

# Radiation of Surface Polaritons from a Charge Rotating Around a Dielectric Cylinder

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**Abstract.** We investigate the radiation of surface polaritons by a charged particle rotating around a dielectric waveguide embedded in a homogeneous medium. A formula is derived for the spectral distribution of the radiation intensity for surface-type modes. It is shown that the corresponding waves are radiated on the eigenmodes of the dielectric cylinder. The number of radiated quanta for surface polaritons of a given harmonic can be essentially larger than that for guiding modes.

**Keywords:** surface polariton, dielectric waveguide, synchrotron radiation

## 1. Introduction

The surface plasmon polaritons (SPPs, for reviews see [1-4]) are evanescent electromagnetic waves propagating along a metal-dielectric interface as a result of collective oscillations of electron subsystem coupled to electromagnetic field. They exist in frequency ranges where the real part of the permittivity has different signs for neighboring media. The incomplete list of fields where the SPPs have found wide applications include surface imaging, data storage, biosensors, plasmonic waveguides, plasmonic solar cells, etc. Remarkable properties of SPPs include the possibility of concentrating electromagnetic fields beyond the diffraction limit of light waves and enhancing the local field strengths by orders of magnitude [1,2]. Depending on the dielectric properties of the active medium, other forms of surface polaritons may exist. In particular, other materials besides metals, such as semiconductors, organic and inorganic dielectrics, ionic crystals, can support surface polariton type waves. An important direction of recent developments is the extension of plasmonics to the infrared and terahertz ranges of frequencies. This can be done by a suitable choice of the active medium such as doped semiconductors and artificially constructed materials (metamaterials) by using various structures [5,6].

From the point of view of an understanding of the fundamental properties and practical applications, of particular interest is the investigation of the effects induced by the curvature of the interface on the generation and propagation of surface polaritons. In the present report we consider the generation of surface polaritons by a charge circulating around a cylindrical waveguide. We are interested in the electromagnetic fields and in the intensity of the radiated surface polaritons.

For a charged particle rotating around a dielectric cylinder, the spectral and angular distribution of the radiation intensity at large distances from the cylinder has been investigated in [7,8]. It was shown both analytically and numerically that if the Cherenkov condition for permittivity of the cylinder and for the velocity of the particle image on the cylinder surface is obeyed then strong narrow peaks appear in the angular distribution of the radiation intensity on a given harmonic. At these peaks the radiated energy density may exceed the corresponding value for the radiation in the absence of the cylinder by several orders of magnitude. Similar features for the radiation from a charge moving along a helical trajectory around a cylinder have been discussed in [9]. The radiation of surface waves on the eigenmodes of a dielectric cylinder by a charge circulating around the cylinder is discussed in [10]. The interference effects between the synchrotron and Smith-Purcell radiations from a charge rotating around a cylindrical grating have been studied in [11]. In all these investigations it has been assumed that the dielectric functions for both the cylinder and surrounding medium are positive and, hence, the cylinder modes corresponding to surface polaritons are absent. The radiation of surface polaritons by a charge rotating round a cylinder has been recently investigated in [12]. The radiation on the guiding modes of the cylinder in the same problem was discussed in [13]. The radiation intensity at large distances from the cylinder in the spectral range where the dielectric permittivity of the cylinder is negative has been considered in [14]. In the present paper we review these recent results.

## 2. Radiation field

Consider a point charge  $q$  circulating around a cylinder with dielectric permittivity  $\varepsilon_0$ . The cylinder is embedded in a homogeneous medium with permittivity  $\varepsilon_1$ . The radii of the rotation orbit and cylinder will be denoted by  $r_q$  and  $r_c$ , respectively. In the absence of the cylinder, for a charge rotating in a homogeneous medium we would have two types of radiations: synchrotron radiation and Cherenkov radiation (if the corresponding condition is satisfied). The presence of the cylinder gives rise to new types of radiations propagating inside the cylinder (guiding modes) and along the surface (surface modes).

