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Multilayer Anisotropic Thin Film with a Twist

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Abstract: We describe a birefringent thin film as a waveplate for polarization conversion, a system of birefringent thin films for photonics bandgaps and achromatization of functionality, birefringent thin film with a twist and multilayer CDW coatings. A theory of a broadband thin-film polarizing beam splitter based on an anisotropic cycloidal diffraction grating is proposed. The achromatization of the grating is provided by creating two-layer chiral structures with an opposite twist sense. Such a system has almost 100% efficiency. The calculations were carried out by the Jones matrix method. The resulting efficiency and bandwidth are higher than the previously known results.

Keywords: achromatic gratings, waveplates, twist structure

1. Introduction

From optical communication and information processing to imaging and metrology, there has been a growing need for analysis and synthesis of the spatial structure of the optical field. The diffraction wave plates (DVP) or the fourth-generation optics technology [1–3] are necessary for creating the thin broadband and efficient optical elements and systems including, for example, beam shapers and switchable lenses. DVPs have a wide range of applications, including the liquid crystal displays (LCD), polarimetry, astronomy, fiber-optic communications, quantum computing, and microscopy. The traditional systems for dividing light into the orthogonal polarizations are large in size/weight and have a limited operating wavelength range. On the contrary, the DVPs have the micron thicknesses, moderate transverse dimensions and are almost weightless. The diffraction wave plates are the effective and polarization-sensitive diffraction thin films [4–11] controlled by electric and magnetic fields [12–14], which can diffract rays into the first order with the $\sim 100\,\%$ efficiency and the polarization selectivity similar to the glass beam splitters. One of such examples is the cycloid diffraction wave plate (CDWP) [4, 6, 7]. It has a spiral-linear birefringence, which is homogeneous over the thickness of the layer. Such structures were used to create the effective thin-film lenses for circular polarizations [15], arrays of all types of optical elements [16], long-wave infrared optical elements [17] and effective waveplates for coronagraph [18]. However, the high diffraction efficiency of the CDWP takes place only in moderate bandwidth, which is a limitation that is applied to almost all gratings. Here we describe a birefringent thin film as a waveplate for polarization conversion, a system of birefringent thin films for photonics bandgaps and achromatization of functionality, birefringent thin film with a twist and multilayer CDW coatings.

This article presents the theory of achromatic CDWP, formed by reactive mesogens (the polymerized LC) [19, 20], which simultaneously performs the chromatic and polarization separations. This design can provide $\sim 100\ \%$ efficiency over the entire visible wavelength range, achieving the achromatic diffraction by compensating the chromatic delay dispersion using the twisting effect. We use the Jones matrix method for computations.

2. Birefringent thin film – a waveplate for polarization conversion

Light beam, generally, is a superposition of two orthogonal polarized beams. Polarization is the attribute of light that provides means to easily and efficiently control its more discernible characteristics such as intensity and propagation direction. Its two orthogonal components, usually considered linear, are differently reflected, and a Brewster plate or a polarizing cube is used to easily separate those components acting as reflective polarizers. Dichroic absorptive film polarizers transmit only one of the linear components why absorbing the orthogonal one. In addition, the best of it all – the state of light polarization can be controlled with a thin layer of an optically anisotropic material. All it takes to reverse the state of polarization is an additional phase shift of one of the polarization components with respect to the other (called as phase retardation) equal to $\pi/2$. This is known as half-waveplate condition since it takes place when the difference in optical paths for the orthogonal polarized beams is equal to the half of the wavelength λ: $(n_{\parallel} - n_{\perp})L = \lambda/2$ where $n_{\parallel}$ and $n_{\perp}$ are the principal refractive indices of the material and L is the layer thickness. Typically, $n_{\parallel} - n_{\perp} \sim 0.2$, and a birefringent film of nearly 1 μm thickness is sufficient for transforming one orthogonal polarization into another thus capable of changing the state of an optical system from transmittive to reflective or absorptive. In general, the phase retardation of a simple parallel plate of single crystal is given by $\Gamma = 2\pi(n_{\parallel} - n_{\perp})L/\lambda$. In addition, the transmission of light through the single crystal placed between parallel polarizers and having optical axis with azimuthal angle $45°$ with respect to the polarizers is given by $T = \cos^2 \Gamma/2$. As we see phase retardation is strongly dependent on the wavelength via the $(1/\lambda)$ factor on the right-hand side. For most transparent crystals, the chromatic dispersion of $(n_{\parallel} - n_{\perp})$ further increases the variation with the wavelength. In many optical applications, including LCDs, it is desirable to have waveplates whose phase retardation is insensitive to the wavelength variation. Such waveplates are known as achromatic.

