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Many Hypotheses Parallel Distributed Detection of the Pair of Families of Probability Distributions

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1 Introduction

There is a considerable literature on the problems of distributed detection and decision in engineering contexts [4, 5, 6]. The decentralized or distributed detection problem was first formulated and studied by Tenney and Sandell [7]. It consists of N geographically dispersed sensors, one-way communication links, and a fusion center. Each sensor makes an observation (denoted by X_i, $i = \overline{1,N}$) of a random source, quantizes X_i into an M-ary message $U_i = g_i(X_i)$, and then transmits U_i, $i = \overline{1,N}$ to the fusion center. Upon receipt of $U_1, U_2, ..., U_N$ the fusion center makes a global decision $U_0 = \mathcal{D}(U_1, U_2, ..., U_N)$ about the nature of the random source. The messages $U_1, U_2, ..., U_N$ are all transmitted to the fusion center which declares hypothesis H_i, $i = \overline{1,N}$ to be true, based on a decision rule $\mathcal{D}$.

Haroutunian [1] investigated the problem of LAO testing of multiple statistical hypotheses. The model of the two-stage LAO testing in multiple hypotheses for a pair of families of distributions is investigated in [2, 3]. In this paper the problem of parallel distributed detection of two-stage multiple hypotheses LAO testing to detect between hypotheses consisting of the pair families of probability distributions (PDs) is studied.

Each sensor observation x takes values in the set $\mathcal{X}$. A deterministic M-ary quantizer is a measurable mapping g from the observation space $\mathcal{X}$ to the message space $\mathcal{U} = \{1, 2, ..., M\}$. Random variable (RV) X characterizing the studied object takes values in the set $\mathcal{X}$ and $\mathcal{P}(\mathcal{X})$ is the space of all distributions on $\mathcal{X}$. The random source have S hypothetical probability distributions (PDs) of X that divided in two disjoint families of distributions. The first family includes R hypotheses $P_1, P_2, ..., P_R$ and the second family consists of $S - R$ hypotheses $P_{R+1}, P_{R+2}, ..., P_S$. The distribution of X under hypotheses H_i is denoted by P_i, $i = \overline{1,N}$. The distributions of the message produced by g are denoted by $P_{i(g)}$, and it is obtainable from P_i and g.

2 The One-stage LAO Multihypotheses Testing of Distributed Detection

We call the procedure of making decision on the base of N-sample the test ϕ^N when it is one-stage. The statistician must detect one among S hypotheses. We study the probabilities of the erroneous acceptance of hypothesis H_l provided that H_s is true $\alpha_{l|s}(\phi^N) \triangleq$

$P_s^N(U_0 = l)$, $l, s = \overline{1, S}$, $l \neq s$ and if the hypothesis H_s is true, but it is not accepted, then the probability of error is $\alpha_{s|s}(\phi^N) \triangleq P_s^N(U_0 \neq s) = \sum_{l \neq s} \alpha_{l|s}(\phi^N)$, $s = \overline{1, S}$. Corresponding "reliabilities", are defined for infinite sequence of tests ϕ as follows:

$$E_{l|s}(\phi) \triangleq \limsup_{N \to \infty} \{-\frac{1}{N} \log \alpha_{l|s}(\phi^N)\}, \; l, s = \overline{1, S}.$$

For construction of the necessary LAO test for preliminarily given positive values $E_{1|1}, \ldots, E_{S-1|S-1}$, we define:

$$\mathcal{R}_s \triangleq \{Q: \; D(Q||P_{s(g)}) \leq E_{s|s}\}, \; s = \overline{1, S-1}, \; \mathcal{R}_S \triangleq \{Q: \; D(Q||P_{s(g)}) > E_{s|s}, \; s = \overline{1, S-}$$

$$E^*_{s|s} \triangleq E_{s|s}, \; s = \overline{1, S-1}, \; E^*_{l|s} \triangleq \inf_{Q: D(Q||P_{l(g)}) \leq E^*_{l|l}} D(Q||P_{s(g)}), \; l, s = \overline{1, S}, \; s \neq l, \; E^*_{S|S} \triangleq \min_{l \neq S} \quad (1)$$

If all distributions P_s, $s = \overline{1, S}$, are different such that the following inequalities hold

$$E_{1|1} < \min_{l = \overline{2, S}} D(P_{l(g)}||P_{1(g)}), \quad E_{s|s} < \min\left[\min_{l = \overline{1, s-1}} E^*_{l|s}, \; \min_{l = \overline{s+1, S}} D(P_{l(g)}||P_{s(g)})\right], \quad s = \overline{2, S-1}, \quad (2)$$

then there exists a *LAO* sequence of tests, all elements of the reliabilities matrix $\mathbf{E}^* = \{E^*_{l|s}\}$ of which are positive and are defined in (1).

