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On the Relations Between Different Forms of the Fuzzy Constructive Logic[1]

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Abstract

A system of fuzzy constructive logic developed in [1] is considered. A partial case of a more general concept of the fuzzy constructive logic (i.e., the logical system based on the A-scale Ω_2 of truth values described in [2]) is investigated. Predicate formulas without functional symbols and object constants are considered. The notions of identically P-valid predicate formula (i.e., predicate formula which is identically true in the framework of the system defined in [1]) and of identically G-valid predicate formula (i.e., predicate formula which is identically true in the framework of the system described in [2]) are defined. It is proved that any identically G-valid predicate formula is also identically P-valid (see below, Theorem 7.1). It is proved also that any predicate formula deducible in the constructive (intuitionistic) predicate calculus is identically G-valid (see below, Theorem 7.2).

Keywords: Fuzzy constructive logic, Predicate formula, Identically P-valid predicate formula, Identically G-valid predicate formula.

1. Introduction

The concepts of fuzzy logic ([3]-[5]) are considered below in the framework of the constructive point of view ([6]-[8]). A. A. Markov's principle ([10]) will not be used. We will compare the properties of the logical system of the extended fuzzy constructive logic ([1], [9]) and of some system of the generalized fuzzy constructive logic ([2]).

In Section 2 below some auxiliary notions are defined and investigated. In Section 3 the notion of fuzzy recursively enumerable set (FRES) defined in [1] is investigated. The equivalence of this notion and the notion of generalized fuzzy enumerable set based on the A-scale Ω_2 (see [2]) is established. The definitions of regular, monotone, open FRES are given. In Section 4 the relations "α P-covers β", "α G-covers β", "α is P-equivalent to β", "α is G-equivalent to β" between FRESes α and β are introduced. In the denotations of these relations

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the prefixes and the footnotes P and G are used in the following sense: the letters P and G mean that the corresponding notion belongs to the system considered correspondingly in [1] and [2]. In Section 5 some operations on FRESes are introduced and investigated, in particular, recursive operators H_n for $n \geq 1$ (these operators are used in what follows), the operations of G-union and P-union on FRESes, the operation of G-generalization and P-generalization of FRESes. The operation of *Cartesian product* of FRESes is defined in different forms in [1] and [2]; in Section 5 only some partial case of this operation is considered (however, actually, only such partial case is used in the consideration in [1] and [2]). In Section 6 the notions of P-ideal and G-ideal are introduced and investigated. In Section 7 the notions of G-assignment and P-assignment for a given predicate formula A as well as the notions of G-interpretation and P-interpretation of A concerning a given assignment and given index majorant for A are introduced similarly to the corresponding notions given in [1] and [2]. The definitions of identically P-valid and identically G-valid predicate formula are given; on the base of these definitions. Main theorems 7.1 and 7.2 are established concerning the properties of these notions.

2. Some Preliminary Definitions

By N we denote the set of all non-negative integers $\{0,1,2,\ldots\}$. By N^n, where $n \geq 1$, we denote the set of all n-tuples $(x_1, x_2, \ldots, x_n)$, where $x_i \in N$ for $1 \leq i \leq n$. By R we denote the set of rational numbers having the form $\frac{m}{2^n}$, where $n \in N$, $m \in N$, $0 \leq \frac{m}{2^n} \leq 1$. The notions of primitive recursive function (shortly PmRF), general recursive function (GRF), partially recursive function (PtRF), primitive recursive set (PmRS), recursive set (RS), recursively enumerable set (RES) are defined in a usual way ([11]-[13]). We will use the notations $x \dot{-} y$, $|x-y|$, $x+y$, $x \cdot y$, 2^x, $sg(x)$, $\overline{sg}(x)$, $[x/y]$, $rest(x,y)$ for some PmRF which are defined as usually ([11]-[13]). The notions of μ-operator and bounded μ-operator are defined as usually ([11]-[12]). The PmRF ρ and θ are defined by the following equalities:

$$\rho(x) = \mu y\ y \leq x\ (x \dot{-} 2^y = 0);$$

$$\theta(x) = \left(\left(2x \dot{-} 2^{\rho(x)}\right) \dot{-} 1\right) + \overline{sg}(|x-1|).$$