3. A system of birefringent thin films: photonics bandgaps

Two and more such films provide new degrees of freedom to obtain the half-waveplate condition practically independent on wavelength in a wide spectral range [21]. A symmetric combination of waveplates is equivalent to single-waveplate. The equivalent phase retardation Γ_e of such a combination of waveplates depends on the azimuth angles as well as phase retardation of

individual plates. Under the appropriate conditions, the equivalent phase retardation Γ_e can be insensitive to the wavelength variation. Such kinds of waveplates are called as Pancharatnam achromatic waveplates. In the case of the combination of three waveplates the transmission of Pancharatnam achromatic waveplate is proportional to $\cos^4 \Gamma_e /2$. That is way for more of wavelength half-wave condition will be close to satisfied. By increasing the number of waveplate layers, we can get much more power of $cos\Gamma_e /2$ and wider range of achromaticity.
A multitude of birefringent layers, each one having its optical axis orientation rotated with respect to its neighbor, produces a photonic bandgap. At the limit of continuous helical rotation of the optical axis orientation that happens naturally in materials known as cholesteric liquid crystals , the bandgap acts as a circular polarizer reflecting light with wavelengths in the spectral range $n_{\perp}P < \lambda < n_{\parallel}P$, where P is the pitch of the helix. Only the component polarized according to the helix is reflected while the orthogonal one propagates as through an isotropic material. The bandgap becomes polarization-insensitive for periodically twisted structure of “swinging nematics”.

4. Birefringent thin film with a twist

Let us reset here the discussion and start over with a single layer of a birefringent material. It is not hypothetical, but a matter of fact, that the optical axis of birefringent materials can be twisted at high spatial frequencies in the plane of the layer. Liquid crystals (LCs) and LC polymers – no surprise indeed, – are the best-suited materials to produce a full rotation of the optical axis over a micrometer distance without breaking it structurally. More simple examples of birefringent materials having optical axis non uniform in the plane of the layer are axial and radial polarizers that can be built as a planar LC cell having thickness and birefringence selected so as to induce a inhomogeneous phase retardation Γ at the working wavelength λ , for light propagation perpendicular to the cell plane walls (z axis). The LC molecular director $\boldsymbol{n}$ is assumed to be uniform in the z direction, but inhomogeneous in the xy plane of the cell, according to a prescribed pattern $\boldsymbol{n}(x,y) = (\cos\delta(x,y) ; sin\delta(x,y); 0)$, where $\delta(x,y)$ is the azimuth angle of the director field. As it was shown in [22] if we will have suitable distribution of phase retardation $\Gamma(x,y)$ then we can design all optical devices allowing to register orientation of Stokes vector or to visualize the light polarization. For this purpose let we consider collimated light with intensity I_0 and unknown polarization propagates in z direction through thin layer of NLC with inhomogeneous optical axes and retardation. Light pass through plane polarizer with the direction of transmission x and register by detector. Then the intensity of light passed through this system can be express by Stokes vector $\mathbf{S}$

$$I(x,y) = \frac{1}{2}[1 + \boldsymbol{\eta}(x,y)\mathbf{S}] \tag{1}$$

were

$$\eta_1(x,y) = \sin^2(\Gamma/2)\sin 4\delta$$

$$\eta_2(x,y) = \sin\Gamma \sin 2\delta \tag{2}$$

$$\eta_3(x,y) = 1 - 2\sin^2(\,\Gamma/2)\sin^2 2\delta$$

It is clearly seen from these expressions that the intensity could rich its maximum or minimum when vector $\boldsymbol{\eta}$ will be parallel or antiparallel to Stokes vector $\boldsymbol{S}$ of light under study. It means that by the distribution of intensity we can reconstruct Stokes vector of light.

