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On the Hierarchies of Some Propositional Systems for Classical and Non-classical Logics *

Anahit Chubaryan

Department of Informatics and Applied Mathematics,
Yerevan State University, Armenia
achubaryan@ysu.am

The families of elimination systems with full substitution rule and with restricted sub-titution rules are introduced for Classical, Intuitionistic and Minimal (Iohansson's) propo-itional logics (CPL, IPL,MPL), and the efficiencies of introduced systems are compared for very mentioned logic.

We use the notions of determinative conjunct and determinative normal forms, introduced or CPL in [1], for IPL and MPL in [2].

Let φ be a propositional formula, $P = \{p_1, p_2, \ldots, p_n\}$ be the set of all variables of φ, nd $P' = \{p_{i_1}, p_{i_2}, \ldots, p_{i_m}\}$ $(1 \leq m \leq n)$ be some subset of P.

Definition 1. *Given $\sigma = \{\sigma_1, \ldots, \sigma_m\} \subset E^m$ (unit Boolean cube), the conjunct $K^\sigma = \{p_{i_1}^{\sigma_1}, p_{i_2}^{\sigma_2}, \ldots, p_{i_m}^{\sigma_m}\}$ is called $\varphi - 1$-determinative ($\varphi - 0$-determinative) if assigning σ_j $(1 \leq j \leq m)$ to each p_{i_j} we obtain the value of φ (1 or 0) independently of the values of the emaining variables.*

Definition 2. *DNF $D = \{K_1, K_2, \ldots, K_r\}$ is called determinative DNF (dDNF) for φ if $\varphi = D$ and every conjunct K_i $(1 \leq i \leq r)$ is 1-determinative for φ.*

The investigated systems are the following systems EC, EI and EM and their general-zations. The axioms of EC are not fixed, but for every formula φ each conjunct from some DNF of φ can be considered as an axiom.

The *elimination rule* (ε-rule) infers $K' \cup K''$ from conjuncts $K' \cup \{p\}$ and $K' \cup \{\overline{p}\}$, where K' and K'' are conjuncts and p is a variable.

DNF $D = \{K_1, K_2, \ldots, K_l\}$ is called full (tautology) if using ε-rule can be proved the mpty conjunction ($\emptyset$) from the axioms $\{K_1, K_2, \ldots, K_l\}$.

The analogies of the determinative conjuncts and dDNF for IPL and MPL (I-dDNF and M-dDNF accordingly) are constructed in [2]. Note that the literals in the latter conjuncts re only variables with negation or with double negations.

By analogy the corresponding proof system EI (EM) can be constructed for IPL (MPL). As axiom is considered every I-determinative (M-determinative) conjunct from some -determinative (M-determinative) DNF.

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For EI (EM) we take the following inference rule

$$\frac{K' \cup \overline{\overline{p}} \quad K'' \cup \overline{p}}{K' \cup K''} \quad I_e - rule$$

$$\left(\frac{K' \cup (p \supset \bot) \supset \bot \quad K'' \cup (p \supset \bot)}{K' \cup K''} \quad M_e - rule\right),$$

where K' and K'' are conjuncts and p is a variable. Let us introduce the substitution rule for the set of conjuncts C as following

$$\frac{C}{S(C)_p^A},$$

where $S(C)_p^A$ denotes the set of results of substitution of formula A instead of variable p everywhere in the conjuncts of the set C, and therefore we have the *generalized* elimination rule for a formula A

$$\frac{C_1 \cup \{A\} \; C_2 \cup \{\overline{A}\}}{C_1 \cup C_2},$$ where A is a literal or on any step substituted formula.

By SEC we denote the system EC with substitution rule and generalized elimination rule. If the number of connectives of substituted formulas is bounded by ℓ, then the corresponding system is denoted by $S_\ell EC$.

The systems SEI, SEM, $S_\ell EI$, $S_\ell EM$ are defined by analogy on the base of the systems EI and EM, using the corresponding generalized elimination rules.

We define the complexity to be the size of a proof (= the total number of symbols).

The minimal complexity of a formula φ (or its representation) in a proof system Φ we denote by l_φ^Φ.

To compare the efficiencies of introduced systems we use the well-known notions of p-simulation, p-equivalence and exponential speed-up from [3].

We use also the well-known notions of Frege systems FC, FI and FM for CPL, IPL and MPL accordingly (see for example in [2]).

Main Theorem

1. For every $l > 0$ the system $S_{\ell+1}EC$ ($S_{\ell+1}EI$, $S_{\ell+1}EM$) has exponential speed-up over the system $S_\ell EC$ ($S_\ell EI$, $S_\ell EM$) in tree form.
2. The systems SEC (SEI, SEM) and FC (FI, FM) are p-equivalent.

References

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