The algorithms Γ and Γ^{-1} are defined by the following equalities:

$$\Gamma(x) = \frac{\theta(x)}{2^{\rho(x)}} \text{ for } x \in N;$$

$$\Gamma^{-1}\big(\Gamma(x)\big) = x \text{ for } x \in N.$$

It is easily seen that $\Gamma(x) \in R$ when $x \in N$, and the algorithm Γ estabilishes a one-to-one correspondence between N and R. The algorithm Γ^{-1} has a similar property.

3. Fuzzy Recursively Enumerable Sets (FRESes)

Fuzzy recursively enumerable set (FRES) having the dimension $n \geq 1$ is defined in [1] as a recursively enumerable set of $(n+1)$- tuples $(x_1, x_2, \ldots, x_n, \varepsilon)$, where $x_i \in N$ for $1 \leq i \leq n$, $\varepsilon \in R$. The notion of *generalized fuzzy recursively enumerable set* (GFRES) is defined in [2] for any given *algorithmic scale of truth values* (shortly, A-scale). Let us recall (see [2]) that any A-scale Ω is described by the following parameters: (1) the set U_Ω of truth values; (2) the relation of equality $\underset{\Omega}{=}$ between truth values; (3) the binary operations $\underset{\Omega}{\bigcup}$ and $\underset{\Omega}{\bigcap}$ on truth values; the constants 0_Ω and 1_Ω. In what follows we will consider only the A-scale Ω_2 described in [2]; we will not consider the other A-scales, and the notation Ω_2 will be replaced in what follows by Ω. Let us recall the definition of parameters of this A-scale. The set U_Ω of truth values for Ω is the set N. The predicate of equality $\underset{\Omega}{=}$ for Ω is the usual equality between the numbers belonging to N. The operations $x \underset{\Omega}{\bigcup} y$ and $x \underset{\Omega}{\bigcap} y$ are defined as, correspondingly, $\Gamma^{-1}(\max(\Gamma(x), \Gamma(y)))$ and $\Gamma^{-1}(\min(\Gamma(x), \Gamma(y)))$. The constants 0_Ω and 1_Ω are defined correspondingly as $0 \in N$ and $1 \in N$. The predicates $x \underset{\Omega}{\leq} y$, $x \underset{\Omega}{\neq} y$, $x \underset{\Omega}{<} y$ considered in [2] are defined as $\Gamma(x) \leq \Gamma(y)$, $\Gamma(x) \neq \Gamma(y)$, $\Gamma(x) < \Gamma(y)$ (see [2]). It is easily seen that all conditions introduced in [2] for A-scales are satisfied for A-scale Ω. The symbol Ω will be sometimes ommitted in the descriptions of notions connected with Ω.

It is easily seen that the notion of FRES given in [1] and the notion of GFRES of the kind described above are actually equivalent: any FRES of the form given in [1] represents a corresponding GFRES, and the reverse statement is also true. Below we will use the notations of such sets described in [1].

We say that an n-dimensional FRES α where $n \geq 1$ is *regular* if any $(n+1)$-tuple of the form $(x_1, x_2, \dots, x_n, 0)$, where $0 \in R$, belongs to α. In what follows we will consider only regular FRESes, so the terms “FRES” and “regular FRES” are admitted as equivalent.

Note. Similar agreement concerning the regularity of all considered FRESes may be introduced also in [1] and [2] without essential changes in the systems of definitions described in [1] and [2].

We say that an n-dimensional FRES α is *monotone* if the following condition holds: if $(x_1, x_2, \dots, x_n, \varepsilon) \in \alpha$ and $\delta \leq \varepsilon$ then $(x_1, x_2, \dots, x_n, \delta) \in \alpha$ (cf. [2]). We say that an n-dimensional FRES α is *open* if it is monotone and for any $(n+1)$- tuple $(x_1, x_2, \dots, x_n, \varepsilon) \in \alpha$, where $\varepsilon > 0$, there exists some $\delta > \varepsilon$ such that $(x_1, x_2, \dots, x_n, \delta) \in \alpha$ (cf. [1]).