More complicated examples of birefringent materials having optical axis non-uniform in the plane of the layer are cycloidal diffraction gratings. The "cycloidal" orientation in LC materials is obtained by imprinting the pattern of boundary orienting forces on substrates that orient the adjacent layer of the LC materials. The imprinting is usually done by subjecting a thin $(10 - 50\,nm)$ photoalignment layer coating to the polarization modulation pattern of overlapping beams of orthogonal circular polarization.

Thus we have cycloidal diffractive waveplate (CDW) confined between $0 < z < L$ with the optical axis $\boldsymbol{n}$ rotating in transverse direction x , so that a periodic structure

$$\mathbf{n}_0(x) = \{\cos qx;\ \sin qx; 0\},\ q = 2\pi/\Lambda \tag{3}$$

is realized. Here Λ is the director's period. The Jones matrix of a such CDW we wright in the form $M = M_0 + M_{+1} + M_{-1}$, were

$$M_0 = \begin{bmatrix} \cos\ \Phi & 0 \\ 0 & \cos\ \Phi \end{bmatrix}, \qquad M_{\pm 1} = \frac{i\sin\Phi}{2}\exp\,(\pm i2qx)\begin{bmatrix} 1 & \pm i \\ \pm i & -1 \end{bmatrix}, \tag{4}$$

where $\Phi = \Gamma/2 = \pi(n_{\parallel} - n_{\perp})L/\lambda$ is half phase retardation, $n_{\parallel}$ and $n_{\perp}$ are parallel and perpendicular to the Ox axis refractive indexes and L is the thickness of the CDW, λ is the wavelength in the vacuum.

Therefore, there are only three waves after the grating – an no diffracted wave (0^{th} order) and two diffracted waves – in the $+1$ and -1 orders. Matrix M_0 determines the 0^{th} order wave and $M_{\pm 1}$ determine the two diffracted waves. To obtain the intensities and the polarization of the waves we have to multiply the Jones vector of the reconstructing wave E_{in} by the matrix M. Let we take E_{in} to be linearly polarized at an angle β with respect to the axis Ox:

$$E_{in} = \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} \tag{5}$$

The field after the grating is given by

$$E_{out} = ME_{in} = (M_0 + M_{+1} + M_{-1})E_{in} = E_0 + M_{+1} + M_{-1} \tag{6}$$

The 0^{th}-order wave is

$$E_0 = \cos\ \Phi \begin{bmatrix} \cos\beta \\ \sin\beta \end{bmatrix} \tag{7}$$

It emerges with the same polarization and its intensity is proportional to $cos^2\ \Phi$. The waves of ± 1 orders are

$$E_{\pm 1} = \frac{isin\Phi}{2} \exp(\pm i2qx) \exp(\pm i\beta) \begin{bmatrix} 1 \\ \pm i \end{bmatrix} \tag{8}$$

The obtained optical component has a number of features quite counter-intuitive if to look at it as a diffraction grating. First, it deflects all the light into $\pm 1^{st}$ diffraction orders, and only those two orders, if its wavelength fulfills the half-waveplate condition ($\Phi = \pi/2$). This is $100\ \%$ diffraction efficiency in a technically thin grating. The diffracted beams are orthogonal circularly polarized if the incident beam is unpolarized or a linearly polarized beam. Only one of the orders is present in case of circular polarized incident beam, and the sign of (circular) polarization of the diffracted beam is reversed with respect to the incident beam. The diffraction caused by such a cycloidal grating may be fully cancelled if the second similar grating is placed after the first one.

$$E'_{\pm 1} = -\frac{1}{2} \exp(\pm i\beta) \begin{bmatrix} 1 \\ \mp i \end{bmatrix} \tag{9}$$

And the diffraction angle is doubled when one of them is rotated 180 degrees around the normal to the grating plane.

$$E'_{\pm 1} = -\frac{1}{2} \exp(\pm i\beta) \exp(\pm i4qx) \begin{bmatrix} 1 \\ \mp i \end{bmatrix} \tag{10}$$

