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Extensions of Markov's Constructive Continuum and Uniform Continuity of Constructive Functions

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We consider everywhere defined constructive functions (c.f.) on the closed unit construc tive interval. As is well-known by the famous Zaslavsky-Tseitin Theorem such a c.f. can b effectively nonuniformly continuous. In this case it can not be extended to a classical contin uous function. In reality, in every known counter-example a singularity could be discovere already on the level of pseudonumbers. Let us recall that a pseudonumber is a recursive se quence of rationals that is a Cauchy sequence classically. Pseudonumbers can be considere as ϕ' (Δ_2)-computable numbers as well. Let D be the set of all constructive real number (Markov's Continuum in the title), D_1 the set of all pseudonumbers. A c.f. f is said to b 1-complete if it can be extended to a computable (and so continuous) function over D_1.

Theorem 1 *There is a 1-complete c.f. that is effectively nonuniformly continuous.*

This result is rather precise as a c.f. continuously extendible to ϕ''-computable numbers i uniformly continuous classically.

As is well known every c.f. can be computed on D by a Kleene operator (partial-recursiv operator). The following result together with Theorem 1 shows that there is an essenti difference between Markov's and Kleene's Computability over D_1.

Theorem 2 *A c.f. f is constructively uniformly continuous iff there is a 1-complete Kleen operator that computes f.*

References

[1] B.A. Kushner, Some Extensions of Markov's Constructive Continuum and their Appl cations to the Theory of Constructive Functions, The L.E.J.Brouwer Centenary Sympo sium, North-Holland Publ. Co., Amsterdam, 1982, Pp. 261–273

[2] B.A. Kushner, Lectures on Constructive Mathematical Analysis. (Translation from th Russian), AMS, Providence, Rhode Island, 1984