega) \le 1$ near these frequencies with the bandwidth

$$\Delta\omega_{n_b} = \frac{\pi v}{d \cdot n_b} \sim \frac{1}{n_b}. \tag{22}$$

## 3. Numerical results and their visual explanation. Ball made of melted quartz

### *3.1. Radiation from a single electron*

In this section we describe the features of the radiation for a ball made of melted quartz. The corresponding dielectric permittivity is given by [27] $\varepsilon_b = \varepsilon_b' + i\varepsilon_b'' = 3.78(1 + 0.0001i)$. In Fig. 3 (see [19]) we display the spectral distribution of the radiated energy from an electron of the energy 2 MeV, flying through the center of the ball with radius $r_b = 4$ cm (full curve).

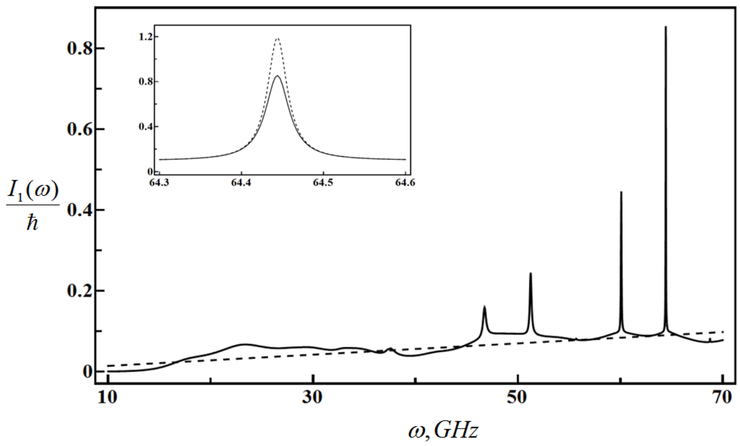


**Figure 3.** Spectral distribution of the radiated energy from an electron of energy 2 MeV flying through the center of the ball of melted quartz (full curve). The dashed curve corresponds to the motion of electron in a homogeneous medium with $\varepsilon_b = \varepsilon_b'$ provided the radiation is accumulated over the length of the path $2r_b$ (ball diameter). The Cherenkov condition is satisfied (for the graph in the upper left corner, see below).

Noteworthy is the fact that

1. sharp peaks are observed on certain "resonant" frequencies (for example, $\omega_0 \approx 64.45$ GHz) with the wavelength of the order $r_b$, whose height is almost an order of magnitude higher than at neighboring frequencies. This circumstance indicates a strong effect of the ball – vacuum interface on the CR generated by the particle in the ball material.

Numerical calculations show that

2. Taking into account the dielectric losses of the material of the ball has practically no effect on the radiation intensity, with the exception of the vicinity of "resonant" frequencies. In the vicinity of these selected frequencies, even small losses of radiation energy in the material of the ball (as, for example, in melted quartz) noticeably reduce the intensity of radiation. This fact is seen in the graph shown in the upper left corner of Fig.3, where the dotted curve corresponds to the case of the absence of dielectric losses.

The foregoing can be explained by the fact that

3. the radiation amplification is caused by the constructive superposition of electromagnetic waves, multiply reflected from the inner surface of the ball.

### *3.2. Radiation from a train of electron bunches*

In Fig. 4 we have plotted the spectral distribution of the radiation energy generated by a train of electron bunches flying through a dielectric ball made of melted quartz.