All these properties, unusual for a diffraction grating, become evident if we note that the cycloidal grating is indeed a waveplate. Due to half-waveplate condition, a linearly polarized incident beam is transformed by the waveplate with cycloidal orientation pattern of its optical axis into a beam with similarly rotating linear polarization pattern. The average field is therefore absent and there is no beam propagating in the direction of the incident beam. This pattern, however, corresponds to the polarization modulation pattern obtained in orthogonal circular polarized beams propagating in the direction of the $\pm 1^{st}$ diffraction orders. Two half-waveplates together make a full waveplate eliminating the polarization modulation, hence cancelling the diffraction. The two cycloidal waveplates combined with opposite signs double the modulation period, hence the diffraction angle.

5. Multilayer CDW coatings

Pairing cycloidal waveplates in opposite orientation considerably widens the diffraction spectrum of the system that is already quite wide for an individual CDW exceeding $100\ nm$ in the visible spectrum even for as high as $95\ \%$ diffraction efficiency.

The Pancharatnam-Berry algorithm allows producing practically achromatic diffraction in a wide spectral range in a multilayer system. In a highly practical approach, the orientation of an individual layer of cycloidally oriented LC was further twisted using a chiral dopant in the direction perpendicular to the plane of the waveplate and a second LC layer was coated, however, with an opposite twist. The diffraction spectrum of such components can extend throughout the visible or near IR spectral range. Such a component diffracts practically $100\ \%$ of a white light. Potentially, two LCP layers, right- and left-twisted, should be sufficient for producing an achromatic CDW. In practice, it is hardly possible, and the required twist angle and the layer thickness are obtained in a larger number of layers. Note that the diffraction spectra change widely with deposition of each layer staying at low efficiency levels up to the very last step.

Combination of two half-wave waveplates into a full-wave waveplate eliminates polarization modulation at the output of the system thus cancelling the diffraction. In case of achromatic CDWs, the diffraction is cancelled for a wide spectrum of wavelengths, essentially, for all visible spectrum. Such a system, consisting of at least 8 layers, while allowing viewing through it as through a glass window, possesses with unusual spectral and angular optical characteristics. It provides important control opportunities due to the feasibility of selectively changing the optical properties of one of the components by light or electric fields. Azobenzene LCPs are used for imparting one or more of the films comprising the system with photoresponsive property.

6. The theory of achromatic cycloidal diffractive waveplates with twisted orientation

We want to discuss the system of two CDW with twist orientations in the direction perpendicular to the plate walls. These two chiral circular waveplates have opposite twist sense. We consider a CDW confined between $0 < z < L$ with the optical axis $\boldsymbol{n}$ rotating in transverse direction x and twisted in direction z, so that a periodic structure

$$\mathbf{n}_0(x) = \{\cos\delta(x,z);\ \sin\delta(x,z); 0\}$$

$$\delta(x,z) = \begin{cases} qx + \Phi_t \frac{z}{L} & when\ 0 < z < L \\ qx - \Phi_t \frac{z}{L} + 2\Phi_t & when\ L < z < 2L \end{cases} \qquad q = 2\pi/\Lambda \qquad (11)$$

is realized. Here Λ is the director's period, δ is the azimuth angle of the director field, L is the thickness, and Φ_t is the twist angle of each chiral layer. The Jones matrix of first cycloidal diffractive waveplate we wright in the form

$$M_{CDW}(\Phi) = R(\delta_0)\begin{bmatrix} e^{-i\Phi} & 0 \\ 0 & e^{i\Phi} \end{bmatrix} R(\delta_0) = \begin{bmatrix} \cos\Phi - i\sin\Phi\cos(2\delta_0) & -i\sin\Phi\sin(2\delta_0) \\ -i\sin\Phi\sin(2\delta_0) & \cos\Phi + i\sin\Phi\cos(2\delta_0) \end{bmatrix} \quad (12)$$

where $\delta_0 = qx$, R is the rotation matrix. Following to [21], the CDW with a twist can be approximated as a stack of multiple (N) thin CDW layers with a small phase shift $\Delta\Phi_t = \Phi_t/N$ in the azimuth. The Jones matrix for this twisted CDW structure can be written as

