Mathematical Problems of Computer Science 38, 93--94, 2012.

# About Order 3 Hypergroups Over Group Which Arise From Groups of Order 18

Pervaneh Zolfaghari

Yerevan State University

Each (unitary) hypergroup M over a group H is isomorphic to a hypergroup over group associated with a complementary set M to a subgroup H of a group G. All unitary hypergroups of order 3 over group, associated with the complementary sets to a subgroup of symmetric groups $S_3$ and $S_4$ was described in [1] and [2]. According to [3] any hypergroup over group is reduced to an irreducible hypergroup over group. Therefore to describe all hypergroups over group first of all it is necessary to find all irreducible hypergroups over group.

Let M be a hypergroup over a group H. Then there exists a structural homomorphism from H to the $S_M$. The hypergroup over group is irreducible if and only if the kernel of this monomorphism is trivial.

Suppose $|M| = 3$. Then we have the following sorts of monomorphisms from H to $S_M$.

1) Trivial $|H| = 1$. In this case the hypergroup over group is a group.
2) $|H| = 2$. In this case the hypergroup M is isomorphic to a hypergroup associated with a subgroup of index 3 in $S_3$. There exist three such hypergroups over group (up to isomorphism). One of this hypergroups is reducible and is reduced to a group. Two others are irreducible.
3) $|H| = 3$. Then the hypergroup over group is reduced to a group, because there does not exist a non-abelian group of order 9.
4) $|H| = 6$. Every such hypergroup over group is associated with a complementary subset M to a subgroup H of index 3 in a group G of order 18.

Up to isomorphism, there exist three non-abelian groups G of order 18.

(a) $G = \langle a, b, c, a^3 = b^2 = c^3 = e, ba = a^2b, ca = ac, cb = bc \rangle$.

This group is the direct product of its symmetric subgroup $N = \langle a, b \rangle \cong S_3$, and cyclic subgroup $\langle c \rangle \cong C_3$. It has four subgroups of index 3: one normal subgroup N and three conjugate cyclic subgroups, which are generated by c and one of elements of order 2 in N.

(b) $G = \langle a, b, a^9 = b^2 = baba = e \rangle$.

Then $G = D_9$ is the dihedral group and is the semidirect product of the normal subgroup $N = \langle a \rangle \cong C_9$, and cyclic subgroup $\langle b \rangle \cong C_2$. This group has three subgroups of order 6. They are conjugate and isomorphic to $S_3$.

(c) $G = \langle a, b, c, a^3 = b^3 = c^2 = e, ba = ab, ca = ac, cb = bc \rangle$.

This group is a semidirect product of the normal subgroup $N = \langle a, b \rangle \equiv C_3 \times C_3$ and cyclic subgroup $\langle c \rangle \equiv C_2$. It has 12 subgroups of index 3. They are generated by one element of order 3 and one element of order 2. Two subgroups of order 6 are conjugate if and only if they have the same generating element of order 3. Thus, there are four classes consisting of three conjugate subgroups of order 6

In case (a) for an arbitrary subgroup H of order 6 of the group G the structural homomorphism from H to $S_M$ has a nontrivial kernel, i. e. in this case we have not irreducible hypergroups over group.

The description of order 3 hypergroups over group, arising in the cases (b) and (c), is more complicate.

## Reference


1. Zolfaghary P., *The hypergroups of order 3, arising from symmetric group* $S_3$. Fourth Group Theory Conference of Iran, Payam Noor University of Isfahan, Isfahan, Iran, March 7-9, 2012 .

2. Zolfaghary P., *The order three right hypergroups over group, arising from symmetric group* $S_4$. Thes. of Conf. of AMU, Yerevan, 25 may -2 June 2012. p. 94-96.

3. Dalalyan S. H., *The reducibility theory for hypergroups over group*. Thes. of Conf. of AMU, Yerevan, 25 may -2 Juin 2012. p. 22 - 24.