When one of the inequalities (2) is violated, then at least one element of the matrix $\mathbf{E}^*$ is equal to zero.

3 The Two-stage LAO Testing of Distributed Detection

Suppose $N = N_1 + N_2$ be such that $N_1 = \lceil \psi N \rceil$, $N_2 = [(1 - \psi)N]$, $0 < \psi < 1$, and $\mathrm{x} = (\mathrm{x}_1, \mathrm{x}_2)$, $\mathrm{x} \in \mathcal{X}^N$, $\mathcal{X}^N = \mathcal{X}^{N_1} \times \mathcal{X}^{N_2}$. And we have vectors of messages as $\mathrm{u} = (\mathrm{u}_1, \mathrm{u}_2)$, $\mathrm{u} \in \mathcal{U}^N$, $\mathcal{U}^N = \mathcal{U}^{N_1} \times \mathcal{U}^{N_2}$. The two-stage test on the base of N-sample we denote by $\Phi^N = (\varphi_1^{N_1}, \varphi_2^{N_2})$ is the parallel distributed detection system depicted in Figure 1. The first stage is for choice of a family of PDs, it is executed by a non-randomized test $\varphi_1^{N_1}(\mathrm{u}_1)$ using messages sample u_1. The next stage is a non-randomized test $\varphi_2^{N_2}(\mathrm{u}_2, U')$ based on messages sample u_2 and the outcome of the first fusion center U'.

First stage of two-stage testing of distributed detection is as follows:
Let us introduce two sets of indices $\mathcal{D}_1 = \{\overline{1, R}\}$ and $\mathcal{D}_2 = \{\overline{R+1, S}\}$ and a pair of disjoint families of PDs are $\mathcal{P}_1 = \{P_s, \; s \in \mathcal{D}_1\}$ and $\mathcal{P}_2 = \{P_s, \; s \in \mathcal{D}_2\}$.

Let $\alpha'_{i|j}(\varphi_1^{N_1})$, $i \neq j$, $i, j = 1, 2$, be the probability of the erroneous acceptance of the i-th family of PDs provided that the j-th family of PDs is true (that is the correct PD is in the j-th family)

$$\alpha'_{i|j}(\varphi_1^{N_1}) \triangleq \max_{s \in \mathcal{D}_j} P_s^{N_1}(U' = i), \;\; i \neq j, \; i, j = 1, 2.$$

We consider reliabilities of the infinite sequence of tests

$$E'_{i|j}(\varphi_1) \triangleq \limsup_{N_1 \to \infty} \{-\frac{1}{N_1} \log \alpha'_{i|j}(\varphi_1^{N_1})\}, \;\; i, j = \overline{1, 2}.$$

Theorem 1. *If all distributions P_s, $s = \overline{1, S}$, are different and the positive values $E'^*_{1|1}$, is such that*

$$E'^*_{1|1} < \min_{s \in \mathcal{D}_1, \; l \in \mathcal{D}_2} D(P_{l(g)}||P_{s(g)}),$$

another element of the reliabilities matrix $E'^{*}_{2|2}$ of which defined as follows:

$$E'^{*}_{2|2} < \min_{s\in\mathcal{D}_2} \inf_{Q:\min\limits_{l\in\mathcal{D}_1} D(Q\|P_{l(g)})\le E'^{*}_{1|1}} D(Q\|P_{s(g)}).$$