In cylindrical coordinates  $(r, \phi, z)$ , with the axis  $z$  along the cylinder axis, for the current density in the Maxwell equations one has  $\mathbf{j} = (0, j_\phi, 0)$ , with

$$j_\phi = \frac{q}{r} v \delta(r - r_q) \delta(\phi - \omega_0 t) \delta(z). \quad (1)$$

Here  $v$  is the velocity of the charge and  $\omega_0 = v/r_q$  is the corresponding angular velocity. Let  $\mathbf{E}(\mathbf{r}, t)$  be the electric field strength generated by the source (2). The corresponding Fourier component  $\mathbf{E}_n(k_z, r)$  is defined by

$$\mathbf{E}(\mathbf{r}, t) = 2 \operatorname{Re} \left[ \sum_{n=0}^{\infty'} e^{in(\phi - \omega_0 t)} \int_{-\infty}^{+\infty} dk_z e^{ik_z z} \mathbf{E}_n(k_z, r) \right], \quad (2)$$

where the prime on the summation sign means that the term  $n=0$  should be taken with an additional coefficient  $1/2$ . In the region outside the cylinder the Fourier component is decomposed into two contributions

$$\mathbf{E}_n(k_z, r) = \mathbf{E}_n^{(0)}(k_z, r) + \mathbf{E}_n^{(c)}(k_z, r), \quad (3)$$

where  $\mathbf{E}_n^{(0)}(k_z, r)$  corresponds to the field in a homogeneous medium with permittivity  $\varepsilon_1$  (the cylinder is absent) and  $\mathbf{E}_n^{(c)}(k_z, r)$  is induced by the presence of the cylindrical waveguide. For  $\varepsilon_0 = \varepsilon_1$  one has  $\mathbf{E}_n^{(c)}(k_z, r) = 0$ .

We are interested in the radiation of the surface polaritons. The corresponding intensity is expressed in terms the work done by the radiation field on the charged particle:

$$I_{(SP)} = - \int_0^\infty dr \int_0^{2\pi} d\phi \int_{-\infty}^\infty dz r \mathbf{j} \cdot \mathbf{E}^{(r)}, \quad (4)$$

where  $\mathbf{E}^{(r)}$  is the electric field corresponding to surface polaritons. Substituting the analog of the expansion (2) for the radiation field we get

$$I_{(SP)} = -2q \operatorname{Re} \left[ \sum_{n=0}^{\infty'} \int_{-\infty}^{+\infty} dk_z \mathbf{v} \cdot \mathbf{E}_n^{(r)}(k_z, r_q) \right]. \quad (5)$$

The only nonzero component of the particle velocity is along the azimuthal direction and, hence, in (5) one needs the azimuthal component of the electric field only.

The Fourier components  $\mathbf{E}_n(k_z, r)$  are found by using the Green function given in [7]. For the azimuthal component the separate contributions are presented as

$$\begin{aligned} E_{n\phi}^{(0)}(k_z, r) &= -\frac{qv}{8n\omega_0\varepsilon_1} \sum_{p, j=\pm 1} \left( j \frac{n^2 \omega_0^2}{c^2} \varepsilon_1 + k_z^2 \right) J_{n+jp}(\lambda_1 r_<) H_{n+p}(\lambda_1 r_>), \\ E_{n\phi}^{(c)}(k_z, r) &= -\frac{iqv}{4\pi n \omega_0 \varepsilon_1} \sum_{p, j=\pm 1} \left( j \frac{n^2 \omega_0^2}{c^2} \varepsilon_1 + k_z^2 \right) B_{n, jp}^{(c)} H_{n+p}(\lambda_1 r), \end{aligned} \quad (6)$$

where  $r_< = \min(r, r_q)$ ,  $r_> = \max(r, r_q)$ ,  $J_\nu(x)$  is the Bessel function,  $H_\nu(x) = H_\nu^{(1)}(x)$  is the Hankel function of the first kind and we have defined

$$\lambda_j^2 = n^2 \omega_0^2 \varepsilon_j / c^2 - k_z^2, \quad j = 0, 1. \quad (7)$$