$$M_{TCDW} = M_N M_{N-1} M_{N-2} \dots M_3 M_2 M_1 = \quad (13)$$

$$\prod_{m=N}^{1} R(-m\Delta\Phi_t) M_m(\Delta\Phi) R(m\Delta\Phi_t).$$

Here $M_m(\Delta\Phi) = M_{CDW}(\Delta\Phi)$, N is the number of CDW layers, $\Delta\Phi$ is the half retardation of each layer. Doing multiplication and, using the property of the rotation matrix, we can get

$$M_{TCDW} = R(-\Phi_t)[M_{CDW}(\Delta\Phi)R(\Delta\Phi_t)]^N \quad (14)$$

This expression can be further simplified by using Chebyshev's identity for unimodular matrices [23]:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^m = \begin{bmatrix} \dfrac{AsinmZ - \sin(m-1)Z}{sinZ} & B\dfrac{sinmZ}{sinZ} \\ C\dfrac{sinmZ}{sinZ} & \dfrac{DsinmZ - \sin(m-1)Z}{sinZ} \end{bmatrix} \quad (15)$$

with

$$Z = arccos\left[\frac{1}{2}(A + D)\right]$$

Thus for twisted cycloidal diffractive waveplate we have Jones matrix in the form

$$M_{TCDW} =$$

$$\begin{bmatrix} \cos X cos\Phi_t + \Phi_t \sin\Phi_t \dfrac{sinX}{X} - i\,\Phi\cos(2\delta_0 + \Phi_t)\dfrac{sinX}{X} & -\cos X sin\Phi_t + \Phi_t \cos\Phi_t \dfrac{sinX}{X} - i\,\Phi\sin(2\delta_0 + \Phi_t)\dfrac{sinX}{X} \\ \cos X sin\Phi_t - \Phi_t \cos\Phi_t \dfrac{sinX}{X} - i\,\Phi\sin(2\delta_0 + \Phi_t)\dfrac{sinX}{X} & \cos X cos\Phi_t + \Phi_t \sin\Phi_t \dfrac{sinX}{X} + i\,\Phi\cos(2\delta_0 + \Phi_t)\dfrac{sinX}{X} \end{bmatrix}$$

(16)

where $X = \sqrt{{\Phi_t}^2 + \Phi^2}$. Here we have exact expression for the Jones matrix of a linearly twisted CDW plate. The Jones matrix can be split into three matrices M_{TCDW}^{0}, M_{TCDW}^{+1}, M_{TCDW}^{-1}.

$$M_{TCDW} = M_{TCDW}^{0} + M_{TCDW}^{+1}\exp\left[i(2\delta_0 + \Phi_t)\right] + M_{TCDW}^{-1}\exp\left[-i(2\delta_0 + \Phi_t)\right] \tag{17}$$

where

$$M_{TCDW}^{0} = \begin{bmatrix} \cos X cos\Phi_t + \Phi_t \sin\Phi_t \frac{sinX}{X} & -\cos X sin\Phi_t + \Phi_t \cos\Phi_t \frac{sinX}{X} \\ \cos X sin\Phi_t - \Phi_t \cos\Phi_t \frac{sinX}{X} & \cos X cos\Phi_t + \Phi_t \sin\Phi_t \frac{sinX}{X} \end{bmatrix}, \tag{18}$$

$$M_{TCDW}^{\pm 1} = \frac{1}{2}\Phi\frac{sinX}{X}\begin{bmatrix} -i & \mp 1 \\ \mp 1 & i \end{bmatrix} \tag{19}$$