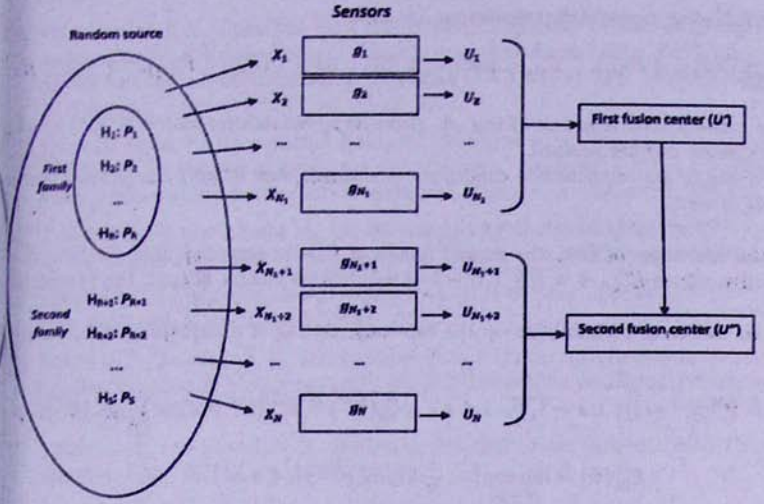

Figure 1: The two-stage multiple hypotheses distributed detection system

Second stage of the two-stage testing of distributed detection is as follows:
The test $\varphi_2^{N_2}(\mathrm{u}_2, U')$ can be defined by using the first fusion center U' and the the second messages sample u_2. The probability of the fallacious acceptance at the second stage of test of PD P_l, when P_s is correct and i-th family of PDs is accepted, is

$$\alpha''_{l|s}(\varphi_2^{N_2}) \triangleq P_s^{N_2}(U''=l|U'=i),\ l\neq s,\ i=\overline{1,2},\ l\in\mathcal{D}_i.$$

The probability to reject P_s, when it is true and i-th family of PDs is accepted, is

$$\alpha''_{s|s}(\varphi_2^{N_2}) \triangleq P_s^{N_2}(U''\neq s|U'=i) = \sum_{l\neq s}\alpha''_{l|s}(\varphi_2^{N_2}),\quad s\in\mathcal{D}_i,\ i=1,2.$$

Corresponding reliabilities for the second stage of test, are

$$E''_{l|s}(\varphi_2) \triangleq \limsup_{N_2\to\infty}\left\{-\frac{1}{N_2}\log\alpha''_{l|s}(\varphi_2^{N_2})\right\},\quad l,s=\overline{1,S}.$$

Theorem 2. *If at the first stage of test the first family of PDs is accepted, then for given positive and finite values $E''_{s|s}$, $s=\overline{1,R-1}$ of the reliabilities matrix $\mathbf{E}''(\varphi_2)$, let us investigate the regions:*

$$\mathcal{R}''_s = \left\{Q: \mathrm{D}\left(Q\parallel P_{s(g)}\right)\le E''_{s|s}\right\},\ s=\overline{1,R-1},\quad \mathcal{R}''_R = \left\{Q: \mathrm{D}\left(Q\parallel P_{s(g)}\right) > E''_{s|s},\ s=\overline{1,R-1}\right.$$

and the following values of elements of the future reliabilities matrix $\mathbf{E}''(\varphi_2^*)$ *of the LAO te*
sequence:

$$E''^{*}_{s|s} = E''_{s|s},\ s=\overline{1,R-1},\ \ E''^{*}_{l|s} = \inf_{Q\in\mathcal{R}''_l} \mathrm{D}\left(Q \,\|\, P_{s(\wp)}\right),\ l,s=\overline{1,R},\ l\neq s,\ \ E^{*}_{R|R} \triangleq \min_{l\neq R} E^{*}_{l|}$$

When the following compatibility conditions are valid

$$E''_{1|1} < \min_{s=\overline{2,R}} \mathrm{D}(P_{s(\wp)} \,\|\, P_{1(\wp)}),\quad E''_{s|s} < \min[\min_{l=\overline{1,s-1}} E''^{*}_{l|s}, \min_{l=\overline{s+1,R}} \mathrm{D}(P_{l(\wp)} \,\|\, P_{s(\wp)})],\ \ 2\leq s\leq R-$$

then there exists a LAO sequence of test φ_2^**, elements of reliabilities matrix* $\mathbf{E}''(\varphi_2^*)$ *of whi*
are defined above and are positive.