The coefficients  $B_{n,p}^{(c)}$  are given by

$$B_{n,p}^{(c)} = -\frac{\pi}{2i} H_{n+p}(\lambda_1 r_q) \frac{V_{n+p}^J}{V_{n+p}^H} + p \frac{\lambda_0 J_n(\lambda_0 r_c)}{2r_c \alpha_n} \frac{J_{n+p}(\lambda_0 r_c)}{V_{n+p}^H} \sum_{l=\pm 1} \frac{H_{n+l}(\lambda_1 r_q)}{V_{n+l}^H}, \quad (8)$$

where for  $F = J, H$  we use the notation

$$V_n^F = \lambda_1 J_n(\lambda_0 r_c) F'_n(\lambda_1 r_c) - \lambda_0 F_n(\lambda_1 r_c) J'_n(\lambda_0 r_c) \quad (9)$$

and  $\alpha_n$  is defined as

$$\alpha_n = \frac{\varepsilon_0}{\varepsilon_1 - \varepsilon_0} - \frac{\lambda_0}{2} J_n(\lambda_0 r_c) \sum_{l=\pm 1} l \frac{H_{n+l}(\lambda_1 r_c)}{V_{n+l}^H}. \quad (10)$$

The eigenmodes of the cylinder are determined from the equation  $\alpha_n = 0$ .

The integration range in (5) is divided into two parts. The first one corresponds  $k_z^2 < n^2 \omega_0^2 \varepsilon_1 / c^2$  and it describes the radiation propagating at large distances from the cylinder,  $r \gg r_q$ . For the second region,  $k_z^2 > n^2 \omega_0^2 \varepsilon_1 / c^2$ , one has  $\lambda_1 = i |\lambda_1|$  and the Hankel functions are reduced to the Macdonald function  $K_\nu(x)$ . The respective electromagnetic fields are exponentially small at large distances from the cylinder. The modes with  $k_z^2 > n^2 \omega_0^2 \varepsilon_1 / c^2$  are further subdivided into guiding modes with  $\lambda_0^2 > 0$  and surface-type modes with  $\lambda_0^2 < 0$ . We are interested in the radiation fields for the second subclass.

For the evaluation of the radiation intensity (5) we must separate the radiation field corresponding to surface-type modes. The exponent in (2) has no stationary points and for large values of  $z$  and for fixed  $r$  the radiation field is determined by the contributions from the poles of the integrand. These poles correspond to the zeros of the function  $\alpha_n$ . For the surface-type modes  $\lambda_j^2 < 0$  and the corresponding formula is presented as

$$\alpha_n(k_z) = \frac{U_n^{(s)}(k_z)}{(\varepsilon_1 - \varepsilon_0)(V_n^{(s)2} - n^2 u^{(s)2})}, \quad u^{(s)} = \frac{u_0}{u_1} - \frac{u_1}{u_0}, \quad (11)$$

with the notations

$$U_n^{(s)}(k_z) = V_n^{(s)} \left( \varepsilon_0 u_1 \frac{I'_n}{I_n} - \varepsilon_1 u_0 \frac{K'_n}{K_n} \right) - \frac{n^4 \omega_0^2 r_c^4 k_z^2}{c^2 u_0^2 u_1^2} (\varepsilon_1 - \varepsilon_0)^2 \quad (12)$$

and

$$V_n^{(s)} = u_1 \frac{I_n'}{I_n} - u_0 \frac{K_n'}{K_n}. \quad (13)$$

Here  $I_n = I_n(u_0)$ ,  $K_n = K_n(u_1)$ ,  $I_n(x)$  is the modified Bessel function and

$$u_j = r_c \sqrt{k_z^2 - n^2 \omega_0^2 \varepsilon_j / c^2}. \quad (14)$$

The expression for the functions  $V_{n+p}^H$  is presented as

$$V_{n+p}^H = \frac{2i}{\pi r_c} K_n I_n (V_n^{(s)} + p n u^{(s)}). \quad (15)$$

We note that  $-iV_{n+p}^H$  is always positive and the function  $V_{n+p}^H$  has no zeros. Consequently, the only poles of the integrand in (2) correspond to the zeros of  $\alpha_n$ . As seen from (11), the equation  $\alpha_n(k_z) = 0$  is reduced to the equation

$$U_n^{(s)}(k_z) = 0. \quad (16)$$

This equation coincides with the equation for surface-type modes (on features of propagation and radiation of surface polaritons in cylindrically curved geometries of the interface see, for instance, [15-22] and references therein). From (16), as necessary condition for the presence of the surface-type modes with  $\lambda_j^2 < 0$ , we get  $\varepsilon_1 / \varepsilon_0 < 0$ . Hence, in order to have surface-type modes the dielectric permittivities of the cylinder and the surrounding medium must have opposite signs. This condition is the same as that for surface-type modes on a planar interface between two media.