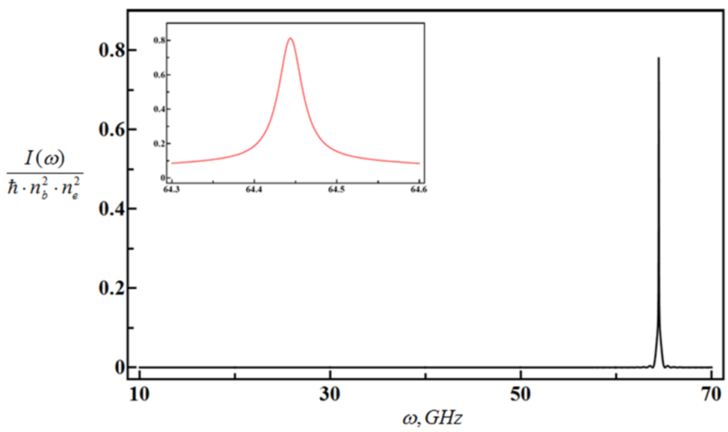


**Figure 4.** Spectral distribution of the radiation energy from a train of electron bunches of energy 2 MeV flying through the center of a ball of melted quartz, $n_e = 10^9$, $n_b = 100$, $d \cong 2{,}8$ cm. Other parameters are the same as those for Fig.3.

The power of quasi-coherent radiation generated by a train of bunches in the range $\omega_0 \pm \Delta\omega/2$ is given by

$$P \cong \frac{\mathrm{v}}{l} \int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} F(\omega) I_1(\omega) d\omega \cong \frac{\mathrm{v}}{l} \Delta W \cdot n_e^2 \cdot n_b^2 \sim 1\,\mathrm{kw}, \tag{23}$$

where

$$\Delta W = \int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} I_1(\omega) d\omega \sim 1.3 \cdot 10^{-8}\ \mathrm{eV}, \qquad \omega_0 \approx 64.45\ \mathrm{GHz}, \tag{24}$$

$\Delta\omega \sim 50\,\mathrm{MHz}$, $n_e \approx 10^9$ is the number of electrons in the bunch, $n_b = 100$ is the number of bunches and $l = n_b d \cong 2{,}8\,\mathrm{m}$ is the length of the train.

The emergence of powerful quasi-coherent radiation is related to the fact that:
a) for given choice of the values of system parameters, one of the resonant frequencies of the ball is equal to the repetition frequency of the bunches, $\omega_0 = \dfrac{2\pi \mathrm{v}}{d}$ (compare with (21)). In this case, the train of bunches will radiate coherently at this resonant frequency of the ball, with the highest spectral density:

$$I(\omega_0) = F(\omega_0) I_1(\omega_0), \qquad \text{where} \qquad F(\omega_0) \approx n_e^2 n_b^2, \tag{25}$$

and
b) is quasi-coherent:

$$I(\omega) = F(\omega) I_1(\omega), \qquad \text{where} \qquad F(\omega) \geq n_e^2 n_b^2 / 2 \tag{26}$$

in the entire frequency range $\omega \in [\omega_0 - \Delta\omega/2, \omega_0 + \Delta\omega/2]$, because

$$\Delta\omega < \Delta\omega_{n_b}, \quad \text{i.e.} \quad n_b < \pi \mathrm{v} / d\Delta\omega. \tag{27}$$

## 4. Conclusions

1. We have investigated the possibility of generating quasi-coherent CR by a train of equidistant one-dimensional electron bunches flying through the center of a ball made of a dielectric, a conductor, or of a composite material.
2. In the case of a melted quartz ball, it was shown that with a special choice of the distance between the bunches $d \cong 2.8\,\mathrm{cm}$, the resonant quasi-coherent CR of 100 bunches is formed near the resonant frequency $\omega_0 \approx 64.45 GHz$ in a narrow frequency band $\Delta\omega \cong 50 MHz$.
3. In a real situation (a) a train of three-dimensional bunches is generated instead of one-dimensional ones, and (b) this train should move along a hollow channel cuted through the ball (to reduce ionization losses). The influence of these factors will be insignificant if the radius of the channel is much smaller than the wavelength of radiation and larger than the transverse dimensions of the bunch.
4. It is proposed to use this phenomenon for the development of high-power and narrow-band sources of electromagnetic waves in the giga-terahertz frequency range.

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## Author Contributions

The authors equally contributed to all steps of the paper preparation.

## Conflicts of interest

The authors declare no conflict of interest.

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