4. Some Relations Between FRESes

The relations between the FRESes considered below are different in the systems of notions given in [1] and [2]. So, we will use the prefix “P” in the detonations of notions considered in the framework of notions in [1] and the prefix “G” in the detonations of notions considered in the framework of notions in [2] (we may say that the prefix “P” is the abbreviation of the expression: “in the sense of notions defined in [1]”, the prefix “G” is the abbreviation of the expression: “in the sense of notions defined in [2]”).

Let α and β be n-dimensional FRESes. We will say that α *P-covers* β and write $\beta \underset{P}{\subseteq} \alpha$ if for any $(n+1)$-tuple $(x_1, x_2, \dots, x_n, \varepsilon) \in \beta$, and any $\varepsilon_1 > 0$ there exists an $(n+1)$-tuple $(x_1, x_2, \dots, x_n, \delta) \in \alpha$ such that $\delta > \varepsilon - \varepsilon_1$. We will say that α *G-covers* β and write $\beta \underset{G}{\subseteq} \alpha$ if for any $(n+1)$-tuple $(x_1, x_2, \dots, x_n, \varepsilon) \in \beta$ there exists an $(n+1)$-tuple $(x_1, x_2, \dots, x_n, \delta) \in \alpha$ such that $\delta \geq \varepsilon$ (cf. [2]). We will say that α and β are *P-equivalent* (correspondingly, *G-equivalent*) and write $\alpha \underset{P}{=} \beta$ (correspondingly, $\alpha \underset{G}{=} \beta$) if $\alpha \underset{P}{\subseteq} \beta$ and $\beta \underset{P}{\subseteq} \alpha$ (correspondingly, $\alpha \underset{G}{\subseteq} \beta$ and $\beta \underset{G}{\subseteq} \alpha$). The negations of the statements $\alpha \underset{P}{=} \beta$ and $\alpha \underset{G}{=} \beta$ will be denoted by $\alpha \underset{P}{\neq} \beta$ and $\alpha \underset{G}{\neq} \beta$, the negations of the statements $\alpha \underset{P}{\subseteq} \beta$ and $\alpha \underset{G}{\subseteq} \beta$ will be denoted by $\alpha \nsubseteq_P \beta$ and $\alpha \nsubseteq_G \beta$.

It is easily seen that if $\alpha \subseteq_G \beta$ then $\alpha \subseteq_P \beta$ but the reverse is in general not true. For example, let α be the set of 2-tuples (x, ε) such that $(x, \varepsilon) \in \alpha$ if and only if $x \in N$ and $\varepsilon \in R$, $0 \leq \varepsilon \leq \frac{1}{2}$. Let β be the set of 2-tuples (x, ε) such that $(x, \varepsilon) \in \beta$ if and only if $x \in N$ and $\varepsilon \in R$, $0 \leq \varepsilon < \frac{1}{2}$. Then $\beta \subseteq_P \alpha$ but $\beta \nsubseteq_G \alpha$.

5. Some Operations on FRESes

For the description of operations considered below we will introduce some auxiliary notions. We will introduce for any $n \geq 1$ the recursive operator H_n transforming any n-dimensional FRES α to some n-dimensional FRES $H_n(\alpha)$ (the notion of recursive operator is defined as in [13]). This operator is defined by the following conditions: 1) If α is any n-dimensional monotone FRES, then $H_n(\alpha) \subseteq_G \alpha$ (hence, $H_n(\alpha) \subseteq_P \alpha$). 2) For any n-dimensional FRES α the following statement holds: if $(x_1, x_2, \ldots, x_n, \varepsilon) \in H_n(\alpha)$ and $\delta \leq \varepsilon$, then $(x_1, x_2, \ldots, x_n, \delta) \in H_n(\alpha)$ (so, the FRES $H_n(\alpha)$ is regular and monotone for any regular α). 3) For any n-dimensional FRES α the following statement holds: if $(x_1, x_2, \ldots, x_n, \varepsilon) \in \alpha$, where $\varepsilon > 0$, then $(x_1, x_2, \ldots, x_n, \varepsilon) \in H_n(\alpha)$ if and only if there exists some $\delta > \varepsilon$ such that $(x_1, x_2, \ldots, x_n, \delta) \in \alpha$.