As we know the transmission or diffraction efficiency for light transmitted through matrix $M_{i,j}$ is equal

$$T = \frac{1}{2}\sum_{i,j=1}^{2}\left|M_{i,j}\right|^2 \tag{20}$$

and we have

$$\eta_0 = \cos^2 X + {\Phi_t}^2 \left[\frac{sinX}{X}\right]^2 \tag{21}$$

$$\eta_{+1} = \eta_{-1} = \frac{1}{2}G^2\left[\frac{sinX}{X}\right]^2 \tag{22}$$

Now we consider the other TCDW with the same twist angle but opposite twist sense. In addition, because of the optical axis of front surface of this TCDW is rotated at the Φ_t angle in respect to it for the first TCDW, then in (17), (18) and (19) we have to take $(\delta_0 + \Phi_t)$ instead of δ_0. Beside this, because of we have the second TCDW with opposite twist sense so we will replace Φ_t with $-\Phi_t$. Than we will have

$$M'_{TCDW} = M_{TCDW}^{01} + M_{TCDW}^{+1}\exp\left[i(2\delta_0 + \Phi_t)\right] + M_{TCDW}^{-1}\exp\left[-i(2\delta_0 + \Phi_t)\right] \tag{23}$$

where

$$M_{TCDW}^{01} = \begin{bmatrix} \cos X cos\Phi_t + \Phi_t \sin\Phi_t \frac{sinX}{X} & \cos X sin\Phi_t - \Phi_t \cos\Phi_t \frac{sinX}{X} \\ -\cos X sin\Phi_t + \Phi_t \cos\Phi_t \frac{sinX}{X} & \cos X cos\Phi_t + \Phi_t \sin\Phi_t \frac{sinX}{X} \end{bmatrix} \tag{24}$$

The Jones matrix M_{ATCDW} for achromatic twisted cycloidal diffractive waveplate composed of two chiral CDW with opposite twist sense can be obtained simply by multiplying the Jones matrices for each TCDW:

$$M_{ATCDW} = M'_{TCDW} M_{TCDW} = M^{0}_{ATCDW} + M^{+1}_{ATCDW} \exp\,[i(2qx)] + M^{-1}_{ATCDW} \exp\,[-i(2qx)] \quad (25)$$

where

$$M^{0}_{ATCDW} = \left\{ \cos^2 X + ({\Phi_t}^2 - \Phi^2) \left[\frac{sinX}{X}\right]^2 \right\} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (26)$$

$$M^{\pm 1}_{ATCDW} = \Phi \frac{sinX}{X} \left(cosX \pm i\Phi_t \frac{sinX}{X} \right) \begin{bmatrix} -i & \mp 1 \\ \mp 1 & i \end{bmatrix} \quad (27)$$

and for diffraction efficiencies we have

$$\eta_0 = \left\{ \cos^2 X + ({\Phi_t}^2 - \Phi^2) \left[\frac{sinX}{X}\right]^2 \right\}^2 \quad (28)$$

$$\eta_{+1} = \eta_{-1} = 2\Phi^2 \left[\frac{sinX}{X}\right]^2 \left\{ \cos^2 X + {\Phi_t}^2 \left[\frac{sinX}{X}\right]^2 \right\} \quad (29)$$

In the general case when we have elliptical polarized light instead of (29) we have

$$\eta_{\pm 1} = 2\Phi^2 (1 \mp S'_3) \left[\frac{sinX}{X}\right]^2 \left\{ \cos^2 X + {\Phi_t}^2 \left[\frac{sinX}{X}\right]^2 \right\} \quad (30)$$

The term $S'_3 = S_3/S_0$ is a normalized Stokes parameter.

If we need to have $\eta_0 = 0$ in the wide range of retardation $2G$ or wavelength. Therefore, we have to solve the transcendent equation

$${\Phi_t}^2 + \Phi^2 \cos 2\left(\sqrt{{\Phi_t}^2 + \Phi^2}\right) = 0 \quad \text{or} \quad \frac{sinX}{X} = \frac{1}{\sqrt{2}\Phi} \quad (31)$$