We will denote by  $k_z = \pm k_{n,s}$ ,  $k_{n,s} > 0$ , the roots of (16) with respect to  $k_z$ , where  $s$  enumerates the roots for a given  $n$ . In the presence of the poles  $k_z = \pm k_{n,s}$ , we need to specify the integration contour in (2). By taking into account that in physically realistic situations  $\varepsilon_j = \varepsilon_j' + i\varepsilon_j''$ , with  $\varepsilon_j''(\omega) > 0$  for  $\omega > 0$ , we can see that the contour should overcome the poles  $k_z = k_{n,s}$  from below and the poles  $k_z = -k_{n,s}$  from above. For the radiation fields in the region  $z > 0$  we close the integration contour by a semicircle with large radius in the upper complex plane and the integral is expressed in terms of the residues of the integrand. The azimuthal component of the radiation field outside the cylinder is presented in the form

$$E_\phi^{(r)}(\mathbf{r}, t) = \sum_{n=1}^{\infty} \frac{q v r_c}{2 n \omega_0 \varepsilon_1} \sum_s \frac{\lambda_{n,s}^{(0)}}{I_n K_n^2 \alpha_n'(k_{n,s})} \sum_{l=\pm 1} \frac{l K_{n+l}(\lambda_{n,s}^{(1)} r_q)}{V_n^{(s)} + l n u^{(s)}} \times \sum_{p,j=\pm 1} p \left( k_{n,s}^2 + j \frac{n^2 \omega_0^2 \varepsilon_1}{c^2} \right) \frac{I_{n+jp} K_{n+p}(\lambda_{n,s}^{(1)} r)}{V_n^{(s)} + j p n u^{(s)}} \cos(n\phi + k_{n,s} z - n\omega_0 t), \quad (17)$$

where  $u^{(s)}$  and  $V_n^{(s)}$  are defined by (11) and (13) with  $u_j = r_c \lambda_{n,s}^{(j)}$ ,  $I_n = I_n(\lambda_{n,s}^{(0)} r_c)$ ,  $K_n = K_n(\lambda_{n,s}^{(1)} r_c)$ , and

$$\lambda_{n,s}^{(j)} = |\lambda_j(k_{n,s})| = \sqrt{k_{n,s}^2 - n^2 \omega_0^2 \varepsilon_j / c^2} \quad (18)$$

The expression (17) corresponding to the surface-type modes is valid for all values of  $z > 0$ .

### 3. Radiation intensity

Having the radiation field (17), the radiation intensity is evaluated by using (4). It is presented as the sum of the intensities on separate harmonics  $I_{(SP)} = \sum_{n=1}^{\infty} I_{(SP)n}$ . After transformations, for the radiation intensity on a given harmonic  $n$  we get

$$I_{(SP)n} = -\frac{q^2 r_q^2 r_c \omega_0}{n(\varepsilon_1 - \varepsilon_0)} \sum_s \frac{\lambda_{n,s}^{(0)2}}{\alpha'_n(k_{n,s}) K_n^2} \left[ \sum_{l=\pm 1} l K_{n+l}(\lambda_{n,s}^{(1)} r_q) \right]^2 \left( \lambda_{n,s}^{(0)} \frac{I'_n}{I_n} - \lambda_{n,s}^{(1)} \frac{K'_n}{K_n} \right). \quad (19)$$

For a given  $\omega_0$ , the radius of the orbit enters in the form  $r_q^2 K_{n+l}^2(\lambda_{n,s}^{(1)} r_q)$ . From here it follows that at distances from the cylinder surface,  $r_q - r_c$ , much larger than the radiation wavelength the radiation intensity is suppressed by the factor  $\exp[-2\lambda_{n,s}^{(1)}(r_q - r_c)]$ . For values of  $\varepsilon_0$  close to  $-\varepsilon_1$  the roots  $k_{n,s}$  are large. The leading term in the radiation intensity is presented as

$$I_{(SP)n} \approx \frac{4q^2 v^2 \varepsilon_0}{(\varepsilon_1^2 - \varepsilon_0^2) r_q^3 \omega_n} \exp \left[ -2 \frac{\omega_n}{c} \frac{\varepsilon_1 (r_q - r_c)}{\sqrt{-\varepsilon_0 - \varepsilon_1}} \right] \quad (20)$$