It is easily seen that for any $n \geq 1$ there exists a recursive operator H_n such that the mentioned conditions are satisfied (see [13]).

Lemma 5.1: *If α is an n-dimensional monotone regular FRES, then $H_n(\alpha)$ is open.*

Lemma 5.2: *If α is an n-dimensional monotone regular FRES, then $H_n(\alpha) =_G \alpha$ if and only if α is open.*

Lemma 5.3: *If α and β are n-dimensional monotone regular FRESes such that $\alpha \subseteq_G \beta$, then $H_n(\alpha) \subseteq_G H_n(\beta)$.*

Lemma 5.4: *If α is an n-dimensional monotone regular FRES and β is an open FRES such that $\alpha \subseteq_G \beta$, then $H_n(\alpha) \subseteq_G \beta$.*

Lemma 5.5: *If α and β are n-dimensional monotone regular FRESes then $H_n(\alpha \cup \beta) =_G H_n(\alpha) \cup_G H_n(\beta)$, $H_n(\alpha \cap \beta) =_G H_n(\alpha) \cap_G H_n(\beta)$.*

The proofs are easily obtained using the definition of $H_n(\alpha)$.

We may say that in the mentioned cases $H_n(\alpha)$ is a maximal open set contained in α.

Now let us consider some operations on FRESes. The *G-union* $\alpha \cup_G \beta$ (correspondingly, *G-intersection* $\alpha \cap_G \beta$) of n-dimensional FRESes α and β is defined as an n-dimensional FRES γ such that $(x_1, x_2, \ldots, x_n, \eta) \in \gamma$ if and only if there exist $\varepsilon \in R$ and $\delta \in R$ such that $(x_1, x_2, \ldots, x_n, \varepsilon) \in \alpha$, $(x_1, x_2, \ldots, x_n, \delta) \in \beta$, $\eta \leq \max(\varepsilon, \delta)$ (correspondingly, $\eta \leq \min(\varepsilon, \delta)$) (cf. [2]).

The *P-union* $\alpha \cup_P \beta$ (correspondingly, *P-intersection* $\alpha \cap_P \beta$) of some n-dimensional FRESes α and β is defined as $H_n(\alpha \cup_G \beta)$ (correspondingly, $H_n(\alpha \cap_G \beta)$).

It is easily seen that there exist recursive operators realizing the operations $\alpha \cup_G \beta$, $\alpha \cap_G \beta$, $\alpha \cup_P \beta$, $\alpha \cap_P \beta$. The operations $\alpha \cup_G \beta$ and $\alpha \cap_G \beta$ for monotone FRESes as well as the operations

$\alpha \bigcup_P \beta$ and $\alpha \bigcap_P \beta$ for open FRESes give the same results as, correspondingly, the union and the intersection of FRESes in the set-theoretical sense.

The notion of *projection* of a given FRES concerning a given coordinate is given in a same way for two mentioned kinds of notions, so, we omit the prefixes P- and G- in the description of this notion. We say that an $(n-1)$-dimensional FRES β is the *projection* of an n-dimensional FRES α, where $n > 1$, concerning i-th coordinate x_i, where $1 \leq i \leq n$, if the following condition holds: $(x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_n, \varepsilon) \in \beta$ if $(x_1, x_2, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_n, \varepsilon) \in \alpha$ for some $x_i \in N$. In the mentioned cases the FRES β will be denoted by $\downarrow_i^n (\alpha)$.

We say that the i-th coordinate x_i (where $1 \leq i \leq n$) is *fictitious* for an n-dimensional FRES α if the following condition holds: $(x_1, x_2, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_n, \varepsilon) \in \alpha$ if and only if $(x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_n, \varepsilon) \in \alpha$.