Even if one of the compatibility conditions is violated, then $\mathbf{E}''(\varphi_2^*)$ *has at least one el ment equal to zero.*

If in the first stage of test, the second family of PDs is accepted, then for $S-R-$ given positive values $E''_{s|s}$, $s=\overline{R+1,S-1}$ of reliabilities matrix $\mathbf{E}''(\varphi_2^*)$, the procedure analogous.

Reliabilities and error probabilities on the two-stage testing of distributed detection are coming:

$$\alpha'''_{l|s}(\Phi^N) \triangleq P_s^N(U''=l),\ l,s=\overline{1,S},\ l\neq s,\quad \alpha'''_{s|s}(\Phi^N) \triangleq P_s^N(U''\neq s) = \sum_{l\neq s}\alpha'''_{l|s}(\Phi^N),\ \ s=$$

$$E'''_{l|s}(\Phi) \triangleq \limsup_{N\to\infty}\{-\frac{1}{N}\log \alpha'''_{l|s}(\Phi^N)\},\ l,s=\overline{1,S}.$$

So we can consider error probabilities as follows

$$\alpha'''_{l|s}(\Phi^{*N}) = P_s^{N_1}(U'=i)\cdot P_s^{N_2}(U''=l|U'=i),\qquad l,s\in\mathcal{D}_i,\ \ i=1,2 \qquad (3)$$

$$\alpha'''_{l|s}(\Phi^{*N}) = P_s^{N_1}(U'=j)\cdot P_s^{N_2}(U''=l|U'=j),\qquad s\in\mathcal{D}_i,\ l\in\mathcal{D}_j,\ \ i,j=1,2,\ \ i\neq j \qquad (4)$$

According to (3)–(4) and definition of reliabilities we obtain

$$E'''_{l|s}(\Phi^*) = (1-\psi)E''^{*}_{l|s},\qquad l,s\in\mathcal{D}_i,\ \ i=1,2,$$

$$E'''_{l|s}(\Phi^*) = \psi E'_{j|s} + (1-\psi)E''^{*}_{l|s},\qquad s\in\mathcal{D}_i,\ l\in\mathcal{D}_j,\ \ i,j=1,2,\ \ i\neq j,$$

$$E'''_{s|s}(\Phi^*) = \min_{l\neq s} E'''_{l|s}(\Phi^*),\qquad s\in\mathcal{D}_i,\ i=1,2.$$

We characterize the optimal error exponents in a pair of stages and we provide compatibilit conditions for this to happen and it is investigated description of characteristics of LA hypotheses testing of distributed detection and the goal is to make a decision on the man possible hypotheses, based on the messages received at the fusion centers.

References

[1] Haroutunian E.A. "Logarithmically asymptotically optimal testing of multiple statistical hypotheses." *Problems of Control and Information Theory*, vol 19, nos 5-6, pp. 413-421, 1990.

[2] Haroutunian E.A., Hakobyan P.M. and Hormozi nejad F. "On two-stage logarithmically asymptotically optimal testing of multiple hypotheses concerning distributions from the pair of families." *Transactions of IIAP of NAS of RA and of YSU, Mathematical Problems of Computer Science*, vol. 37, pp. 34-42, 2012.

[3] Hormozi nejad F., Haroutunian E.A. and Hakobyan P.M. "On LAO testing of multiple hypotheses for the pair of families of distributions." *Proceeding of the Conference "Computer Science and Information Technologies"*, Yerevan, Armenia, pp. 135-138, 2011.

[4] Tsitsiklis J. N. and Athans M., On the complexity of decentralized decision making and detection problems, IEEE Trans. *Automat. Contr., vol. ACT30*, pp. 440-446; 1985.

[5] Tsitsiklis J. N., Decentralized detection by a large number of sensors,*Math. Contr., Signals, Syst.*, vol. 1, no. 2, pp. 167-182, 1988.

[6] Kreidl O. P., Tsitsiklis J. N., and Zoumpoulis S. I. On decentralized detection with partial information sharing among sensors". *IEEE Transactions on Signal Processing*, vol. 59, No. 4, April 2011.

[7] Tenney R. R. and Sandell N. R., Detection with distributed sensors, *IEEE Trans. Aerosp. Electron. Syst.*, vol. 17, pp. 501-510, 1981.