and, hence, the radiation intensity tends to zero in the limit  $\varepsilon_0 \rightarrow -\varepsilon_1$ . In the limiting case  $\varepsilon_0 \ll -\varepsilon_1$  the surface polariton type modes are present for the main harmonic  $n=1$  only. In this range the roots for  $k_z$  are close to the limiting value  $\omega_n \sqrt{\varepsilon_1} / c$  and one gets

$$I_{(SP)1} \approx \frac{q^2 c |\varepsilon_0|}{2\varepsilon_1^{5/2} r_q^2} \left( r_q^2 / r_c^2 - 1 \right)^2 \exp \left( -\frac{c \sqrt{|\varepsilon_0|}}{\varepsilon_1 v_c} \right). \quad (21)$$

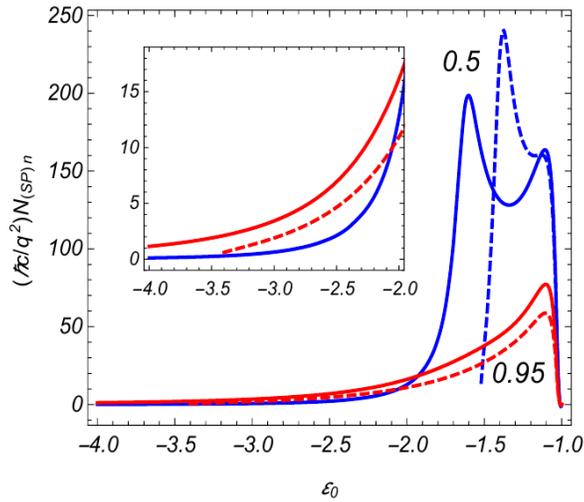
Hence, the radiation intensity vanishes in the limit  $\varepsilon_0 \rightarrow -\infty$ . For  $n > 1$  and  $k_z \rightarrow \omega_n \sqrt{\varepsilon_1} / c$ , the dielectric permittivity of the cylinder, determined from the mode equation for surface polaritons,

tends to a finite limiting value,  $\varepsilon_0 \rightarrow \varepsilon_{0n}^{(1)}$ . Consequently, the quantity  $\lambda_{n,s}^{(0)}$  in (19) tends to the finite limit  $(\omega_n / c)\sqrt{\varepsilon_1 + |\varepsilon_{0n}^{(1)}|}$ . In this case the radiation intensity  $I_{(SP)n}$  approaches a finite limiting value.

In Fig. 1 we have plotted the number of the radiated quanta in the form of surface polaritons on a given harmonic  $n$  per period of the charge rotation  $T = 2\pi / \omega_0$ ,

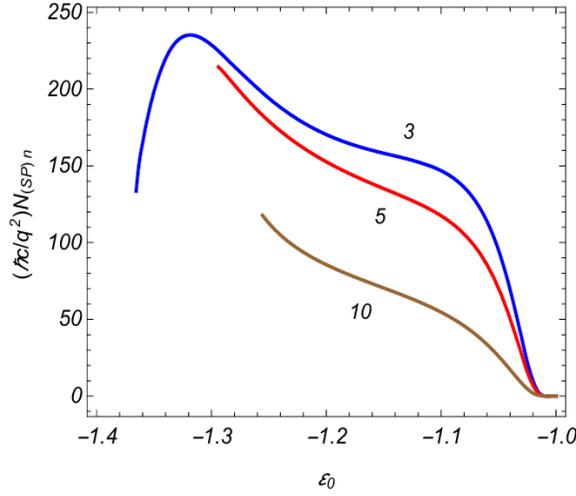
$$N_{(SP)n} = T \frac{I_{(SP)n}}{\hbar n \omega_0} \quad (22)$$

as a function of  $\varepsilon_0$  for  $\varepsilon_1 = 1$ ,  $r_c / r_q = 0.95$ . The numbers near the curves correspond to the values of  $v/c$  and the full (dashed) curves correspond to  $n = 1$  ( $n = 2$ ). For the critical values of the cylinder dielectric permittivity in the case  $n = 2$  one has  $\varepsilon_{0n} \approx -1.52$  for  $v/c = 0.5$  and  $\varepsilon_{0n} \approx -3.44$  for  $v/c = 0.9$ .