By V_G^n (correspondingly, V_P^n), where $n \geq 1$, we denote the FRES such that $(x_1, x_2, \dots, x_n, \varepsilon) \in V_G^n$ (correspondingly, $(x_1, x_2, \dots, x_n, \varepsilon) \in V_P^n$) when $x_i \in N$ for $1 \leq i \leq n$ and $0 \leq \varepsilon \leq 1$ (correspondingly, $0 \leq \varepsilon < 1$). By Λ^n where $n \geq 1$, we denote the FRES such that $(x_1, x_2, \dots, x_n, \varepsilon) \in \Lambda^n$ if $x_i \in N$ for $1 \leq i \leq n$, $\varepsilon = 0$.

The G-*generalization* of an n-dimensional FRES α, where $n > 1$ concerning i-th coordinate is defined as an n-dimensional FRES β such that $(x_1, x_2, \dots, x_{i-1}, y, x_{i+1}, \dots, x_n, \varepsilon) \in \beta$ if $(x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_n, \varepsilon) \in \downarrow_i^n (\alpha)$ (for any $y \in N$). The G-*generalization* of an 1-dimensional FRES α concerning its single coordinate is defined as the set of all pairs having the form (y, ε), where $y \in N$ and $(x_1, \varepsilon) \in \alpha$ for some $x_1 \in N$. The G-generalization of an n-dimensional FRES α (both for $n > 1$ and for $n = 1$) concerning its i-th coordinate will be denoted as $\uparrow_{i,G}^n (\alpha)$. The P-*generalization* of an n-dimensional FRES α concerning its i-th coordinate is defined as $H_n(\uparrow_{i,G}^n (\alpha))$; it will be denoted by $\uparrow_{i,P}^n (\alpha)$.

The *Cartesian product* of FRESes α and β is defined in different ways in [1] and [2]; we will consider some partial cases of the corresponding notions when the second set β has the form V_G^n or V_P^n for $n \geq 1$. Actually only such partial cases are used in [1] and [2] in the further considerations. As it will be seen from the definitions given below, the FRESes $\alpha \times V_P^n$ and $\alpha \times V_G^n$ are G-equivalent for any FRES α (though V_G^n and V_P^n are not G-equivalent!). So, we will omit the symbols G and P in the notations of these FRESes. Now let α be an n-dimensional FRES, where $n \geq 1$. By $\alpha \times V^m$, where $m \geq 1$, we will denote the $(n+m)$-dimensional FRES defined by the following condition: if $(x_1, x_2, \dots, x_n, \varepsilon) \in \alpha$, then $(x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_m, \varepsilon) \in \alpha \times V^m$, where $x_i \in N$, $y_j \in N$ for $1 \leq i \leq n$, $1 \leq j \leq m$. (Clearly the transformation of α to $\alpha \times V^m$ may be characterized as the introducing of m fictious variables in α).

The operation T_{ij}^n of *transposition* of i-th and j-th coordinates in an n-dimensional FRES α and the operation of *substitution* S_{ij}^n of the j-th coordinate for the i-th coordinate in an n-dimensional FRES α are defined in an obvious way similar to the corresponding definitions given in [1] and [2].

6. Ideals

A non-empty constructive set Δ of n-dimensional monotone FRESes is said to be n-dimensional G-*ideal* if the following conditions hold: (1) If $\alpha \in \Delta$ and $\beta \underset{G}{\subseteq} \alpha$, then $\beta \in \Delta$. (2) If $\alpha \in \Delta$ and $\beta \in \Delta$, then $\alpha \bigcup_G \beta \in \Delta$ (cf. [2]).

A non-empty constructive set Δ of n-dimensional open FRESes is said to be n-dimensional P-*ideal* if the following conditions hold: (1) If $\alpha \in \Delta$ and $\beta \underset{P}{\subseteq} \alpha$, then $\beta \in \Delta$. (2) If $\alpha \in \Delta$ and $\beta \in \Delta$, then $\alpha \underset{P}{\cup} \beta \in \Delta$ (cf. [1]).

