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Pauly Matrix and Transformation Operators for Dirac System

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Let $\sigma_1 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ are well known Pauly matrix and $E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. It is known that the solution $y = \varphi(x, \lambda, \alpha)$ of Causchy problem

$$\{\sigma_1 \frac{1}{i}\frac{d}{dx} + \sigma_2 p(x) + \sigma_3 q(x)\}y = \lambda y, \quad \lambda \in C$$

$$y(0) = \begin{pmatrix} sin\alpha \\ -cos\alpha \end{pmatrix},$$

can be represented in the form $\left(\varphi_0(x, \lambda, \alpha) = \begin{pmatrix} sin(\lambda x + \alpha) \\ -cos(\lambda x + \alpha) \end{pmatrix}\right)$

$$\varphi(x, \lambda, \alpha) = \varphi_0(x, \lambda, \alpha) + \int_0^x K(x,t)\varphi_0(t, \lambda, \alpha)dt = (E + K)\varphi_0.$$

Operator $E + K$ is called the transformation operator. Under different conditions on scalar functions p and q this operator and his kernel $K(x, t)$ was investigated in different papers (see [1]-[6]).

Theorema. *Let $p, q \in L^1_{loc}(0, \infty)$. Then the kernel $K(x,t)$ and the kernel $H(x,t)$ of inverse operator $\varphi_0(x, \lambda) = \varphi(x, \lambda) + \int_0^x H(x,t)\varphi(t, \lambda)dt$ can be represented in the form*

$$K(x,t) = a\sigma_1 + b\sigma_2 + c\sigma_3 + d \cdot E$$

$$H(x,t) = \tilde{a}\sigma_1 + \tilde{b}\sigma_2 + \tilde{c}\sigma_3 + \tilde{d} \cdot E,$$

where the functions (of two variables (x, t))) a, b, c, d and $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}$ are represented by functions p and q.

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