Particularly, in the case of half CDW ($\Phi = \pi/2$) we have $X = 2.0103$ and $\Phi_t = 1.2546 = 71.88°$. Following [24], to quantitatively evaluate the diffraction bandwidth, we introduce the spectral range $\Delta\lambda$ (units of wavelength) for high diffraction efficiency as the range of wavelengths over which the total first-order diffraction $\sum \eta_{\pm 1}$ is $\geq 99.5\,\%$. The normalized bandwidth $\Delta\lambda/\lambda_{center}$ (units of %) is defined as the ratio of the spectral range to its center wavelength λ_{center}. CDW have a modest diffraction bandwidth given by $\Delta\lambda/\lambda_{center} \cong 6.8\%$. In the case of exact 0 of diffraction efficiency η_0 or $\sum \eta_{\pm 1} = 1$, when $\Phi_t = 1.2546 = 71.88$ at the central wavelength the bandwidth $\Delta\lambda/\lambda_{center} \cong 34.23\ \%$. In the case of [24] $\Phi_t = 1.22175 = 70°$. And the bandwidth is $\Delta\lambda/\lambda_{center} \cong 41.12\ \%$. If we will do the calculation more roughly we get the result of [24] $\Delta\lambda/\lambda_{center} \cong 34.3\ \%$. We can get the maximum bandwidth for $\Phi_t = 1.20498 = 69.04°$. Moreover, the bandwidth will be $\Delta\lambda/\lambda_{center} \cong 44.15\ \%$. Note that this is a 6.5-fold enhancement in the maximum diffraction bandwidth as compared with CDW. For twist angles smaller than $69.04°$, the diffraction bandwidth increases gradually as the twist angle increases. On the other hand, the diffraction bandwidth decreases drastically for higher twist angles. The achromaticity of the diffraction can be explained by the counter chromatic dispersions of retardation by linear birefringence and induced circular birefringence due to twist. The former becomes larger for shorter wavelengths while the latter becomes larger for longer wavelengths and vice versa. When $\Delta nd = \lambda/2$ and $\Phi_t = 69.04°$, the retardation compensation occurs by balancing out both effects and the achromatic diffraction is achieved.

7. Conclusion

While it comes as no surprise that a micron-thick material layer can be used for controlling the phase, hence, polarization of light, it used to be counter-intuitive to suggest that it also could deflect a beam like a thick prism or a thick Bragg grating. While DWs and multilayer DW coatings challenge Bragg gratings in some of their applications due to their ability to provide same efficiency in orders of magnitude thinner layers, the broadband nature of their diffraction both in angular as well as spectral space allowing them to act as high transmission broadband circular polarizers and beam splitters, steering of uncollimated and non-monochromatic light, including white light, open up new opportunities for information displays, spatial light modulators, high power lasers, and high –efficiency thin-film optics for infrared and THz applications.

Light transmission or reflection from multilayer dielectric coatings exhibits spectral and angular characteristics rather different from that of Fresnel reflection from a single material layer. The opportunity of reducing reflection for given wavelength or wavelength range is one of their most important applications. One of the most interesting properties of multilayer CDW coatings is the opportunity of full cancellation of diffraction.

The need for using two LCP layers to reach $+70$ or -70 degree twist is due to the photoaligning ability of the photoalignment material and LCP itself available at this point. Namely, achromaticity requires obtaining ± 70-degree twist structures. The twist is induced by doping the LCP by chiral dopants of positive and negative helical twisting power. The required twist angle is obtained by setting a given thickness for the layer. It turned out, however, that such a layer of the currently available LCP is not oriented well on the photoalignment layer. Therefore, we use two layers. The first thin layer then plays two roles: enhancing the orienting ability of the photoalignment layer, and providing fine-tuning for the twist angle.

The diffraction bandwidth of the ACDW, defined as the range of wavelengths when the diffraction efficiency exceeds 95% , spans from 410 to $640\ nm$ for the visible ACDW (Figure 2(a)), and from $510 - 850\ nm$ for the near IR ACDW.

The fabrication process of ACDWs turned to be a highly challenging task both from the standpoint of making them in large area and high efficiency, as well as from the standpoint of finding the material compositions and coating regimes that would result in achromaticity at the very last, the $5th$ stage of the process. Figure 3 shows the change in the diffraction spectrum with each coating. These are transmission spectra obtained with a fiber optics spectrometer. Low transmission T is obtained not due to absorption, but due to the diffraction of the beam out of the receiving aperture of the spectrometer. Thus, the higher the diffraction, the lower transmission is registered. The diffraction efficiency η is approximately equal $\eta = 1 - T$. Thus, just before the last coating is applied, the diffraction efficiency is near 70%, and the last LCP layer provides the fine-tuning increasing the diffraction efficiency to near 100% in a large portion of the spectrum.

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