The notion of constructive set is interpreted following to the principles of constructive mathematics ([4]-[6]).

Lemma 6.1: *If Δ_1 is an n-dimensional G-ideal and Δ_2 is the set of FRESes having the form $H_n(\alpha)$, where $\alpha \in \Delta_1$, then Δ_2 is a P-ideal.*

The proof is easily obtained using the corresponding definitions.

An n-dimensional G-ideal (correspondingly, P-ideal) Δ is said to be *complete* if $V_G^n \in \Delta$ (correspondingly, $V_P^n \in \Delta$). Clearly, in this case any n-dimensional monotone (correspondingly, open) FRES belongs to Δ. An n-dimensional G-ideal or P-ideal is said to be *null-ideal* if $\alpha \in \Delta$ only when $\alpha \underset{G}{=} \Lambda^n$ or $\alpha \underset{P}{=} \Lambda^n$ (clearly, in this case such statements are equivalent).

7. Predicate Formulas and Their Interpretations

We consider predicate formulas (as in [1] and [2]) on the base of logical operations $\&, \vee, \supset, \neg, \forall, \exists$ without functional symbols and object constants. The symbols of truth and falsity, T and F, are considered as elementary formulas. Predicate formulas will be denoted as A, B, C, D (probably, with indexes). We suppose that the sequence $x_1, x_2, \dots, x_n, \dots$ is fixed including all object variables contained in all predicate formulas.

Index majorant for a predicate formula A is defined as any natural number which is greater or equal to all indexes i of object variables x_i contained in A and the dimensions of all predicate symbols contained in A (cf. [2]).

By $H^{(con)}$ we denote the constructive (intuitionistic) predicate calculus on the base of predicate formulas of the kind mentioned above ([14]).

Let A be a predicate formula, let $p_1, p_2, \dots, p_l$ be the list of all predicate symbols contained in A. Let us denote the dimensions of $p_1, p_2, \dots, p_l$ correspondingly by $k_1, k_2, \dots, k_l$. Let k be an index majorant for the formula A. We define the G-*assignment* (correspondingly, P-*assignment*) for A as the sequence $\varphi_1, \varphi_2, \dots, \varphi_l$, where any φ_i for $1 \le i \le l$ is an k_i-dimensional G-idelal (correspondingly, P-ideal). The sequence $\varphi_1, \varphi_2, \dots, \varphi_l$ we will denote in what follows by Φ.

Let A be a predicate formula, k be an index majorant for A. We define the G-*interpretation* $\Pi_{G,\Phi,k}(A)$ (correspondingly, P-*interpretation* $\Pi_{P,\Phi,k}(A)$) of the formula A concerning G-assignment Φ (correspondingly, P-assignment Φ) for A and for index majorant k of A by induction using the construction of A. The G-interpretation $\Pi_{G,\Phi,k}(A)$ is defined similar to the definition of $\Pi_{\Omega,\varphi,k}(A)$ given in [2] (see [2], pp. 272-273). The P-interpretation $\Pi_{P,\Phi,k}(A)$ is defined similar to the definition of $\Pi_{\varphi,k}(A)$ given in [1] (see [1] pp. 50-51).

We say that a predicate formula A is idenitcally G-valid (correspondingly, identically P-valid) if the G-interpretation (correspondingly, P-interpretation) of the formula A concerning any G-assignment (correspondingly, P-assignment) Φ for A and any sufficiently great index majorant k for A is a complete G-ideal (correspondingly, complete P-ideal).

Let us introduce some auxiliary notations. If Φ is a G-assignment or P-assignment for a formula A, and Φ has the form $\varphi_1, \varphi_2, \dots, \varphi_l$ where any ideal φ_i has the dimension k_i, then by $H(\Phi)$ we denote the sequence $\psi_1, \psi_2, \dots, \psi_l$, where any ideal ψ_i is the set of k_i-dimensional FRESes having the form $H_m(\theta)$, where $m = k_i$, $\theta \in \varphi_i$. It is easily seen that if Φ is a G-

assignment or P-assignment for a formula A then $H(\Phi)$ is a P-assignment for A. Besides, if Φ is a P-assignment for A, then $H(\Phi)$ is equal to Φ (in some natural sense).