**Fig. 1.** The number of the radiated quanta in the form of surface polaritons versus the dielectric permittivity of the cylinder. The graphs are plotted for  $v/c = 0.5, 0.95$  (numbers near the curves) and for  $n = 1, 2$  (full and dashed curves, respectively).

In Fig. 2 we have plotted the dependence of the number of the radiated surface polaritons on the dielectric permittivity of the cylinder for  $v/c = 0.5$  and for higher harmonics  $n = 3, 5, 10$  (numbers near the curves). Again we see the presence of critical values of the dielectric permittivity for the radiation of surface polaritons on the harmonics with  $n > 1$ . The critical value increases with increasing  $n$ . For  $n = 3, 5, 10$  one has  $\varepsilon_{0n} \approx -1.37, -1.29, -1.26$ , respectively. In these cases the critical values  $\varepsilon_{0n}$  coincide with the limiting values  $\varepsilon_{0n}^{(1)}$ .



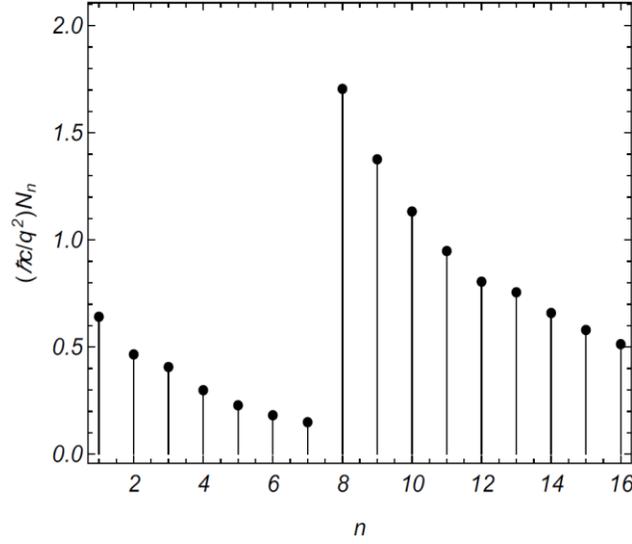
**Fig. 2.** The same as in Fig. 1 for  $v/c = 0.5$  and  $n = 3, 5, 10$  (numbers near the curves).

It is of interest to compare the radiation intensity for surface polaritons with the radiation for guiding modes ( $\lambda_1^2 < 0 < \lambda_0^2$ ) from a charge rotating around a cylinder having dielectric permittivity  $\varepsilon_0 > 0$  in a given spectral range. The intensity for the part of the corresponding radiation propagating in the region outside the cylinder has been investigated in [10] (the radiation of guiding modes by a charge circulating inside the dielectric cylinder has been discussed in [23]). The numerical results were presented for a cylinder made of quartz. Note that for guiding modes, with increasing harmonic number  $n$  the number of roots  $k_{n,s}$  increases for some critical values of the harmonic. For the example considered in [10], the number of roots was 1 for  $1 \leq n \leq 7$ , 2 for  $8 \leq n \leq 12$ , and 3 for  $13 \leq n \leq 16$ . For the corresponding number of the quanta radiated on the guiding modes per period of the rotation by an electron of energy 2 MeV and for  $r_c / r_q = 0.99$  one has  $N_{(\text{GM})n} < 0.25q^2 / (\hbar c)$ . The numerical data for these values of the energy and the ratio  $r_c / r_q$ , similar to those depicted in Fig. 1, show that for  $-5 < \varepsilon_0 < 0$  the number of the radiated surface polaritons is essentially larger. For example, in the case of  $n = 1$  one gets  $N_{(\text{SP})n} \approx 4.23q^2 / (\hbar c)$  for  $\varepsilon_0 = -3$  and  $N_{(\text{SP})n} \approx 45.49q^2 / (\hbar c)$  for  $\varepsilon_0 = -1.5$ .