Lemma 7.1: *Let A be a predicate formula of the kind mentioned above, let Φ be a G-assignment for A, let k be an index majorant for A, let $\Pi_{G,\Phi,k}(A)$ be the G −interpretation of A concerning Φ and k. Let $\Pi_{\Phi,H(\Phi),k}(A)$ be the P-interpretation of A concerning $H(\Phi)$ and k. Then $\Pi_{P,H(\Phi),k}(A)$ has the form $H_m(\Pi_{G,\Phi,k}(A))$ where m is the dimension of $\Pi_{G,\Phi,k}(A)$.*

The proof is obtained using the induction on the construction of the formula A.

Theorem 7.1: *If a predicate formula A is identically G-valid, then it is also identically P-valid.*

The proof is obtained using Lemma 7.1 and the definitions of identically P-valid and identically G-valid predicate formula.

Theorem 7.2: *If a predicate formula of the kind mentioned above is deducible in the constructive (intuitionistic) predicate calculus $H^{(con)}$, then it is identically G-valid*

The proof is similar to the proof of Theorem 5.1 in [1] (see [1] pp. 57-63).

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Ոչ պարզորոշ կոնստրուկտիվ տրամաբանության տարբեր ձևերի միջև փոխհարաբերությունների մասին

Ի. Զասլավսկի

Ամփոփում

Դիտարկվում է ոչ պարզորոշ կոնստրուկտիվ տրամաբանության համակարգը, որը նկարագրված է [1]-ում: Դիտարկվում է նաև ոչ պարզորոշ կոնստրուկտիվ տրամաբանության ավելի ընդհանուր գաղափարի որոշ մասնավոր դեպք (այսինքն՝ տրամաբանական համակարգ, որը հիմնվում է [2]-ում ներկայացված տրամաբանական արժեքների Ω_2 A-սանդղակի վրա): Դիտարկվում են պրեդիկատային բանաձևեր առանց ֆունկցիոնալ նշանների և առարկայական հաստատունների: Սահմանվում են նույնաբար P-ճշմարիտ (այսինքն՝ [1]-ում նկարագրված տրամաբանական համակարգի տեսակետից նույնաբար ճշմարիտ) և նույնաբար G-ճշմարիտ (այսինքն՝ [2]-ում նկարագրված համակարգի տեսակետից նույնաբար ճշմարիտ) պրեդիկատային բանաձևի հասկացությունները: Ապացուցվում է (թեորեմ 7.1), որ նույնաբար G-ճշմարիտ ցանկացած պրեդիկատային բանաձև կլինի նաև նույնաբար P-ճշմարիտ: Ապացուցվում է նաև (թեորեմ 7.2), որ կոնստրուկտիվ (ինտուիցիոնիստական) պրեդիկատային հաշվում արտածվող ցանկացած բանաձև նույնաբար G-ճշմարիտ է:

О соотношениях между различными формами нечеткой конструктивной логики

И. Заславский

Аннотация

Рассматривается система нечеткой конструктивной логики, представленная в [1]. Исследуется также некоторый частный случай обобщенной концепции нечеткой конструктивной логики, представленной в [2] (а именно, логическая система, основанная на алгоритмической шкале (А-шкале) логических значений Ω_2, описанной в [2]). Рассматриваются предикатные формулы без функциональных символов и предметных констант. Определяются понятия тождественно P-истинной предикатной формулы (то есть предикатной формулы, тождественно истинной с точки зрения системы понятий, введённых в [1]) и тождественно G-истинной предикатной формулы (то есть предикатной формулы, тождественно истинной с точки зрения системы понятий, введённых в [2]). Доказывается (теорема 7.1), что всякая тождественно G-истинная предикатная формула является также тождественно P-истинной. Кроме того, доказывается (теорема 7.2), что всякая предикатная формула, выводимая в конструктивном (интуиционистском) исчислении предикатов, является тождественно G-истинной.