The radiation on the guiding modes for a charge rotating around the dielectric cylinder is investigated in [13]. This modes are radiated in the spectral range where  $\varepsilon_0(\omega) > 0$ . The corresponding intensity is given by the formula

$$\begin{aligned}
 I_{(\text{GM})n} = & -\frac{q^2 v^2}{2\varepsilon_1 n \omega_0} \sum_{s=1}^{s_n} \frac{r_c \lambda_{n,s}}{\alpha'_n(k_{n,s}) J_n K_n^2} \sum_{l=\pm 1} l K_{n+l}(\lambda_{n,s}^{(1)} r_q) \\
 & \times \sum_{p=\pm 1} K_{n+p}(\lambda_{n,s}^{(1)} r_q) \left[ \left( \frac{n^2 \omega_0^2 \varepsilon_1}{c^2} + k_{n,s}^2 \right) \frac{J_{n+p}}{V_n - pnu} - \frac{\lambda_{n,s}^{(1)2} J_{n-p}}{V_n + pnu} \right], \quad (23)
 \end{aligned}$$

where  $\lambda_{n,s} = \sqrt{n^2 \omega_0^2 \varepsilon_0 / c^2 - k_{n,s}^2}$ ,  $J_n = J_n(\lambda_{n,s} r_c)$ . The maximum value  $s_n$  in the summation over  $s$  is determined from the condition  $k_{n,s} < n\omega_0 \sqrt{\varepsilon_0} / c$ . For a given angular frequency  $\omega_0$ , the contribution of the mode with a given  $k_{n,s}$  is exponentially suppressed for large values of the cylinder radius.



**Fig. 3.** The number of quanta emitted on the guiding modes of a cylinder made of fused silica, as a function of the radiation harmonic.

The corresponding number of quanta of a given harmonic  $n$ ,  $N_n = TI_{(\text{GM})n} / (\hbar n \omega_0)$ , emitted per period of rotation of the charge is depicted in Fig. 3 as a function of the harmonic number. The numerical data are given for an electron with the energy of  $E_e = 2$  MeV and for the values of the parameters  $\varepsilon_1 = 1$ ,  $\varepsilon_0 = 3.74$  (the dielectric constant of fused silica), and  $r_c / r_q = 0.99$ . As seen, for the examples considered the number of the radiated surface polaritons is essentially larger than the number of the guiding modes.

For the problem under consideration, in addition to the radiation of surface polaritons, there is also radiation propagating at large distances from the cylinder. The latter corresponds to synchrotron radiation modified by the presence of the cylinder. For the respective electromagnetic fields one has  $k_z^2 < \omega_n^2 \varepsilon_1 / c^2$  and their radial dependence is given by the Hankel functions  $H_{n+p}(\lambda_1 r)$  with positive  $\lambda_1$ . The features of synchrotron radiation for a cylinder made of material with negative dielectric permittivity have been investigated in [14]. It has been shown that the radiation intensity on a given harmonic, integrated over the angles, can be essentially amplified by the presence of the cylinder.

#### 4. Conclusion

We have investigated the radiation of surface polaritons by a charge rotating around a dielectric cylinder with permittivity  $\varepsilon_0$ . For the corresponding waves the component of the wave vector along the cylinder axis obey the condition  $k_z^2 > \omega_n^2 \varepsilon_1 / c^2$  and they are radiated on the eigenmodes of the cylinder, determined by the zeros of the function  $\alpha_n(k_z)$  for a given harmonic. For the existence of solutions to this equation the dielectric permittivities for the cylinder and surrounding medium should have opposite signs. We have considered the case  $\varepsilon_0 < 0 < \varepsilon_1$ . The radiation intensity for surface waves on a given harmonic  $n$  is given by (19). For the main harmonic  $n=1$ , the surface polaritons are radiated for all values of the cylinder dielectric function in the range  $\varepsilon_0 < -\varepsilon_1$ . For higher harmonics  $n > 1$ , there exists a critical value  $\varepsilon_{0n}$  with the absence of surface polariton radiation in the range  $\varepsilon_0 < -\varepsilon_{0n}$ . The radiation wavelength increases with approaching  $\varepsilon_0$  to the limiting value  $-\varepsilon_1$ . In that range, the wavelength of surface polaritons is much smaller than the wavelength of electromagnetic radiation in free space with the same frequency. We have also demonstrated that, for a given harmonic, the number of radiated quanta for surface polaritons can be essentially larger than that for guiding modes of the cylinder.

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#### Conflict of Interest

The authors declare no conflict of interest.

#### Author Contributions

The authors equally contributed to the study.

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